ISSN 1472-2739 (on-line) 1472-2747 (printed)

Algebraic & Geometric Topology
Volume 3 (2003) 1275–1276
Published: 20 December 2003

Correction to "Open books and configurations of symplectic surfaces"

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Abstract We correct the main theorem in "Open books and configurations of symplectic surfaces" [1] and its proof. As originally stated, the theorem gave conditions on a configuration of symplectic surfaces in a symplectic 4-manifold under which we could construct a model neighborhood with concave boundary and describe explicitly the open book supporting the contact structure on the boundary. The statement should have included constraints on the areas of the surfaces.

AMS Classification 57R17; 57N10, 57N13

Keywords Symplectic, contact, concave, open book, plumbing, fillable

In the paper being corrected [1], we considered symplectic configuration graphs where each vertex was decorated with a triple (g_i, m_i, a_i), g_i being the genus of a surface, m_i its self-intersection, and a_i its symplectic area. Theorem 1.1 in [1] is false as stated. However we have the following:

Correction 1 *The conclusions of theorem 1.1 (parts A and B) are true if we add the hypothesis that there exists a constant $\lambda > 0$ such that, for each i, $a_i = \lambda(m_i + d_i)$.*

An easy counterexample to the theorem as originally stated is given by two surfaces Σ_1 and Σ_2, with $\Sigma_1 \cdot \Sigma_1 = \Sigma_2 \cdot \Sigma_2 = 1$ and $\Sigma_1 \cdot \Sigma_2 = 1$. If $\int_{\Sigma_1} \omega = a_1 \neq \int_{\Sigma_2} \omega = a_2$, then $[\omega] \neq 0$ when restricted to the boundary of a neighborhood of $\Sigma_1 \cup \Sigma_2$, because $[\omega]$ is nonzero when evaluated on the class $[\Sigma_1] - [\Sigma_2]$, which lives on the boundary. In general, this problem occurs when the intersection matrix for the configuration of surfaces has determinant equal to zero.

The error is on line 4 of page 583 in [1], in the calculation of α_H^-. The correct statement is:

$$\alpha_H^- = kc_i(d\mu + d\lambda) - \alpha_H^+$$

Thus, to arrange that this agree with the contact pair we already have on a neighborhood of $K \subset \partial X$, we are forced to choose $c_i = 1$, as opposed to

$c_i = a_i/(2\pi K(m_i + d_i))$, which was the choice made on page 582 line 22. With $c_i = 1$, we get that the area of Σ_i is $2\pi(m_i + d_i)k$. After the construction we can rescale by a constant to get the area to be $\lambda(m_i + d_i)$.

Note that, if the configuration $\Sigma_1 \cup \ldots \cup \Sigma_n$ is contained in a closed symplectic 4-manifold (X, ω), then the area condition added in this correction is equivalent to the condition that $[\omega]$ is Poincaré dual to some multiple of $[\Sigma_1] + \ldots + [\Sigma_n] + \gamma$, where $\gamma \in H_2(X; \mathbb{Z})$ is a class with $\gamma \cdot \Sigma_i = 0$ for $i = 1, \ldots, n$. This is because $m_i + d_i = \Sigma_1 \cdot \Sigma_i + \ldots + \Sigma_n \cdot \Sigma_i$.

Here we emphasize that this correction does not affect the applications in section 2 of [1], or forthcoming applications in [2].

References

[1] **D T Gay** *Open books and configurations of symplectic surfaces*, Alg. Geom. Top. 3 (2003), 569–586

[2] **D T Gay and R Kirby** *Constructing symplectic forms on 4-manifolds which vanish on circles*, to appear

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Received: 1 December 2003