Covering a nontaming knot by the unlink

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There exists an open 3–manifold M and a simple closed curve $\gamma \subset M$ such that $\pi_1(M \setminus \gamma)$ is infinitely generated, $\pi_1(M)$ is finitely generated and the preimage of γ in the universal covering of M is equivalent to the standard locally finite set of vertical lines in $\mathbb{R}^3$, that is, the trivial link of infinitely many components in $\mathbb{R}^3$.

57N10; 57M10, 57N45

0 Introduction

Definition 0.1 We say that the locally finite collection of proper lines $\Gamma \subset \mathbb{R}^3$ is a *trivial* $\mathbb{R}^3$*–link* if there exists a homeomorphism of $\mathbb{R}^3$ taking Γ to a subset of $(\mathbb{Z}, 0) \times \mathbb{R} \subset \mathbb{R}^2 \times \mathbb{R}$.

For example, if L is a locally finite union of geodesics in $\mathbb{H}^3$, then L is a $\mathbb{R}^3$–trivial link, as seen by applying Morse theory to the distance function from any point in $\mathbb{H}^3$.

The main result in this paper is the following:

Theorem 0.2 *There exists a simple closed curve γ in an open 3–manifold M such that*

(1) *$\pi_1(M - \gamma)$ is infinitely generated,*

(2) *$\pi_1(M)$ is finitely generated,*

(3) *the universal covering $\widetilde{M}$ of M is $\mathbb{R}^3$ and*

(4) *the preimage Γ of γ in $\widetilde{M}$ is $\mathbb{R}^3$–trivial.*

Addendum 0.3 A simple closed curve ω can be chosen in the above manifold M satisfying the above properties as well as the following additional ones:

(1) ω is algebraically disc busting in $\pi_1(M)$ and

(2) $0 = [\omega] \in H_1(M, \mathbb{Z}_2)$.

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Definition 0.4 A *nontaming knot* is a smooth simple closed curve k in a 3–manifold M such that $\pi_1(M)$ is finitely generated and $\pi_1(M-k)$ is infinitely generated.

Remarks 0.5 By Tucker [8], the condition $\pi_1(M-\gamma)$ is infinitely generated implies that the manifold M is not tame, that is, not the interior of a compact manifold. There are lots of examples of nontame manifolds with finitely generated fundamental group whose universal covers are $\mathbb{R}^3$, for example, see Theorem 2.1. This paper provides the first example of a knot in such a manifold, which is sufficiently complicated to be nontaming, yet sufficiently straight to lift to an $\mathbb{R}^3$–unlink.

Our manifold M is obtained as a nested union of handlebodies of genus 2, $V_1 \subset V_2 \subset V_3 \subset \cdots$ where the inclusion $V_i \subset V_{i+1}$ is as in Figure 1. Let $\gamma \subset V_1$ be the knot also shown in Figure 1.

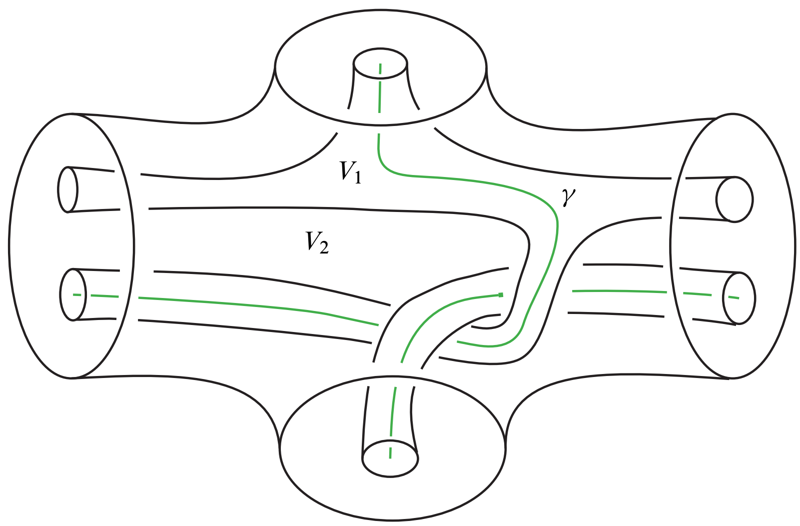

Figure 1: Glue the top disc to the bottom one and the left disc to the right one to obtain the embedding of V_1 into V_2.

The paper is organized as follows. In Section 1 we show that M is homotopy equivalent, but not homeomorphic to the standard open genus–2 handlebody and that $\pi_1(M-\gamma)$ is infinitely generated. In Section 2 we show that Γ is the trivial $\mathbb{R}^3$–link of infinitely many components. In Section 3 we prove Addendum 0.3.

Historical Remarks In the early 1990s the first author showed that the nonexistence of a knot having the properties stated in our main result implies the Tame Ends conjecture (also known as the Marden conjecture [4]) for hyperbolic 3–manifolds. See Myers [5].

In the fall of 1996 the authors found the knot $\gamma \subset M$. We are finally presenting its proof. Very recently, we found the example of Addendum 0.3.

Ian Agol [1] and independently Danny Calegari and the second author [2] have obtained proofs of the Tame Ends conjecture.

Notation 0.6 If $X \subset Y$, then $N(X)$ denotes a regular neighborhood of X in Y. If X is a topological space, then $|X|$ denotes the number of components of X.

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1 $\pi_1(M-k)$ is infinitely generated

Since the inclusion of each V_i into V_{i+1} is a homotopy equivalence, it follows that the inclusion of V_1 into M is a homotopy equivalence and hence M is an open genus–2 homotopy handlebody.

To show that $\pi_1(M-k)$ is infinitely generated it suffices to show that ∂V_3 is incompressible in $V_3-\gamma$ and for each i, $\partial(V_{i+1}-\mathring{V}_i)$ is incompressible in $V_{i+1}-\mathring{V}_i$ and $V_{i+1}-\mathring{V}_i$ is not a product. See Figure 2. These facts, together with the work of Stallings [7] show that the induced map $\pi_1(\partial V_i) \to \pi_1(V_{i+1}-\mathring{V}_i)$ is injective but not surjective. The standard Seifert–Van Kampen argument completes the proof.

Lemma 1.1 *∂V_3 is incompressible in $V_3-\gamma$.*

Proof It suffices to show that if $W_0 = V_3 - \mathring{N}(\gamma)$, then $R_0 := \partial V_3$ is incompressible in W_0. Let $D_3 \subset V_3$ (resp. $E \subset V_3$) be the disc obtained by gluing D_3' to D_3'' (resp. E_1 to E_2). By considering boundary compressions it suffices to show that if W_1 is W_0 split along D_3 and R_1 is R_0 split along D_3, then R_1 is incompressible in W_1. Let W_2 (resp. R_2) denote W_1 (resp. R_1) split along E. We abuse notation by now viewing D_3', D_3'' (resp. E_1, E_2) as compact annuli (resp. pants). Note that R_2 is incompressible in W_2, for any essential compressing disc H would nontrivially separate the set $\{D_3', D_3'', E_1, E_2\}$. On the other hand by considering $\partial N(\gamma) \cap W_2$ we see that all of these components must lie in the same component of $W_2 - H$.

Therefore to show that R_1 is incompressible in W_1 it suffices to show that there exists no essential, properly embedded disc $(F, \partial F) \subset (W_2, E_1 \cup E_2 \cup R_2)$ such that $\partial F \cap (E_1 \cup E_2)$ is connected. We now show that $F \cap E_1 = \varnothing$. A similar argument will show that $F \cap E_2 = \varnothing$. In the natural way write W_2 as $P \times [1,2]$ where P is a

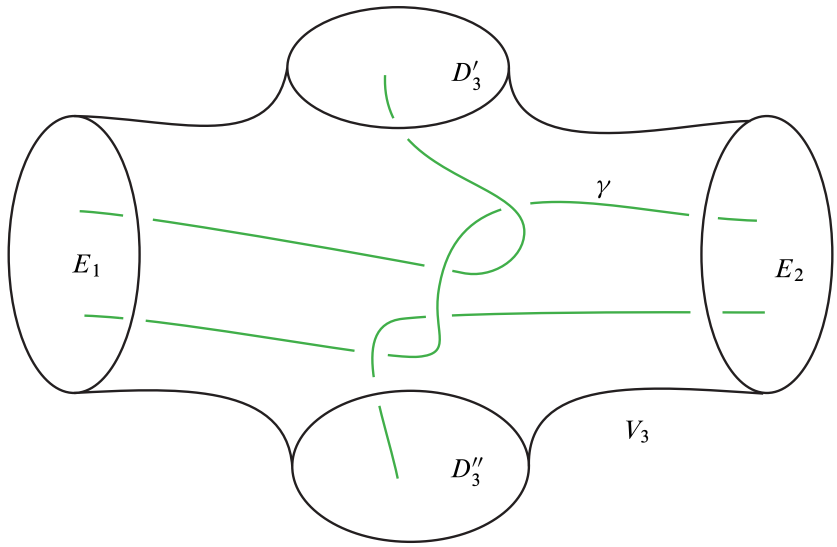


Figure 2: The knot γ viewed inside V_3

disc with 3 open discs removed and $D_3'' \cup E_1 \subset \mathring{P} \times 1$ and $D_3' \cup E_2 \subset \mathring{P} \times 2$. Here P_i denotes $P \times i$. Assume that F was chosen so that $|F \cap P_2|$ is minimal and that $F \cap (R_2 \cap \partial P \times [1,2])$ are arcs from P_1 to P_2. Isotope F to be Morse with respect to projection onto the $[1,2]$ factor. Arguing as in Roussarie [6] we can assume that the only critical points are of index -1. Since F is disjoint from $E_2 \cup D_3'$ and F is a disc it follows that $F \cap P_2$ is a finite union of parallel arcs and the closest saddle point to P_2 must involve distinct such arcs. Therefore, if F contained a saddle tangency, then by considering a boundary compression we could have found another essential F as above, with $|F \cap P \times 2|$ reduced. It follows that F has no saddle tangencies and hence $|F \cap E_1| \geq 3$, which is a contradiction. See Figure 3. $\square$

Remark 1.2 Another way to prove Lemma 1.1 is to show that the manifold obtained by doubling $V_3 - \mathring{N}(\gamma)$ along ∂V_3 is irreducible. One can prove irreducibility of the double by constructing a taut sutured manifold hierarchy.

Note that ∂V_2 is compressible in $V_2 - \gamma$.

Since the inclusion of V_i into V_{i+1} is a homotopy equivalence we obtain the following:

Lemma 1.3 *For each $i \geq 1$, $V_{i+i} - \mathring{V}_i$ has incompressible boundary.* $\square$

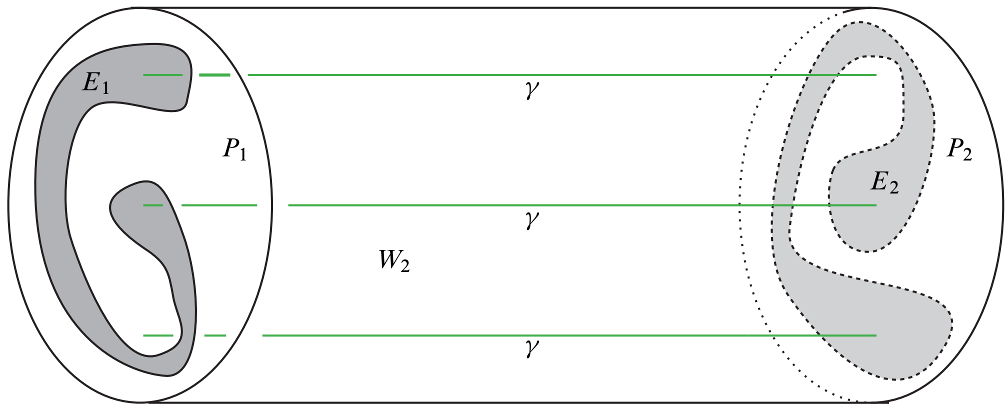
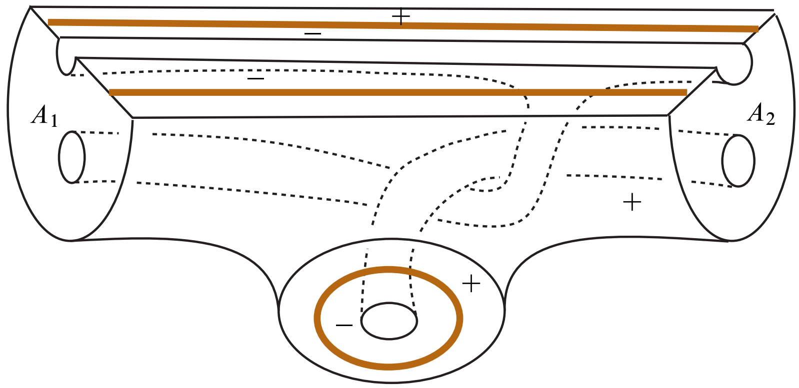


Figure 3

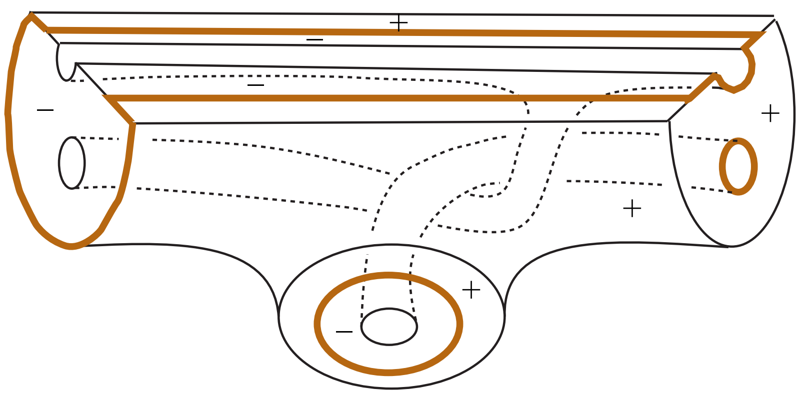
Lemma 1.4 *For $i \geq 1$, $V_{i+1} - \mathring{V}_i$ is not a product.*

Proof It suffices to consider the case $i = 2$. If $V_3 - \mathring{V}_2$ is a product, then the pair (V_3, γ) is homeomorphic to (V_2, γ). On the other hand, the note after Remark 1.2 implies that ∂V_2 is compressible in $V_2 - \gamma$, while Lemma 1.1 implies that ∂V_3 is incompressible in $V_3 - \gamma$.

Here is a second proof. Let $W = V_3 - \mathring{V}_2$. Let (W, σ) be the sutured manifold with $R_-(\sigma) = \partial V_2$ and $R_+(\sigma) = \partial V_3$. It suffices to construct a taut sutured manifold hierarchy $(W, \sigma) = (N_0, \sigma_0) \to (N_1, \sigma_1) \to \cdots \to (D^2 \times I, \partial D^2 \times I)$ such that for some j, $R_+(\sigma_j)$ is not homeomorphic to $R_-(\sigma_j)$, since by [3] a taut sutured manifold decomposition of a product always yields a product. (Products are exactly those taut sutured manifolds of minimal complexity, and taut splittings do not increase complexity.) Figure 4 shows a step in such a hierarchy. The top sutured manifold (N_2, σ_2) is (W, σ) split along the product annulus $D_3 - \mathring{V}_2$ followed by splitting along a product disc (that is, a disc crossing the sutures twice) meeting $E_2 - \mathring{V}_2$ in a single arc. The thick brown lines denote the sutures. Note that each of $R_+(\sigma_2)$, $R_-(\sigma_2)$ is a pant. To obtain (N_3, σ_3) split along the annulus corresponding to A_2 and A_1, so that A_2 is given the $+-$orientation. Note that $R_+(\sigma_3)$ is not homeomorphic to $R_-(\sigma_3)$. Splitting (N_3, σ_3) along a product disc yields (N_4, σ_4) where $N_4 = D^2 \times S^1$ and the sutures of σ_4 are 4 parallel longitudes. One more splitting yields, $(D^2 \times I, \partial D^2 \times I)$. □



Glue annuli A_1 to A_2 to obtain (N_2, σ_2).



(N_3, σ_3)

Figure 4

2 Γ is $\mathbb{R}^3$–trivial

The following is well known.

Theorem 2.1 *If the open 3–manifold N is exhausted by compact irreducible manifolds $W_1 \subset W_2 \subset \cdots$ such that for each i, $\mathrm{in}_*\colon \pi_1(W_i) \to \pi_1(W_{i+1})$ is injective, then $\tilde{N} = \mathbb{R}^3$.*

Proof The universal covering space $\widetilde{N}$ of N is exhausted by the universal covering spaces of the various W_i's. By Waldhausen [9], the universal covering space of W_i is $B^3 - K_i$, where $K_i \subset \partial B^3$ is compact. Since a space is $\mathbb{R}^3$ if every compact set lies in a 3–cell, the result follows. □

Lemma 2.2 *If L is a smooth locally finite link in the open unit 3–ball $B \subset \mathbb{R}^3$, such that away from exactly one point, each component is transverse to the concentric 2–spheres, then L is $\mathbb{R}^3$–trivial.* □

Corollary 2.3 *A locally finite collection of geodesics in $\mathbb{H}^3$ is $\mathbb{R}^3$–trivial.* □

Definition 2.4 Let T denote the *standard infinite* $\mathbb{R}^3$*–link* $(\mathbb{Z}, 0) \times \mathbb{R} \subset \mathbb{R}^2 \times \mathbb{R}$. Let X be the 3–manifold with boundary obtained by removing small open regular neighborhoods of the rays $\mathcal{R}_X := (\mathbb{Z}, 0) \times [1, \infty) \cup (\infty, -1]$. Let T_X be the restriction of T to X.

Remark 2.5 If X_1 is the 3–manifold with boundary obtained from the standard infinite $\mathbb{R}^3$–link by removing small open regular neighborhoods of the rays $\mathcal{R}_{X_1}$ defined by $\{(n, 0) \times [n, \infty) \cup (-\infty, n-1] \mid n \in \mathbb{Z}\}$ and T_{X_1} is the restriction of T to X_1, then (X_1, T_{X_1}) is diffeomorphic to (X, T_X).

The pair (X, T_X) can be viewed geometrically via the following lemma.

Lemma 2.6 *Let G be the 2–dimensional Schottky group generated by length 10 translations g_1, g_2 along orthogonal geodesics $A, B \subset \mathbb{H}^2$. Extend G to act on $\mathbb{H}^3$. Let $Q \subset \mathbb{H}^3$ be the totally geodesic plane orthogonal to B at distance 5 from $A \cap B$ and $\mathcal{Q}$ be the orbit GQ. Let Y be the closure of a component of $\mathbb{H}^3 - \mathcal{Q}$ and GB_Y the restriction of the orbit GB to Y. Then there is a diffeomorphism $(X, T_X) \to (Y, GB_Y)$.*

Let $\pi\colon \widetilde{M} \to M$ denote the universal covering projection and let Γ denote the link $\pi^{-1}(\gamma)$. By Theorem 2.1 $\widetilde{M}$ is homeomorphic to $\mathbb{R}^3$.

The construction of M gives rise to a properly embedded plane $P \subset M$ which intersects each V_i in a single disc D_i and intersects V_3 in the disc D_3. Furthermore $P \cap \gamma = D_3 \cap \gamma$. Let $\mathcal{P} = \pi^{-1}(P)$.

Proposition 2.7 *There exists a diffeomorphism $(\widetilde{M}, \Gamma) \to (\mathbb{H}^3, GB)$.*

Assuming for the moment Proposition 2.7 we obtain the following proof:

Proof that Γ is $\mathbb{R}^3$–trivial It follows from the Proposition 2.7 that the pair $(\widetilde{M}, \Gamma)$ is diffeomorphic to $(\mathbb{H}^3, \Delta)$ where Δ is a locally finite union of pairwise disjoint geodesics. By Lemma 2.2, Γ is $\mathbb{R}^3$–trivial. $\square$

Proof of Proposition 2.7 It suffices to show that if W is the closure of a component of $\widetilde{M} - \mathcal{P}$ and Γ_W is the restriction of Γ to W, then there is a diffeomorphism $(W, \Gamma_W) \to (Y, GB_Y)$ where Y and GB_Y are as in Lemma 2.6. By Lemma 2.6 it suffices to show that (W, Γ_W) is diffeomorphic to (X, T_X), where T_X is defined as in Definition 2.4. Let W_i denote the compact manifold obtained by splitting V_i open along D_i. Then W is exhausted by the manifolds $\widetilde{V}_i$.

Consider the $\mathbb{R}^3$–link Σ shown in Figure 5. It has infinitely many components and is invariant under a rigid $\mathbb{R}^3$–translation g. Each component has an end which is a vertical ray and another that forever spirals down. Let $\mathcal{R}$ be the union of the (thick) blue rays, two for each component of Σ. Let $\mathring{N}(\mathcal{R})$ be a union of small open regular neighborhoods of the components of $\mathcal{R}$ and $Z = \mathbb{R}^3 - \mathring{N}(\mathcal{R})$. The pair (W, Γ_W) is diffeomorphic to $(Z, \Sigma \cap Z)$. Indeed, Z can be exhausted by manifolds diffeomorphic to $\widetilde{V}_i$ in a manner compatible with the inclusion $\widetilde{V}_i \subset \widetilde{V}_{i+1}$. Furthermore, the quotient $Z/\langle g\rangle = W/\mathbb{Z}$ where $\mathbb{Z}$ is the group of covering translations of W and $\widetilde{V}_i/\mathbb{Z} = V_i$. Figure 6 shows three fundamental domains V_1 within Z. Figure 7 shows one fundamental domain of V_2. Notice that the curves α and β bound discs in the boundary of this fundamental domain which lie in ∂Z. Again, just translate by g to get the entire embedding of $\widetilde{V}_2 \subset Z$. In a similar manner construct $\widetilde{V}_i$, $i \geq 3$.

Consider the collection $\{H_i\}$, $i \in \mathbb{Z}$ of horizontal planes shown as lines in Figure 8. Coordinates on $\mathbb{R}^3$ could have been chosen so that $H_i = \mathbb{R}^2 \times i$ and $g(H_i) = H_{i+1}$. If S_i is the slab $\mathbb{R}^2 \times [i, i+1]$, then $\Sigma | S_i$ is equivalent to the link $(\mathbb{Z}, 0) \times [i, i+1]$. Putting these slabs together, we conclude that Σ is $\mathbb{R}^3$–trivial.

The diffeomorphism H of $\mathbb{R}^3$ which takes Σ to the standard link T could have been chosen to fix H_0 pointwise and setwise fix the various horizontal planes. Therefore it could have been chosen to take $\mathcal{R}$ to the rays $\mathcal{R}_{X_1}$. This shows that (W, Γ_W) is diffeomorphic to (X_1, T_{X_1}) and hence is diffeomorphic to (X, T_X). $\square$

3 Another example

Theorem 0.2 answers in the negative a conjecture of the first author. Myers [5] asked whether a more restrictive version of that conjecture holds. The example of this section

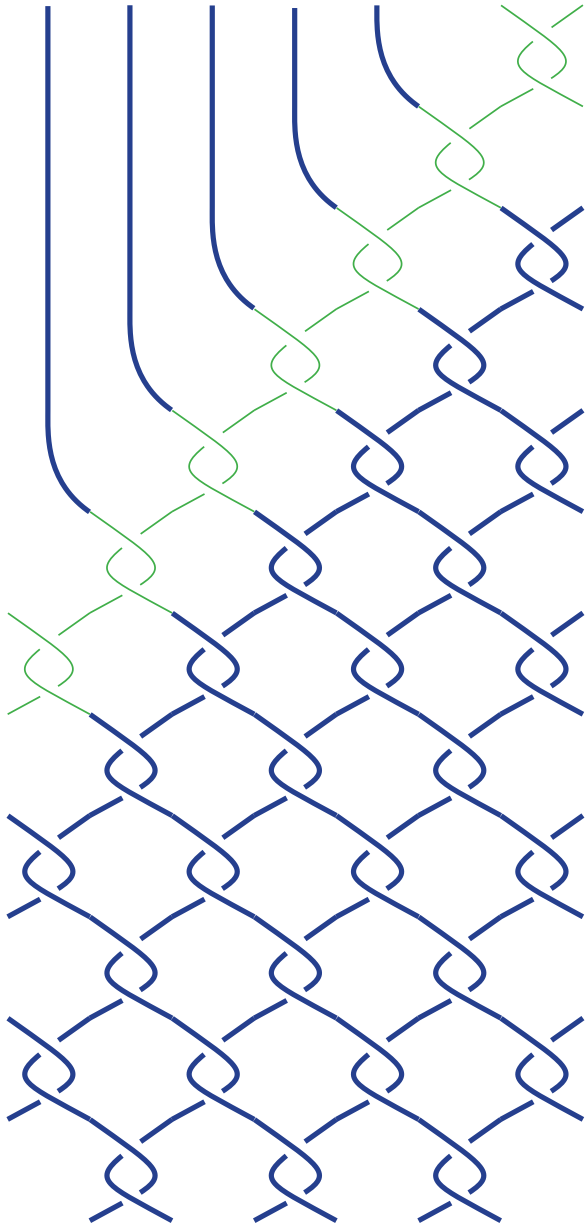


Figure 5: This infinite $\mathbb{R}^3$–link is rigidly $\mathbb{Z}$–translation invariant. The top part of each strand is straight and the bottom part is infinitely twisted in the helical pattern indicated.

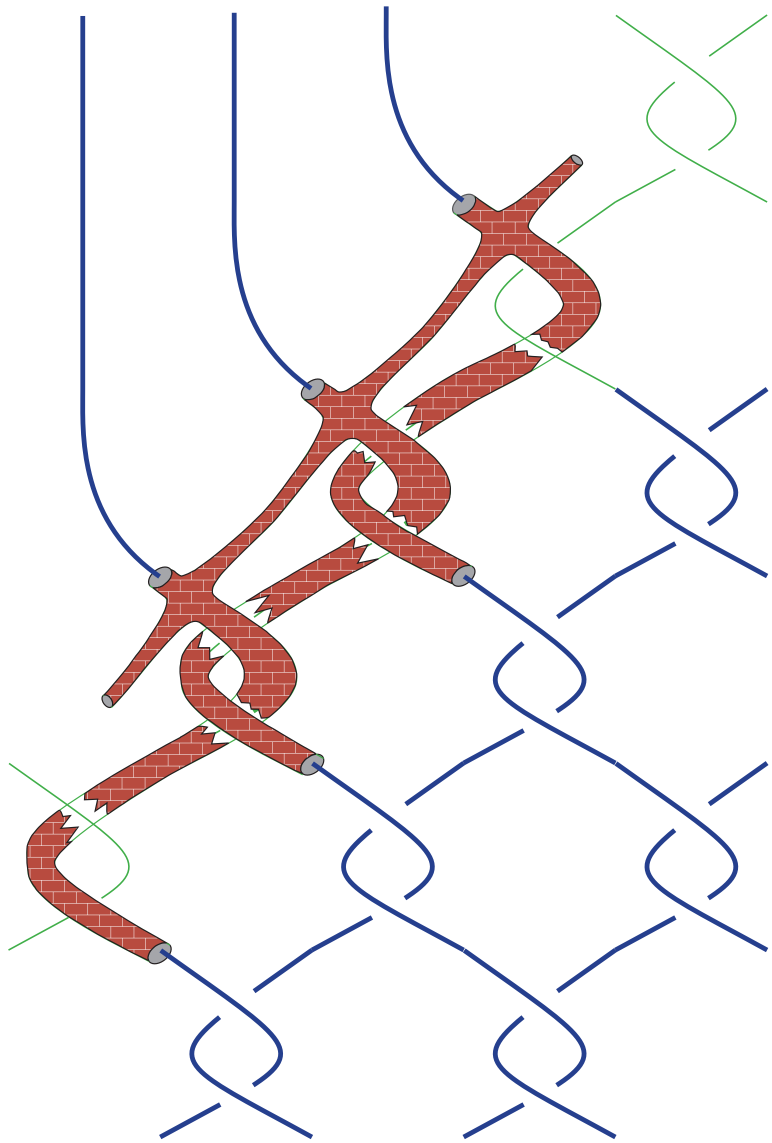


Figure 6: Three fundamental domains of V_1 lifted to Z

provides a similar answer to that question. Let the manifold M be as in Section 2, with the knot $\omega \subset V_1$ presented as in Figure 9.

Proof of Addendum 0.3 With respect to the standard generators of $\pi_1(V_1)$, ω represents the element a^2b^2 which according to Myers [5] is algebraically disc busting in

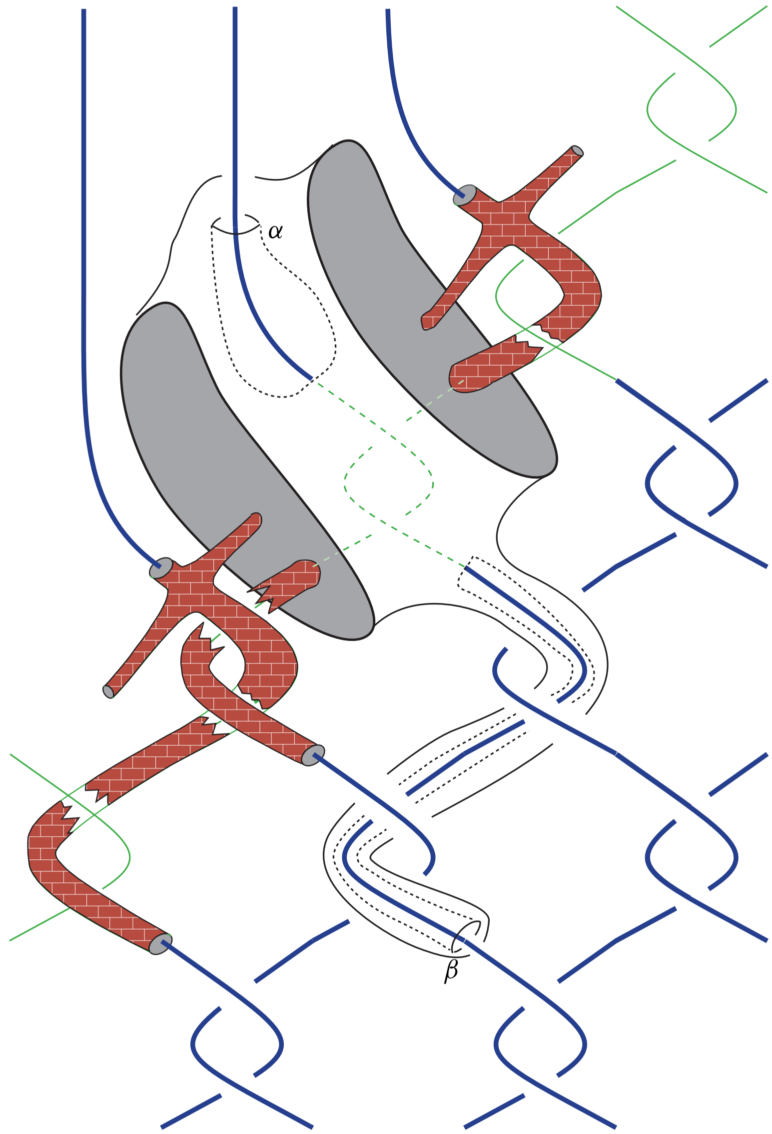


Figure 7: One fundamental domain of $\widetilde{V}_2$ lifted to Z

V_1 and hence in M. that is, $\pi_1(M)$ cannot be expressed as a nontrivial free product such that $[\omega]$ can be conjugated to lie in a single factor. Since ω is algebraically disc busting, ∂V_1 is incompressible in $V_1 - \omega$. As in Section 1, $\pi_1(M-\omega)$ being infinitely generated then follows from Lemma 1.3 and Lemma 1.4.

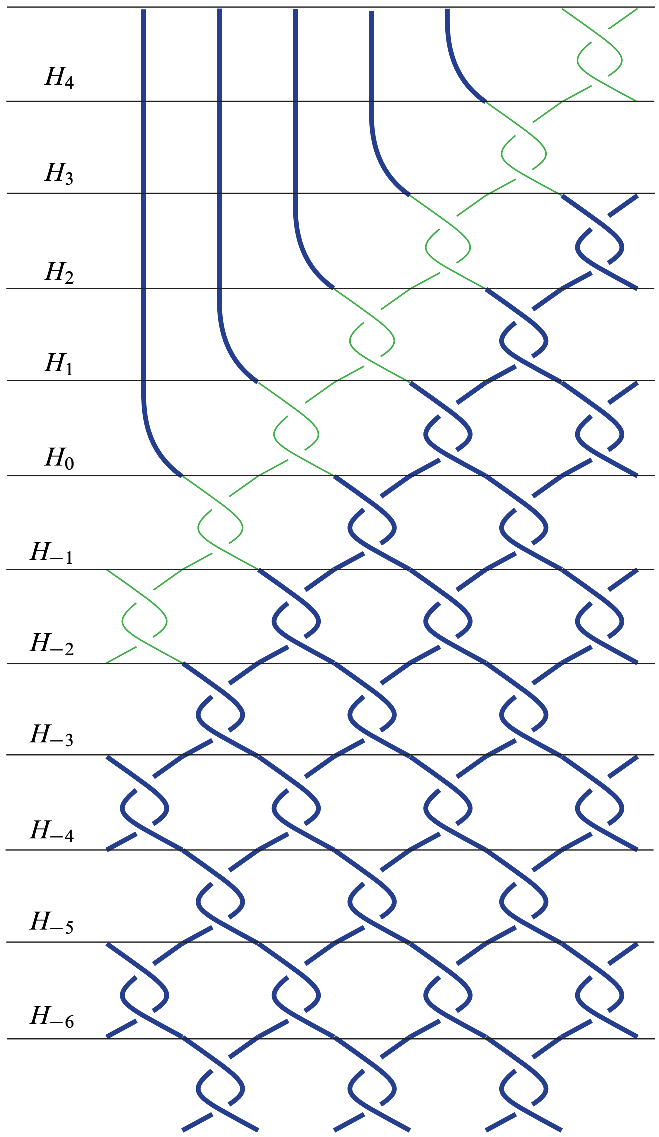


Figure 8

An argument similar to that of Section 2 shows that the restriction of Ω to W is the union of properly embedded arcs Ω_W as drawn in Figure 10. Figure 10 can be decoded with the help of Figure 11, for example to unclutter the picture, certain pairs of thin green arcs are drawn as one arc. Note that W is $\mathbb{R}^3$ with open regular neighborhoods

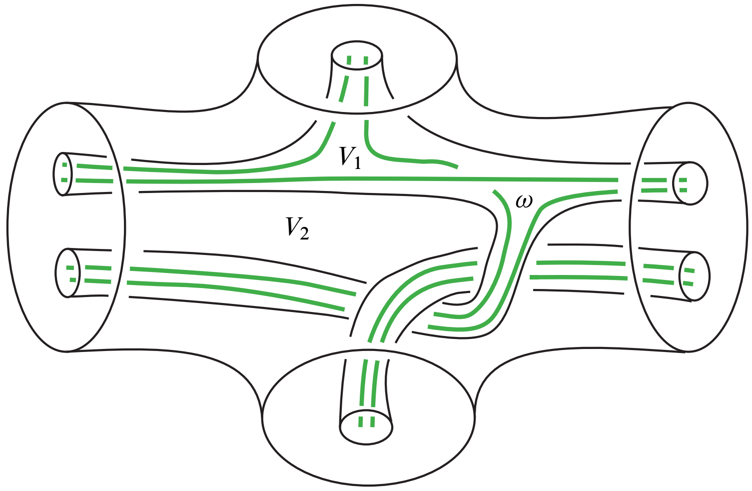


Figure 9

of a countable collection of rays deleted. These neighborhoods are denoted by the thick blue lines. Finally the boxes drawn in Figure 10 coordinatize $\mathbb{R}^3$ and will be useful for the next paragraph. Imagine that both ∂W and Ω_W lie very close to the xy–plane. Let $\{E_i\}$ denote the components of ∂W.

To each component E of ∂W we define a foliation $\mathcal{F}_E$ of W which is the restriction of a topologically concentric foliation on $\mathbb{R}^3$ with center in the component of $\mathbb{R}^3 - W$ separated off by E. For each i, $\mathcal{F}_E|E_i$ will be a topologically concentric foliation by circles with center point the dot shown in Figure 12. $\mathcal{F}_E$ will have exactly one tangency with each component of Ω_W except for the two components Ω_E which hit E and $\mathcal{F}_E$ will be transverse to Ω_E. Suppose that E is the component containing the point $(0,0)$ shown in Figure 10. The leaves S_t of $\mathcal{F}_E$ will be parametrized by $[0,\infty)$, where S_0 is a point. Define S_i, $i \in \mathbb{N}$ according to the pattern given in Figure 12. Next modify these spheres as in Figure 13. In particular if $S_i \cap E_j \neq \varnothing$, then $S_i \cap E_j$ is a circle. The other spheres get modified in a similar way. For example, the modified S_3 has three tube like extensions. One passes by $(1,1)$ and the others at $(2,2)$ and $(3,3)$. It is an exercise to show that the desired foliation $\mathcal{F}_E$ can be constructed to contain these integral spheres. Note that near $(0,0)$, but not including $(0,0)$, all the leaves of $\mathcal{F}_E$ are discs.

In a similar way construct a foliation $\mathcal{F}_0$ on W to have all the properties of $\mathcal{F}_E$ except that the center point of the concentric foliation lies in $\mathrm{int}(W - \Omega_W)$, nearby leaves are spheres and each component of Ω_W is tangent to $\mathcal{F}_0$ at exactly one point.

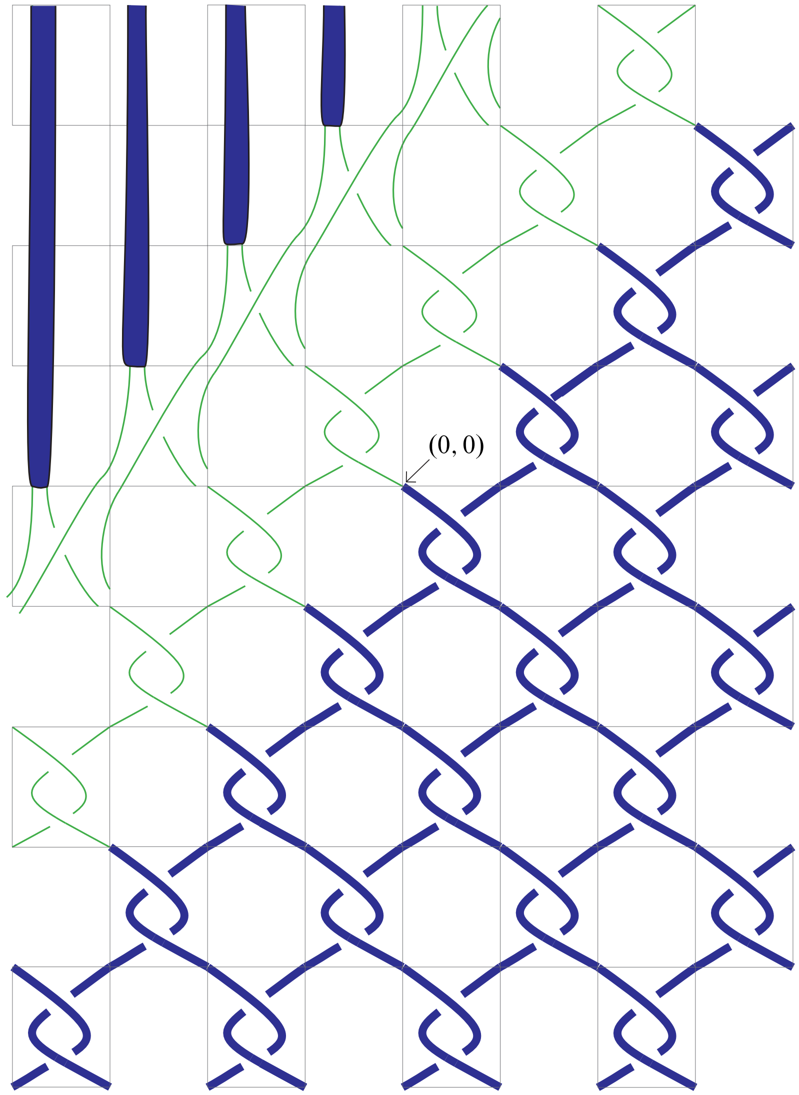


Figure 10

To show that Ω is $\mathbb{R}^3$–trivial we describe a foliation $\mathcal{F}$ on $\widetilde{M} = \mathbb{R}^3$ which satisfies the hypothesis of Lemma 2.2 with respect to the link Ω. $\widetilde{M}$ is built by gluing copies of W in a treelike fashion. Let T be the tree dual to $\mathcal{P} \subset \widetilde{M}$ with base vertex v_0. Let v_0 also denote the corresponding copy of W. Define $\mathcal{F}|v_0 = \mathcal{F}_0$. If v_i and v_0 have an edge in common and v_i is glued to v_0 along the plane $E_{g(i)} \subset \partial v_i$, then give v_i the foliation

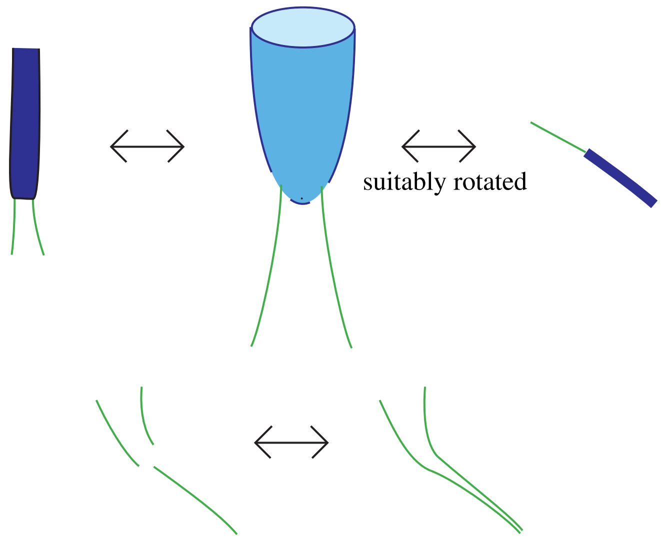


Figure 11

$\mathcal{F}_{E_{g(i)}}$. In what follows we denote $\mathcal{F}_{E_{g(i)}}$ by $\mathcal{F}_{g(i)}$. Since each foliation restricts to a concentric foliation on $E_{g(i)}$ the identification of $\mathcal{F}_0$ and $\mathcal{F}_{g(i)}$ is determined by a homeomorphism $h_{0i}\colon [0,\infty) \to [0,\infty)$. Similarly if v_j and v_k share an edge with v_j closer to v_0, then give v_k the foliation $\mathcal{F}_{g(k)}$ where the plane $E_{g(k)} \subset \partial v_k$ glues to v_j. So $\mathcal{F}$ is determined by the various homeomorphisms $h_{ij}\colon [0,\infty) \to [0,\infty)$ where v_i and v_j share an edge. Any choice of functions gives rise to a foliation by spheres and planar surfaces of possibly infinite Euler characteristic. Furthermore, since each leaf of $\mathcal{F}_{E_i}$ is compact and hits E_i in exactly one component, it follows that each leaf of $\mathcal{F}$ hits v_0 in exactly one component. If the leaves of $\mathcal{F}_0$ are parametrized by $[0,\infty)$ and $T_0 \subset T_1 \subset \cdots$ is an exhaustion of T by compact connected sets, then pass to a subsequence of the T_i's and choose the functions h_{ij} so that if $t \le n-1 \in \mathbb{N}$ and L_t is the leaf of $\mathcal{F}$ passing through the leaf of $\mathcal{F}_0$ parametrized by $t \in [0,\infty)$, then L_t is a sphere contained in $\widetilde{M}_n$, where $\widetilde{M}_n$ is the submanifold of $\widetilde{M}$ corresponding to T_n. Assume that $\mathcal{F}$ has been inductively constructed on $\widetilde{M}_{n-1}$ and satisfies the above conditions for $t \le n-2$. Let $\mathcal{G}$ denote those leaves of $\mathcal{F}|\widetilde{M}_{n-1}$ which restrict to leaves $L_t \subset \mathcal{F}_0$ with $t \in [0, n-1]$. There is a finite set $F = \{F_1, \cdots, F_k\}$ of components of $\partial \widetilde{M}_{n-1}$ so that $\mathcal{G} \cap \partial \widetilde{M}_{n-1} \subset F$ and lies in a compact subset C of F. By passing to a subsequence we can suppose that each F_i glues to a vertex of T_n. If v_j glues to $v_i \in T_{n-1}$ along F_p, where $1 \le p \le k$, then choose the function h_{ij} so that each circle c of $\mathcal{G}|F_p$ is capped off by a disc of $\mathcal{F}_{g(j)}$. $\square$

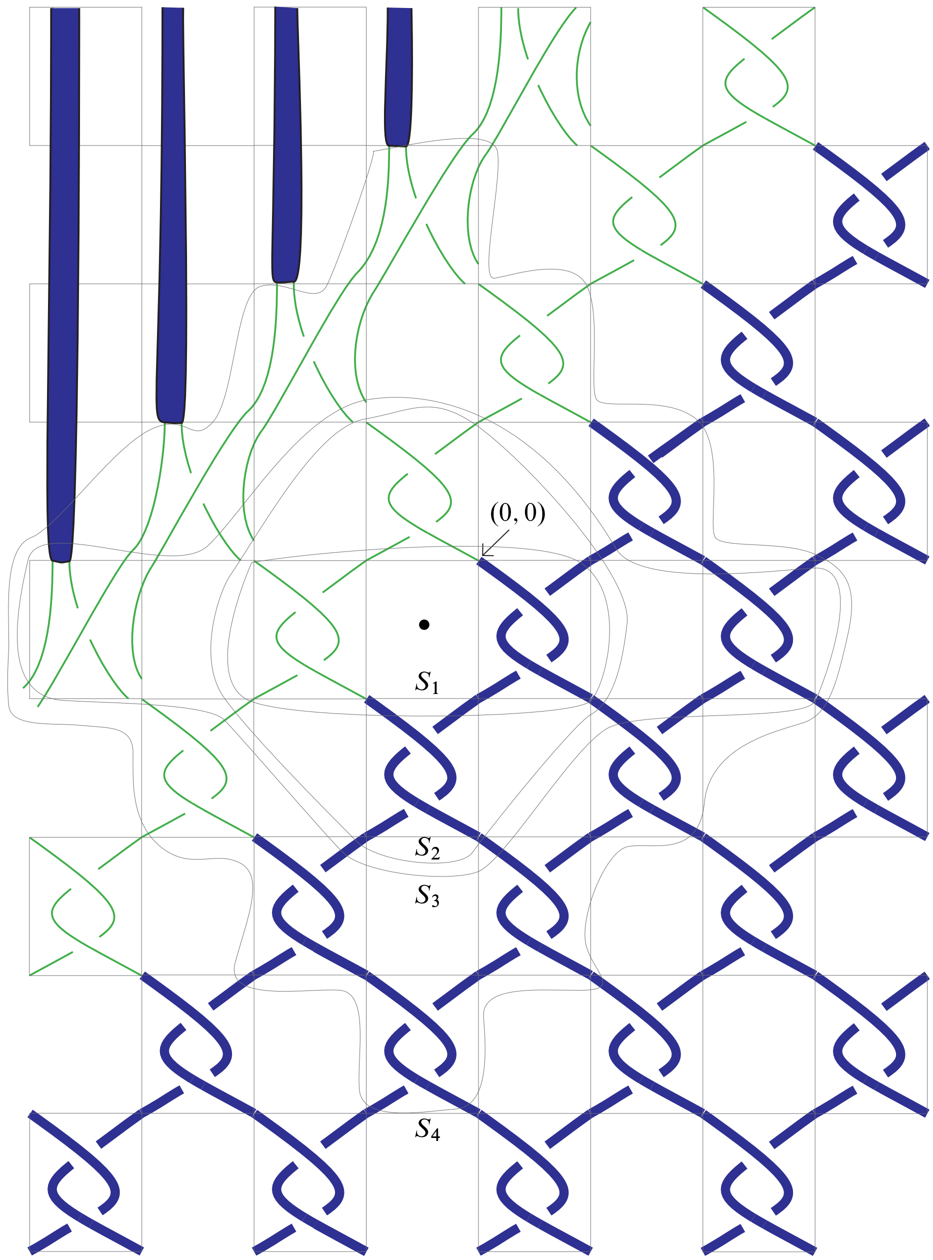


Figure 12

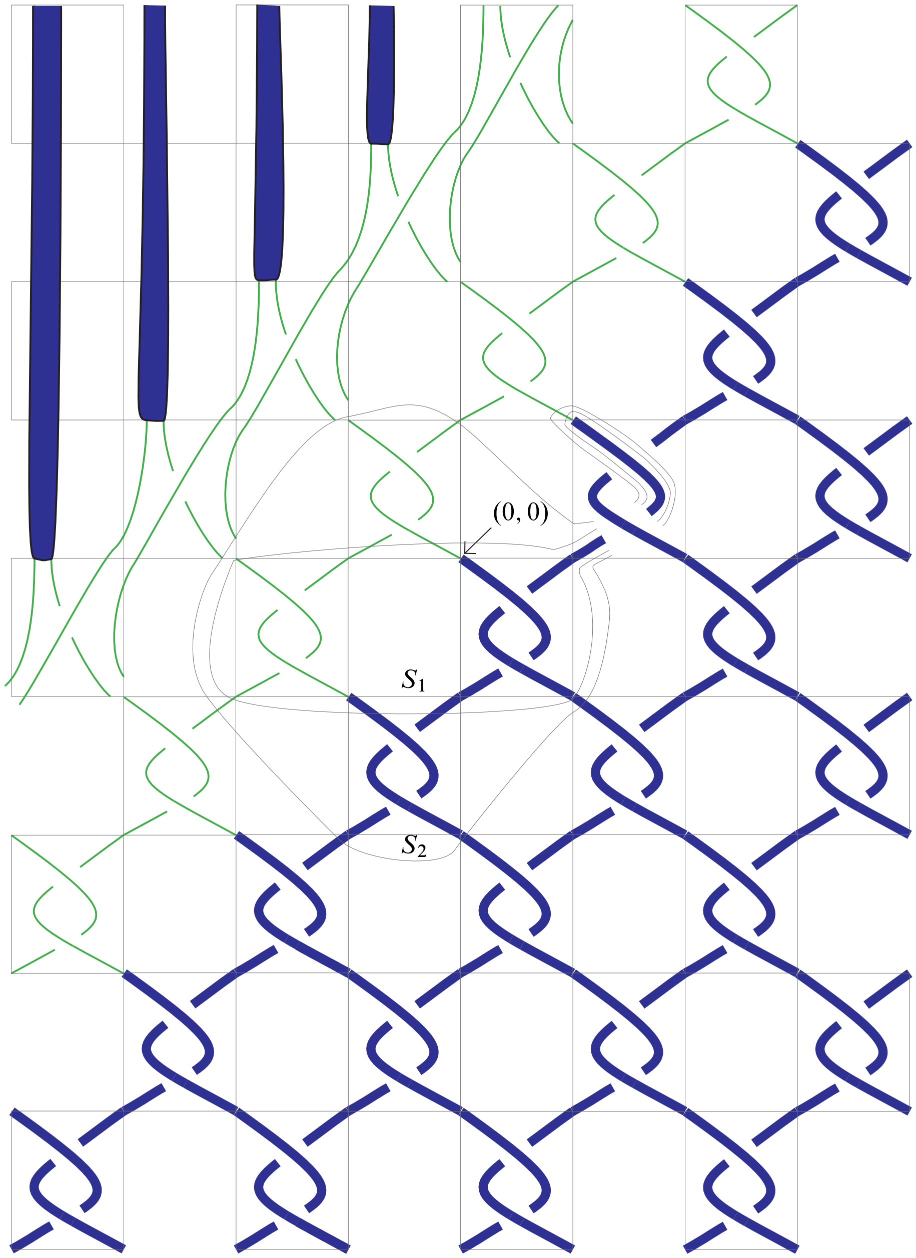


Figure 13

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