Lens spaces obtainable by surgery on doubly primitive knots

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In this paper, we consider which lens spaces are obtainable by Dehn surgery described by Berge on doubly primitive knots. An algorithm is given to decide whether a given lens space is obtainable by such a surgery. Also included is a complete characterization of such surgeries yielding lens spaces with Klein bottles.

57M25; 57M27

1 Introduction

Given a knot K in a 3–manifold, the following operation is called *Dehn surgery* on K: Remove an open regular neighborhood of K, and glue a solid torus back. Dehn surgeries on the trivial knot in the 3–sphere S^3 give the well-known class of 3–manifolds, the so-called *lens spaces*. On the other hand, it is known that only restricted Dehn surgeries on nontrivial knots yield lens spaces. Such a nontrivial example was found by Fintushel and Stern in [5], and since then, these surgeries have been widely studied.

Based on the pioneering work of Berge [1], Gordon conjectured in [7, Problem 1.78] that the knots admitting Dehn surgeries yielding lens spaces are all *doubly primitive*. In this paper, we concentrate our attention on doubly primitive knots and consider the problem: Which lens spaces are obtainable by Dehn surgery on doubly primitive knots? See Section 2 for the definitions in detail.

We first give an algorithm to decide whether a given lens space is obtainable by Dehn surgery described by Berge on a doubly primitive knot. Our algorithm depends on the work on the triviality of three-bridge knots presented by Homma and Ochiai [6], and so it is quite effective.

Next we consider the class of lens spaces which Klein bottles. In the study of Dehn surgeries giving 3–manifolds with Klein bottles, Teragaito asked the following question: Which nontrivial knot admits Dehn surgery yielding a lens space with a Klein bottle? In particular, can a hyperbolic knot admit such a surgery? Our result gives a partial answer to the question.

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Theorem 1.1 *Dehn surgery described by Berge on a nontrivial doubly primitive knot yields a lens space containing a Klein bottle if and only if the knot is either of the* $(\pm 5, 3)$*– or* $(\pm 7, 3)$*–torus knots. The lens spaces so obtained are of type* $(16, 7)$ *or* $(20, 9)$*. In particular, no hyperbolic doubly primitive knots admit such Dehn surgeries.*

We remark that a characterization of lens spaces containing Klein bottles was already established by Bredon and Wood [3].

Also, it is already completely known which lens spaces are obtainable by surgery on torus knots by Moser [9]. Furthermore, by Bleiler and Litherland [2], Wang [13] and Wu [14], the first homology group of any lens space produced by Dehn surgery on a satellite knot has odd order. It is an easy consequence of homology theory that such a 3–manifold does not contain a closed nonorientable surface. Consequently Dehn surgery on a nontrivial nonhyperbolic knot yields a lens space containing a Klein bottle if and only if the knot is either of the $(\pm 5, 3)$– or $(\pm 7, 3)$–torus knots. The lens spaces so obtained are of type $(16, 7)$ or $(20, 9)$.

Tange recently announced that Dehn surgeries on nontrivial knots only result in lens spaces of type $(16, 7)$ or $(20, 9)$. However he could not determine the types of the knots admitting such surgeries.

2 Preliminaries

In this section, we will set up our terminology. In the following, $E(B; A)$ denotes the *exterior* of a subset B in a topological space A, ie, $E(B; A) = A \setminus \eta(B; A)$, where $\eta(B; A)$ means an open regular neighborhood of B in A.

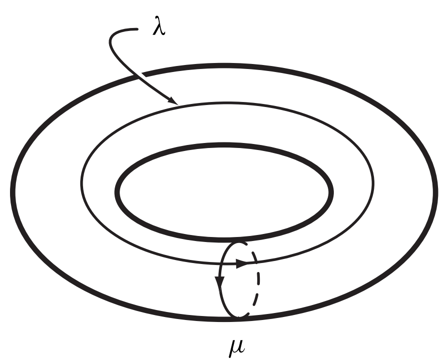

Figure 1

2.1 Lens space

Let V_1 be a standard solid torus in S^3, μ_1 a meridian of V_1 and λ a longitude of V_1 such that λ bounds a disk in $\mathrm{cl}(S^3 \setminus V_1)$. We fix an orientation of μ_1 and λ as illustrated in Figure 1. Let p and q be coprime integers and μ_2 a meridian of V_2. Then by attaching another solid torus, say V_2, to V_1 so that μ_2 is isotopic to a representative of the homology class $p[\lambda]+q[\mu_1]$, we obtain the *lens space of type* (p,q), denoted by $L(p,q)$.

2.2 Dehn surgery

Let K be a knot in a connected, compact, orientable 3–manifold N. We fix an oriented meridian–longitude system (μ,λ) for K as in Figure 1. When $K \subset S^3$, we always take the preferred longitude for K as λ. Recall that a *Dehn surgery* on a knot K is an operation to attach a solid torus $\bar{V}$ to $E(K;N)$ by a homeomorphism $\varphi\colon \partial\bar{V} \to \partial E(K;N)$. If $\varphi(\bar{\mu})$ is isotopic to a representative of the homology class $p[\mu]+q[\lambda]$ for a meridian $\bar{\mu}$ of $\bar{V}$, then the surgery is called (p/q)*–surgery*. Note that the lens space $L(p,q)$ is obtained by $(-p/q)$–surgery on a trivial knot. By an *integral surgery*, we mean an r–Dehn surgery with r an integer. Set $N_\varphi = E(K;N)\cup_\varphi \bar{V}$ and let $K^* \subset N_\varphi$ be a core loop of $\bar{V}$. We call K^* the *dual knot* of K in N_φ. We remark that $E(K;N)$ is homeomorphic to $E(K^*;N_\varphi)$ and that if a Dehn surgery on K in N yields a 3–manifold N_φ, then K^* admits a Dehn surgery yielding N.

2.3 Doubly primitive knots and dual knots

Let H be a genus two handlebody standardly embedded in S^3, ie, $E(H;S^3)$ is also a genus two handlebody. A simple closed curve on the boundary ∂H is in a *doubly primitive position* if it represents a free generator both of $\pi_1(H)$ and of $\pi_1(E(H;S^3))$. A knot in S^3 is called a *doubly primitive knot* if it is isotopic to a simple closed curve in a doubly primitive position. Let K be a doubly primitive knot with a peripheral torus T, ie, $T = \mathrm{cl}(\eta(K;S^3)) \setminus \eta(K;S^3)$. When K is isotoped into a doubly primitive position, $\partial H \cap T$ can be assumed to consist of two essential simple closed curves which are mutually isotopic on T. The isotopy class is called a *surface slope* of K. We remark that a Dehn surgery along a surface slope of K is always an integral surgery. Berge [1] then proved that any Dehn surgery along a surface slope of K yields a lens space. Moreover, he showed the dual knot of K in the lens space is isotopic to a knot defined as follows.

Definition 2.1 Let V_1 be a standard solid torus in S^3, μ_1 a meridian of V_1 and λ a longitude of V_1 such that λ bounds a disk in $\mathrm{cl}(S^3 \setminus V_1)$. We fix an orientation of

μ_1 and λ like in Figure 1. By attaching a solid torus V_2 to V_1 so that a meridian μ_2 of V_2 is isotopic to a representative of $p[\lambda]+q[\mu_1]$, we obtain a lens space $L(p,q)$, where p and q are coprime positive integers. The intersection points of μ_1 and μ_2 are labeled by $P_0,\ldots,P_{p-1}$ successively along the positive direction of μ_1. Let t_i^u $(i=1,2)$ be simple arcs in the disks D_i in V_i bounded by μ_i joining P_0 to P_u $(u=1,2,\ldots,p-1)$. Then the notation $K(L(p,q);u)$ denotes the knot $t_1^u \cup t_2^u$ in $L(p,q)$. See Figure 2.

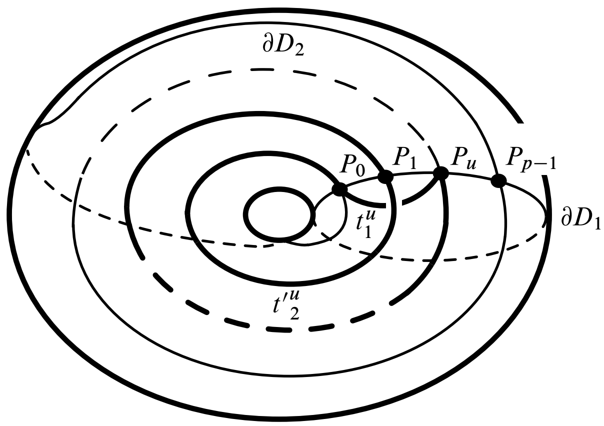

Figure 2: Here, t'^u_2 is a projection of t_2^u on ∂V_1.

If $K(L(p,q);u)$ is the dual knot of some doubly primitive knot in S^3, then it admits a Dehn surgery yielding S^3. Note that this Dehn surgery is an integral surgery for a natural meridian–longitude system in the lens space. We remark that the converse does not hold in general; it is not necessary for any knot represented by $K(L(p,q);u)$ to admit a integral surgery yielding S^3.

Remark 2.2 It is known that two lens spaces $L(p,q)$ and $L(p',q')$ are (possibly orientation reversingly) homeomorphic if and only if $|p|=|p'|$, and $q \equiv \pm q' \pmod{p}$ or $qq' \equiv \pm 1 \pmod{p}$. Also, we easily see that $K(L(p,q);u)$ is isotopic to $K(L(p,q);p-u)$. Hence for $K(L(p,q);u)$, we assume that $0<q<p/2$ and $1 \le u \le p/2$ in the remainder of the paper.

2.4 Lemma on dual knots

In this subsection, we prepare a lemma which will be used in the later sections.

Lemma 2.3 *Let K be a knot in the lens space $L(p,q)$ represented as $K(L(p,q);u)$. Suppose that an integral surgery on K yields S^3. Let K^* be the dual knot of K in S^3. Then the following holds.*

(1) *The knot K^* is a doubly primitive knot in S^3.*

(2) *The knot K^* is trivial in S^3 if and only if $q = u = 1$.*

To prove this lemma, we use a result by the second author. To state this, we prepare the following notation.

Definition 2.4 Let p and q be a pair of positive coprime integers. Let $\{s_j\}_{1 \le j \le p}$ be the finite sequence such that $0 \le s_j < p$ and $s_j \equiv q \cdot j \pmod{p}$. For an integer k with $0 < k < p$, $\Psi_{p,q}(k)$ denotes the smallest integer j with $s_j = k$, and $\Phi_{p,q}(k)$ the number of elements of the set $\{s_j \mid 1 \le j < \Psi_{p,q}(k),\ s_j < k\}$.

Given this notation, the following is our key lemma.

Lemma 2.5 [12, Theorem 2.5] *Let p and q be coprime integers with $0 < q < p$ and u an integer with $1 \le u \le p-1$. If $K(L(p,q);u)$ admits integral surgery yielding S^3, then we have*

$$p \cdot \Phi_{p,q}(u) - u \cdot \Psi_{p,q}(u) = \pm 1 \quad \text{or} \quad \pm 1 - p.$$

In particular, p and u are coprime.

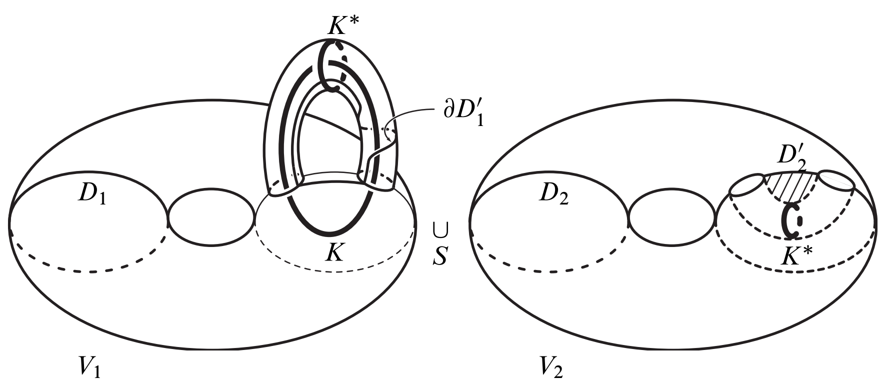

Figure 3

Proof of Lemma 2.3 (1) Let S be the genus two Heegaard surface of the lens space $L(p,q)$ illustrated in Figure 3. Note that this S can be regarded as a Heegaard surface of genus two also in the surgered manifold, ie, S^3 in this case. Let V_1 and V_2 be the genus two handlebodies bounded by S in S^3. Then since the surgery we are now considering on K is integral, the dual knot K^* can be isotopic on S, in particular, lying on the boundary of a cocore of the 1–handle as shown in Figure 3. It then follows that there are discs D_1, D_1' in V_1, D_2, D_2' in V_2 in S^3 such that $K^* \cap \partial D_i = \varnothing$ and $K^* \cap \partial D_i'$ is a single point for $i = 1, 2$. See Figure 3. This indicates that the dual knot K^* is a doubly primitive knot in S^3.

(2) In general, as a dual fact remarked in the previous subsection, if a knot K in the lens space $L(p,q)$ yields S^3 by an integral surgery, then the dual knot K^* in S^3 admits an integral surgery yielding $L(p,q)$.

Now assume that the dual knot K^* is a trivial knot. Then since $L(p,q)$ is obtained by an integral surgery on the trivial knot, we have $q = 1$. Hence the knot K is described as $K(L(p,1);u)$ for some p and u. For this K, note that $\Phi_{p,1}(u) = u - 1$ and $\Psi_{p,1}(u) = u$ hold. Then by Lemma 2.5, we have

$$p \cdot (u-1) - u^2 = \pm 1 \quad \text{or} \quad \pm 1 - p.$$

Since we are assuming $1 \le u \le p/2$, the only possibility is $u = 1$.

Conversely, we assume that $q = u = 1$, ie, K is represented as $K(L(p,1);1)$. Then we have $\Psi_{p,1}(1) = 1$, equivalently, K is isotopic to a core of the Heegaard solid torus. See the article by the second author [12] for more explanations. This means that the exterior of K is homeomorphic to a solid torus, which is also the exterior of the dual knot K^* in S^3. Thus we conclude that the dual knot K^* is trivial. □

3 Algorithm to detect obtainable lens spaces

In this section, we will describe an algorithm to decide whether a given lens space is obtainable by Dehn surgery on a doubly primitive knot along its surface slope.

As stated before, Berge [1] showed that if a Dehn surgery along a surface slope on a doubly primitive knot in S^3 yields a lens space $L(p,q)$, then its dual knot in $L(p,q)$ is isotopic to a knot described as $K(L(p,q);u)$ with some u. Conversely, as shown in Lemma 2.3 (1), if a knot in a lens space $L(p,q)$ represented as $K(L(p,q);u)$ admits an integral surgery yielding S^3, then the dual knot is always a doubly primitive knot in S^3. This implies that a lens space $L(p,q)$ is obtainable by Dehn surgery on a doubly primitive knot along its surface slope if and only if $L(p,q)$ contains a knot represented as $K(L(p,q);u)$ admitting an integral surgery yielding S^3. Therefore the key of our algorithm is how to check whether a given knot represented as $K(L(p,q);u)$ admits an integral surgery yielding S^3 or not.

As an instructive example, let us check that the knot $K(L(5,1);2)$ in $L(5,1)$ can admit integral surgery creating S^3. This implies that the lens space $L(5,1)$ is obtainable by Dehn surgery on some doubly primitive knot in S^3.

Example 3.1 We consider a Heegaard splitting of genus two of $L(5,1)$ illustrated as in Figure 4, which is obtained from the standard Heegaard splitting of genus one by stabilization. Note that the knot $K(L(5,1);2)$ is isotopic to the dotted knot in Figure 4 below. Take the quotient of $L(5,1)$ by involution as illustrated in Figure 4. It follows from Figure 5 that the quotient space is S^3.

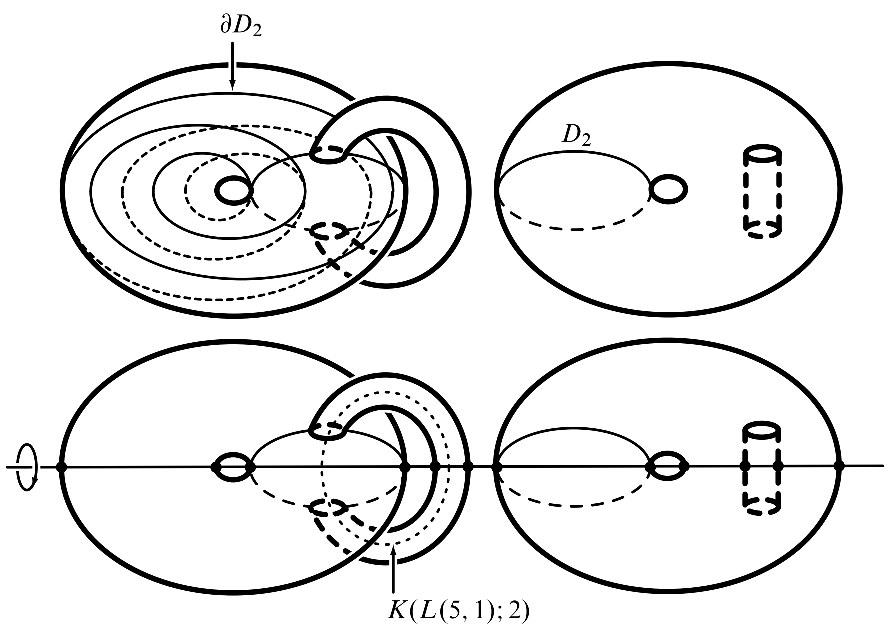

Figure 4

Let B be the 3–ball in Figure 5, which appears as the quotient of an equivariant regular neighborhood of $K(L(5,1);2)$ in $L(5,1)$. Also the quotient of the axis of the involution gives a knot in S^3 as shown in Figure 6.

If $K(L(5,1);2)$ admits a Dehn surgery yielding S^3, then, from the knot shown in Figure 6, the corresponding untangle surgery at the 3–ball B must give the trivial knot. This follows from the so-called *Montesinos trick* [8].

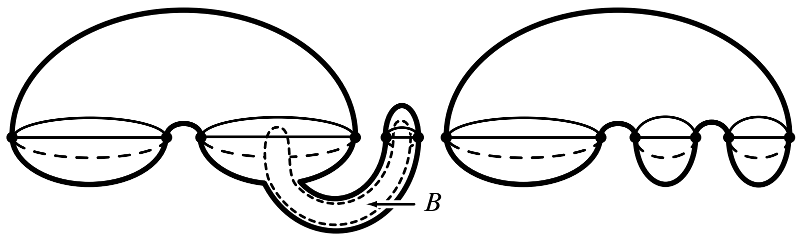

Figure 5

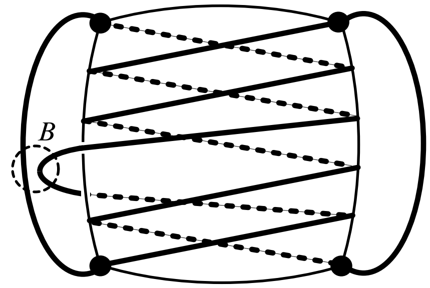

Figure 6

In fact, it suffices to check the only two links shown in Figure 7, due the result of the second author given in [12, Theorem 2.5]. We can see that the knot depicted in the left side of Figure 7 is actually trivial. Therefore $K(L(5,1);2)$ can admit Dehn surgery yielding S^3.

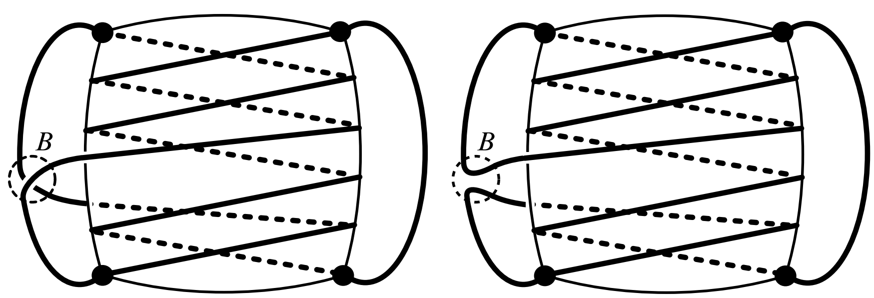

Figure 7

In general, by the following procedure, we can determine whether the given $L(p,q)$ is obtainable by a Dehn surgery on a nontrivial doubly primitive knot along a surface slope or not.

(1) Consider the two-bridge link of type (p,q) represented by the Schubert form $\mathrm{b}(p,q)$. This knot has the diagram illustrated as in Figure 8. See Burde and Zieschang [4] for example.

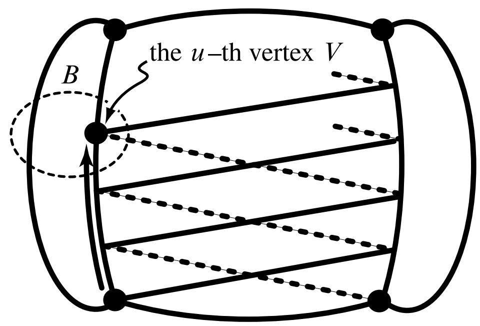

Figure 8

For any integer u with $1 \leq u \leq p/2$, we do the following steps (2)–(4) repeatedly.

(2) Put a vertex V on the diagram as illustrated in Figure 8, that is, put V on the u–th "wedge" from the left-bottom side of the pillowcase.

(3) In the neighborhood of V, depicted as the encircled region in Figure 8, make a crossing (the left) or do smoothing (the right) as in Figure 9.

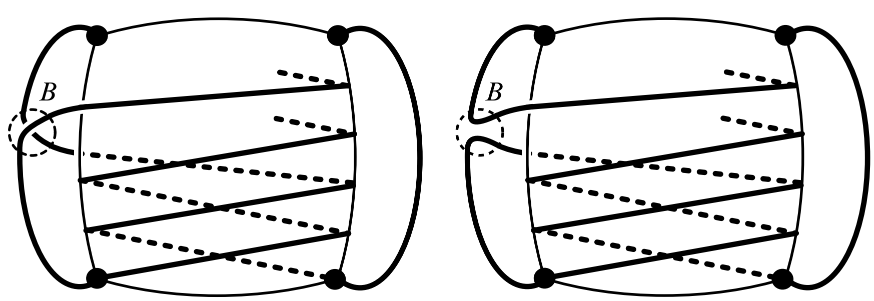

Figure 9

(4) Exactly one of the two diagrams so obtained gives a 3–bridge knot (not a link). By the algorithm given in Homma and Ochiai [6], we check the knot is trivial or not. The knot is trivial if and only if $K(L(p,q);u)$ admits a Dehn surgery yielding S^3, that is, $L(p,q)$ is obtainable by a Dehn surgery on a doubly primitive knot along a surface slope. As shown in Lemma 2.3 (2), the doubly primitive knot so obtained is trivial if and only if $q = u = 1$. Otherwise $L(p,q)$ is obtainable by a Dehn surgery on a nontrivial doubly primitive knot along a surface slope.

In Table 1, we list lens spaces obtainable by a Dehn surgery on a doubly primitive knot along a surface slope. In the table, we collect the values of (p,q,u) with $1 \le q < p/2, 1 \le u \le p/2, p \le 30$, for which $K(L(p,q);u)$ admits an integral surgery yielding S^3. There, to describe the dual knots, $T(a,b)$ denotes the torus knot of type (a,b) and $C(a,b;K_0)$ the (a,b)–cable knot on a knot K_0. Note that we permit duplicate descriptions of such knots: Some in the list might be ambient isotopic to or mirror image of another one.

4 Proof of Theorem 1.1

In this section, we consider the lens spaces containing Klein bottles, and give a proof of Theorem 1.1. In this case, by virtue of Lemma 2.5, we can determine the possible position of the vertex V, ie, the possible values of u, in the algorithm in the previous section. Based on this, we prove our theorem as follows.

Proof of Theorem 1.1 Let K be a doubly primitive knot with a Dehn surgery along a surface slope yielding a lens space M containing a Klein bottle. Then M has to be of type $(4n, 2n-1)$, where n is a positive integer, up to (possibly orientation-reversing) homeomorphism [3]. Hence the dual knot K^* of K in M is represented by $K(L(4n,2n-1);u)$ for some positive integer u. Note that this K^* has to admit an integral surgery yielding S^3.

Suppose that $n = 1$. Then the lens space is of type $(4,1)$, and u must be 1 or 2 by assumption on p and u. However, $u \neq 2$, for p and u have to be coprime by Lemma 2.5. If $u = 1$, then by Lemma 2.3, the knot K is trivial.

Thus in the following, we assume $n \ge 2$.

Claim 4.1 *If $K(L(4n,2n-1);u)$ with $n \ge 2$ admits integral surgery yielding S^3, then $\Phi_{4n,2n-1}(u) = 0$ and one of the following holds:*

$$(n,u) = (4,3), (4,5), (5,3), (5,7)$$

p	q	u	K^*	p	q	u	K^*
5	1	2	$T(3,2)$	21	5	4	$T(5,4)$
7	2	3	$T(3,2)$	21	5	10	$T(11,2)$
7	3	2	$T(3,2)$	22	5	7	$T(7,3)$
9	2	4	$T(5,2)$	22	9	3	$T(7,3)$
9	4	2	$T(5,2)$	23	4	2	$T(11,2)$
11	2	3	$T(4,3)$	23	5	8	$T(8,3)$
11	3	5	$T(5,2)$	23	6	11	$T(11,2)$
11	4	2	$T(5,2)$	23	7	4	$C(11,2;T(3,2))$
11	5	4	$T(4,3)$	23	9	3	$T(8,3)$
13	3	4	$T(4,3)$	23	10	6	$C(11,2;T(3,2))$
13	3	6	$T(7,2)$	25	4	2	$T(13,2)$
13	4	2	$T(7,2)$	25	6	12	$T(13,2)$
13	4	3	$T(4,3)$	25	9	3	$T(8,3)$
14	3	5	$T(5,3)$	25	9	4	$C(13,2;T(3,2))$
14	5	3	$T(5,3)$	25	11	6	$C(13,2;T(3,2))$
15	4	2	$T(7,2)$	25	11	8	$T(8,3)$
15	4	7	$T(7,2)$	27	4	2	$T(13,2)$
16	7	3	$T(5,3)$	27	5	7	$T(7,4)$
16	7	5	$T(5,4)$	27	7	13	$T(13,2)$
17	4	2	$T(9,2)$	27	8	10	hyperbolic
17	4	8	$T(9,4)$	27	10	8	hyperbolic
18	5	7	hyperbolic	27	11	4	$T(7,4)$
18	7	5	hyperbolic	29	4	2	$T(15,2)$
19	3	4	$T(5,4)$	29	4	5	$T(6,5)$
19	4	2	$T(9,2)$	29	7	6	$T(6,5)$
19	5	9	$T(9,2)$	29	7	14	$T(15,2)$
19	6	5	$T(5,4)$	29	9	3	$T(10,3)$
19	7	8	hyperbolic	29	9	7	$T(7,4)$
19	8	7	hyperbolic	29	13	4	$T(7,4)$
20	9	3	$T(7,3)$	29	13	10	$T(10,3)$
20	9	7	$T(7,3)$	30	11	7	hyperbolic
21	4	2	$T(11,2)$	30	11	13	hyperbolic
21	4	5	$T(5,4)$				

Table 1: The values of (p,q,u) for which $K(L(p,q);u)$ admits an integral surgery yielding S^3 with $p \leq 30$

Proof Since $K(L(4n,2n-1);u)$ admits an integral surgery yielding S^3, it follows from Lemma 2.5 that $4n$ and u are coprime. Hence u is an odd integer. By Remark

2.2, we can assume that $u < 2n$. Then the sequence $\{s_j\}_{1\le j\le 4n}$ for $(4n, 2n-1)$ is

$$s_j \equiv \begin{cases} 2n-j \pmod{4n} & \text{if } j \text{ is odd} \\ 4n-j \pmod{4n} & \text{if } j \text{ is even.} \end{cases}$$

In particular, a subsequence $\{s_j\}_{1\le j\le 2n-1}$ of $\{s_j\}_{1\le j\le 4n}$ satisfies the following.

(1) s_j is odd if j is odd, and s_j is even if j is even.

(2) Each of subsequences $\{s_{2k-1}\}_{1\le k\le n}$ and $\{s_{2k}\}_{1\le k\le n}$ is monotonically decreasing.

(3) $\max\{s_{2k-1} \mid 1\le k\le n\} = s_1 = 2n-1$ and $\min\{s_{2k} \mid 1\le k\le n\} = s_{2n} = 2n$. Hence we have $\max\{s_{2k-1} \mid 1\le k\le n\} < \min\{s_{2k} \mid 1\le k\le n\}$.

Let m be the integer satisfying $s_m = u$, that is, $m = \Psi_{4n,2n-1}(u)$. Then $u = 2n-m$. Since u is an odd integer less than $2n$, we see that $1\le m\le 2n-1$. This implies that $s_j > u$ for any integer j satisfying $1\le j\le m-1$ and hence $\Phi_{4n,2n-1}(u) = 0$. Therefore we have the first conclusion of the claim.

By Lemma 2.5, we also have

$$u\cdot m = \pm 1 \quad \text{or} \quad 4n\pm 1.$$

Case 1 $u\cdot m = \pm 1$.

Since u and m are positive integers, we have $u = m = 1$. This implies that $s_1 = 1$ and hence $n = 1$. This contradicts that $n\ge 2$.

Case 2 $u\cdot m = 4n\pm 1$.

In this case, we first note that $m\ne 2$. Hence we have the following.

$$\begin{aligned} u\cdot m &= 4n\pm 1 \\ (2n-m)\cdot m &= 4n\pm 1 \\ 2n &= m+2+\frac{4\pm 1}{m-2} \end{aligned}$$

Since m and n are positive integers, we have the desired conclusion. □

By this claim, the possible type of the obtained lens space M is $(16,7)$ or $(20,9)$.

In fact, by applying our algorithm, we can directly check that $K(L(4n,2n-1);u)$ actually admits integral surgery yielding S^3 for $(n,u) = (4,3), (4,5), (5,3), (5,7)$.

Let us consider these cases in detail.

Case 1 $K^* = K(L(16,7);3)$ or $K(L(16,7);5)$.

Since $\Phi_{16,7}(3) = \Phi_{16,7}(5) = 0$, we can see that K^* is a torus knot, ie, a knot lying the standard Heegaard torus (cf [10, Proposition 5.2]). Since $E(K^*;M) \cong E(K;S^3)$, $E(K;S^3)$ admits Seifert fibration and hence K is a torus knot. It then follows from Van Kampen's theorem (cf [11, Section 5]) that

$$\begin{aligned}\pi_1(E(K;S^3)) &\cong \pi_1(E(K^*;M)) \\ &\cong \langle x, y \mid x^5 = y^3 \rangle.\end{aligned}$$

This implies that K is the $(\pm 5, 3)$–torus knot, and the obtained lens space M is homeomorphic to $L(16,7)$.

Case 2 $K^* = K(L(20,9);3)$ or $K(L(20,9);7)$.

In the same way as above, we see that K is a torus knot. It also follows from Van Kampen's theorem that

$$\begin{aligned}\pi_1(E(K;S^3)) &\cong \pi_1(E(K^*;M)) \\ &\cong \langle x, y \mid x^7 = y^3 \rangle.\end{aligned}$$

This implies that K is the $(\pm 7, 3)$–torus knot, and the obtained lens space M is homeomorphic to $L(20,9)$. □

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References

[1] **J Berge**, *Some knots with surgeries yielding lens spaces*, unpublished manuscript

[2] **S A Bleiler**, **R A Litherland**, *Lens spaces and Dehn surgery*, Proc. Amer. Math. Soc. 107 (1989) 1127–1131 MR984783

[3] **G E Bredon**, **J W Wood**, *Non-orientable surfaces in orientable 3–manifolds*, Invent. Math. 7 (1969) 83–110 MR0246312

[4] **G Burde**, **H Zieschang**, *Knots*, de Gruyter Studies in Mathematics 5, Walter de Gruyter & Co., Berlin (1985) MR808776

[5] **R Fintushel**, **R J Stern**, *Constructing lens spaces by surgery on knots*, Math. Z. 175 (1980) 33–51 MR595630

[6] **T Homma**, **M Ochiai**, *On relations of Heegaard diagrams and knots*, Math. Sem. Notes Kobe Univ. 6 (1978) 383–393 MR512641

[7] **R Kirby**, *Problems in low-dimensional topology*, from: "Geometric topology (Athens, GA, 1993)", (R Kirby, editor), AMS/IP Stud. Adv. Math. 2, Amer. Math. Soc., Providence, RI (1997) 35–473 MR1470751

[8] **J M Montesinos**, *Surgery on links and double branched covers of* S^3, from: "Knots, groups, and 3–manifolds (Papers dedicated to the memory of R. H. Fox)", (L P Neuwirth, editor), Ann. of Math. Studies 84, Princeton Univ. Press, Princeton, N.J. (1975) 227–259 MR0380802

[9] **L Moser**, *Elementary surgery along a torus knot*, Pacific J. Math. 38 (1971) 737–745 MR0383406

[10] **T Saito**, *The dual knots of doubly primitive knots*, to appear in Osaka J. Math.

[11] **T Saito**, *Knots in lens spaces with the 3–sphere surgery*, preprint

[12] **T Saito**, *Dehn surgery and* $(1,1)$*–knots in lens spaces*, Topology Appl. 154 (2007) 1502–1515 MR2310483

[13] **S C Wang**, *Cyclic surgery on knots*, Proc. Amer. Math. Soc. 107 (1989) 1091–1094 MR984820

[14] **Y Q Wu**, *Cyclic surgery and satellite knots*, Topology Appl. 36 (1990) 205–208 MR1070700

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