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Étale contractible varieties in positive characteristic

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Unlike in characteristic 0, there are no nontrivial smooth varieties over an algebraically closed field k of characteristic $p > 0$ that are contractible in the sense of étale homotopy theory.

1. Introduction

Homotopy theory is founded on the idea of contracting the interval, either as a space, or as an actual homotopy, that is, a path in a space of maps. In algebraic geometry, the affine line $\mathbb{A}^1_k$ serves as an algebraic equivalent of the interval, at least in characteristic 0, where $\mathbb{A}^1_k$ is contractible.

Matters differ in characteristic $p > 0$, where $\pi_1(\mathbb{A}^1_k)$ is an infinite group: a group G occurs as a finite quotient of $\pi_1(\mathbb{A}^1_k)$ precisely if G is a quasi-p-group due to Abhyankar's conjecture for the affine line as proven by Raynaud. This raises the question whether there is an étale contractible variety in positive characteristic.

Theorem 1. *Let k be an algebraically closed field of characteristic $p > 0$. A smooth variety U/k is étale contractible if and only if $U = \operatorname{Spec}(k)$ is the point.*

It turns out that our discussion in positive characteristic depends only on H^1 and H^2. By the étale Hurewicz and Whitehead theorems (see [Artin and Mazur 1969, §4]), we might therefore replace "étale contractible" with "étale 2-connected" in Theorem 1. Further, our proof covers more than just smooth varieties. Here is the more precise result, which proves Theorem 1 because smooth varieties have big Cartier divisors.

Theorem 2. *Let k be an algebraically closed field of characteristic $p > 0$ and let U/k be a normal variety such that*

(i) *the group $\mathrm{H}^1_{\text{ét}}(U, \mathbb{F}_p)$ vanishes,*

(ii) *there is a prime number $\ell \neq p$ such that $\mathrm{H}^2_{\text{ét}}(U, \mu_\ell) = 0$,*

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(iii) *U has a big Cartier divisor or* $\dim U \leq 2$.

Then U has dimension 0.

In order to show the range of varieties to which Theorem 2 applies, we list in Proposition 4 properties of varieties that imply the existence of big Cartier divisors, including quasiprojective varieties and locally $\mathbb{Q}$-factorial (in particular smooth) varieties. The proof of Theorem 2 in the presence of a big Cartier divisor will be given in Section 2.3. The case of normal surfaces will be treated in Section 3.

In the proof of Theorem 2, one would like to work with a compactification $U \subseteq X$ and the geometry of line bundles on U versus X. For that strategy to work, we need a compactification that is locally factorial along $Y = X \setminus U$. Since in characteristic $p > 0$, resolution of singularities is presently absent in dimension ≥ 4, we resort to desingularisation by alterations due to de Jong. Unfortunately, the alteration typically destroys the étale contractibility assumption. We first deduce more *coherent* properties from étale 2-connectedness that transfer to the alteration.

The key difference with characteristic 0 comes from Artin–Schreier theory relating $\mathrm{H}^1_{\text{ét}}(U, \mathbb{F}_p)$ to regular functions on U.

Remark 3. Let us illustrate the situation in characteristic 0 in contrast to Theorem 1.

(1) There are contractible complex smooth surfaces other than $\mathbb{A}^2_{\mathbb{C}}$. The first such example is due to Ramanujam [1971, §3]; see also [tom Dieck and Petrie 1990] for explicit equations. All of them are affine and have rational smooth projective completions.

(2) Smooth varieties $U/\mathbb{C}$ different from affine space $\mathbb{A}^n_{\mathbb{C}}$ but with $U(\mathbb{C})$ diffeomorphic to $\mathbb{C}^n$ are known as exotic algebraic structures on $\mathbb{C}^n$. These varieties are contractible and we recommend the Bourbaki talk on $\mathbb{A}^n$ by Kraft [1996], or the survey by Zaĭdenberg [1999]. A remarkable nonaffine (but quasiaffine) example U was obtained by Winkelmann [1990] as a quotient $U = \mathbb{A}^5/\mathbb{G}_a$, and more concretely as the complement in a smooth projective quadratic hypersurface in $\mathbb{P}^5_{\mathbb{C}}$ of the union of a hyperplane and a smooth surface.

(3) The notion of $\mathbb{A}^1$-contractibility is a priori stronger than contractibility in the complex topology. Asok and Doran [2007] construct, for every $d \geq 6$, continuous families of pairwise nonisomorphic, nonaffine smooth varieties of dimension d that are even $\mathbb{A}^1$-contractible.

Notation. Throughout the note, k will be an algebraically closed field. By definition, a variety over k is a separated scheme of finite type over k. We will denote the étale fundamental group by π_1 and its maximal abelian quotient by π_1^{ab}. The sheaf μ_ℓ for ℓ different from the characteristic denotes the (locally) constant sheaf of ℓ-th roots of unity.

2. Big Cartier divisors

2.1. ***Existence of big divisors.*** Recall that a Cartier divisor D on a normal (but not necessarily proper) variety U/k is big if the rational map associated to the linear system $|mD|$ is generically finite for $m \gg 0$.

Proposition 4. *Let k be an algebraically closed field and let U/k be a normal variety such that one of the following holds*:

(a) *U is quasiprojective.*

(b) *U is a product of varieties with big divisors.*

(c) *U is locally $\mathbb{Q}$-factorial everywhere.*

Then U has a big Cartier divisor.

Proof. Since any ample divisor is big, the conclusion holds if we assume (a). In case (b), the sum of the pullbacks of big Cartier divisors on the factors is again big.

If (c) holds, then we first choose a dense affine open $V \subseteq U$ and an effective big Cartier divisor D on V by (a). Let $B = U \setminus V$ be the boundary, in fact a Weil divisor since V is affine, and let D' be the Zariski closure of D as a Weil divisor on U. By assumption, mD' and mB are both effective Cartier divisors for $m \gg 0$, and there are sections $s_0, \dots, s_d \in \mathrm{H}^0(V, mD)$ such that the induced map $V \to \mathbb{P}^d_k$ is generically finite. For $r \gg 0$, the sections s_i extend to sections of

$$\mathrm{H}^0(U, mD + mrB),$$

so that $mD + mrB$ is the desired big Cartier divisor on U. □

2.2. ***Geometry of varieties with vanishing*** $\mathbf{H^1}$ ***and*** $\mathbf{H^2}$**.** Let ℓ be a prime number different from the characteristic of k and let U/k be a variety with $\mathrm{H}^2_{\text{ét}}(U, \mu_\ell) = 0$. It follows from the Kummer sequence in étale cohomology that $\mathrm{Pic}(U)$ is an ℓ-divisible abelian group.

The following crucially depends on k being a field of positive characteristic.

Proposition 5. *Let k be of characteristic $p > 0$. If U/k is a connected reduced variety such that $\pi_1^{\mathrm{ab}}(U) \otimes \mathbb{F}_p$ is finite, then $\mathrm{H}^0(U, \mathcal{O}_U) = k$.*

Proof. We argue by contradiction. If $f : U \to \mathbb{A}^1_k$ is a dominant map, then the induced map

$$f_* : \pi_1^{\mathrm{ab}}(U) \otimes \mathbb{F}_p \to \pi_1^{\mathrm{ab}}(\mathbb{A}^1_k) \otimes \mathbb{F}_p$$

has image of finite index in the infinite group $\pi_1^{\mathrm{ab}}(\mathbb{A}^1_k) \otimes \mathbb{F}_p$, a contradiction. □

By the duality $\mathrm{H}^1_{\text{ét}}(U, \mathbb{F}_p) = \mathrm{Hom}(\pi_1^{\mathrm{ab}}(U), \mathbb{F}_p)$, the vanishing of $\mathrm{H}^1_{\text{ét}}(U, \mathbb{F}_p)$ implies the assumption of Proposition 5.

2.3. ***Using alterations.*** Section 2.2 reduces the proof of Theorem 2 in the presence of a big Cartier divisor to the following proposition.

Proposition 6. *Let k be an algebraically closed field and let U/k be a connected normal variety with a big Cartier divisor and such that*

(i) $\mathrm{H}^0(U, \mathbb{O}_U) = k$ *and*

(ii) *there is a prime number ℓ such that* $\mathrm{Pic}(U)$ *is ℓ-divisible.*

Then U has dimension 0.

Proof. By [de Jong 1996, Theorem 7.3], there exists an alteration, that is, a generically finite projective map $h : \tilde{U} \to U$ such that $\tilde{U}$ can be embedded into a connected smooth projective variety $\tilde{X}$.

Step 1. The maximal open $V \subset U$, such that the restriction $\tilde{V} = h^{-1}(V) \to V$ is a finite map, has boundary $U \setminus V$ of codimension at least 2, since U is normal.

The k-algebra $\mathrm{H}^0(\tilde{V}, \mathbb{O}_{\tilde{V}})$ is an integral domain inside the function field of $\tilde{V}$. The minimal polynomial for $s \in \mathrm{H}^0(\tilde{V}, \mathbb{O}_{\tilde{V}})$ with respect to the function field of V has coefficients that are regular functions on V by normality and uniqueness of the minimal polynomial. Hence these coefficients are elements of $\mathrm{H}^0(V, \mathbb{O}_V) = \mathrm{H}^0(U, \mathbb{O}_U) = k$, and so

$$\mathrm{H}^0(\tilde{V}, \mathbb{O}_{\tilde{V}}) = k.$$

Step 2. By the theorem of the base [Kleiman 1971, Theorem 5.1], the Néron–Severi group

$$\mathrm{NS}(\tilde{X}) = \mathrm{Pic}(\tilde{X}) / \mathrm{Pic}^0(\tilde{X})$$

is a finitely generated abelian group. Since the restriction map $\mathrm{Pic}(\tilde{X}) \twoheadrightarrow \mathrm{Pic}(\tilde{U})$ is surjective, the induced composite map

$$h^* : \mathrm{Pic}(U) \to \mathrm{coker}\big(\mathrm{Pic}^0(\tilde{X}) \to \mathrm{Pic}(\tilde{U})\big) \tag{2-1}$$

maps an ℓ-divisible group to a finitely generated abelian group, and hence has finite image.

Step 3. Let D be a big Cartier divisor on U. Since $h : \tilde{U} \to U$ is generically finite, the divisor h^*D is also a big Cartier divisor. Moreover, as in the proof of Proposition 4, there is a big divisor $\tilde{D}$ on $\tilde{X}$ that restricts to h^*D on $\tilde{U}$. Upon replacing D and $\tilde{D}$ by a positive multiple, we may assume, by the finiteness of the image of the map (2-1), that $\tilde{D}$ is algebraically and thus numerically equivalent to a divisor B on $\tilde{X}$ that is supported in $\tilde{X} \setminus \tilde{U}$.

Since bigness on projective varieties only depends on the numerical equivalence class, see [Lazarsfeld 2004, Corollary 2.2.8], the divisor B is also big. Restriction to $\tilde{V}$ yields

$$\bigcup_{n \geq 0} \mathrm{H}^0(\tilde{X}, \mathbb{O}_{\tilde{X}}(nB)) \subseteq \mathrm{H}^0(\tilde{V}, \mathbb{O}_{\tilde{V}}) = k,$$

by Step 1 above. We conclude that $\dim U = \dim \tilde{X} = 0$ by the bigness of B. □

2.4. ***Complementing example.*** We illustrate the importance of the presence of a big divisor in Theorem 2 or Proposition 6 by an example from toric geometry.

We first recall two facts about complete toric varieties that are standard analytically over $\mathbb{C}$ and that have étale counterparts for toric varieties over arbitrary algebraically closed base fields, in particular of characteristic $p > 0$.

Lemma 7. *Let k be an algebraically closed field. Any complete toric variety X/k is étale simply connected*: $\pi_1(X) = \mathbf{1}$.

Proof. By toric resolution, see [Fulton 1993, §2.6], there is a resolution of singularities $\tilde{X} \to X$ with a smooth projective toric variety $\tilde{X}$. Birational invariance of the étale fundamental group shows $\pi_1(\tilde{X}) = \pi_1(\mathbb{P}^n_k) = \mathbf{1}$, and the surjection $\pi_1(\tilde{X}) \twoheadrightarrow \pi_1(X)$ shows that X is étale 1-connected. □

Lemma 8. *Let k be an algebraically closed field of characteristic p, and let X/k be a complete toric variety. Then for all $\ell \neq p$ we have*

$$\mathrm{H}^2_{\text{ét}}(X, \mathbb{Z}_\ell(1)) \simeq \mathrm{Pic}(X) \otimes \mathbb{Z}_\ell.$$

Proof. In the context of toric varieties over $\mathbb{C}$ and with respect to singular cohomology, this is [Fulton 1993, Corollary in 3.4]. The ℓ-adic case for toric varieties over an algebraically closed field k of characteristic $\neq \ell$ follows with a parallel proof. □

Example 9. Let $U = X$ be a complete normal nonprojective toric variety X of dimension 3 with trivial Picard group. Such toric varieties have been constructed in [Eikelberg 1992, Example 3.5; Fulton 1993, pp. 25–26, 65]. These sources construct X over $\mathbb{C}$ but the constructions work mutatis mutandis over any algebraically closed base field k. Then

(i) $\mathrm{H}^1_{\text{ét}}(X, \mathbb{F}_p) = 0$ by Lemma 7, and

(ii) $\mathrm{H}^2_{\text{ét}}(X, \mathbb{Z}_\ell(1)) = 0$ for all $\ell \neq p$ by Lemma 8, and since there is nontrivial torsion in ℓ-adic cohomology only for finitely many primes [Gabber 1983], we conclude that $\mathrm{H}^2(X, \mu_\ell) = 0$ for almost all $\ell \neq p$.

Therefore the assumptions of Theorem 2 hold, with the exception of the presence of a big Cartier divisor. Nevertheless, these toric varieties are not étale contractible since $\mathrm{H}^6_{\text{ét}}(X, \mathbb{Z}_\ell(3)) = \mathbb{Z}_\ell$.

3. Normal surfaces

In this section, Proposition 10 completes the proof of Theorem 2 for surfaces. Not every normal surface admits a big Cartier divisor, so something needs to be done. Examples of proper normal surfaces with trivial Picard group, in particular without big divisors, can be found in [Nagata 1958; Schröer 1999]. However, on a hypothetical normal 2-contractible surface, a specialisation argument allows us to conclude the existence of a big Cartier divisor in general.

Proposition 10. *There is no normal connected surface U/k over an algebraically closed field k of characteristic $p > 0$ such that*

(i) $\mathrm{H}^1_{\text{ét}}(U, \mathbb{F}_p) = 0$ *and*

(ii) $\mathrm{H}^2_{\text{ét}}(U, \mu_\ell) = 0$ *for some prime number $\ell \neq p$.*

Proof. We argue by contradiction and assume that U is a surface as in the proposition. By Nagata's embedding theorem and resolution of singularities for surfaces, U is a dense open in a normal proper surface X/k with boundary $Y = X \setminus U$ being a normal crossing divisor. Hence, X is smooth in a neighbourhood of Y.

By limit arguments, we may spread out over an integral scheme S of finite type over $\mathbb{F}_p$, that is, there is a proper flat $f : \mathscr{X} \to S$, a relative Cartier divisor $\mathscr{Y}$ in $\mathscr{X}/S$ with normal crossing relative to S and complement $\mathscr{U} = \mathscr{X} \setminus \mathscr{Y}$ such that

(a) all fibres are normal proper surfaces;

(b) there is a point $\eta : \mathrm{Spec}(k) \to S$ over the generic point of S such that the fibre over η agrees with the original $\mathscr{X}_\eta = X$ together with $\mathscr{U}_\eta = U$ and $\mathscr{Y}_\eta = Y$;

(c) the set of irreducible components of the fibres of $\mathscr{Y}$ forms a constant system, and each component of $\mathscr{Y}$ is a Cartier divisor; and

(d) the higher direct image $\mathrm{R}^2 f|_{\mathscr{U}*}\mu_\ell$ is locally constant and commutes with arbitrary base change by [Deligne 1977, Finitude, Theorem 1.9].

Since the generic stalk $(\mathrm{R}^2 f|_{\mathscr{U}*}\mu_\ell)_\eta = \mathrm{H}^2_{\text{ét}}(U, \mu_\ell) = 0$ vanishes, we conclude that for all geometric points $\bar{s} \in S$, we have $\mathrm{H}^2_{\text{ét}}(\mathscr{U}_{\bar{s}}, \mu_\ell) = 0$, where $\mathscr{U}_{\bar{s}}$ is the fibre of $\mathscr{U} \to S$ in $\bar{s}$. As in the proof of Proposition 6, this implies that for every Cartier divisor D on $\mathscr{X}_{\bar{s}}$, there are an $m \geq 1$ and a Cartier divisor E on $\mathscr{X}_{\bar{s}}$ supported in $\mathscr{Y}_{\bar{s}}$ such that $mD \equiv E$ are numerically equivalent.

We apply this insight to a geometric fibre $\mathscr{X}_{\bar{t}}$ above a closed point $t \in S$. Since by [Artin 1962, Corollary 2.11], all proper normal surfaces over the algebraic closure of a finite field are projective, we conclude that there is a very ample Cartier divisor $H_{\bar{t}}$ on $\mathscr{X}_{\bar{t}}$ with support contained in $\mathscr{Y}_{\bar{t}}$.

Let $\mathscr{H} \hookrightarrow \mathscr{X}$ be the relative Cartier divisor with support in $\mathscr{Y}$ that specialises to $H_{\bar{t}}$. By [Grothendieck 1961, Théorème 4.7.1], the divisor $\mathscr{H}$ is ample relative to S in an open neighbourhood of $t \in S$. Consequently, the normal proper surface X is projective, and in particular, U admits a big divisor. The part of Theorem 2 proven in Section 2.3 leads to a contradiction. □

Remark 11. It follows from the proof of Proposition 10 that any proper nonprojective normal surface X with trivial Picard group, in particular the examples of [Nagata 1958; Schröer 1999], must have $\mathrm{H}^2_{\text{ét}}(X, \mu_\ell) \neq 0$ and a fortiori must contain nontrivial ℓ-torsion classes in the cohomological Brauer group $\mathrm{Br}(X)$ for all ℓ different from the characteristic. The existence of nontrivial torsion classes in $\mathrm{Br}(X)$ under the above assumptions was proven by different methods in [Schröer 2001, proof of Theorem 4.1].

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