

# SUPPLEMENT TO “MODULAR CURVES OF PRIME-POWER LEVEL WITH INFINITELY MANY RATIONAL POINTS”

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This section includes tables that list data for all the groups  $G$ , up to conjugacy in  $\mathrm{GL}_2(\hat{\mathbb{Z}})$ , from Theorem 1.1. The genus 0 groups are given in Tables 1, 2 and 3. The genus 1 groups are given in Table 4.

We now describe how to read Tables 1–4. Each row corresponds to a unique group  $G$  from Theorem 1.1 up to conjugacy; it is given a unique label of the form  $MZ^g\text{-}Nz$ , where  $M$ ,  $N$  and  $g$  are integers and  $Z$  and  $z$  are letters. The integers  $N$  and  $g$  are the level and genus of  $G$ , respectively. Let  $\Gamma$  be the congruence subgroup consisting of matrices in  $\mathrm{SL}_2(\mathbb{Z})$  whose image modulo  $N$  lies in the image of  $G$  modulo  $N$ . The integer  $g$  is also the genus of the Riemann surface obtained by taking the quotient of the complex upper-half plane by the action of  $\Gamma$ . The integer  $M$  is the level of  $\Gamma$  and  $Z$  is an uppercase letter that distinguishes  $\Gamma$  up to conjugacy in  $\mathrm{GL}_2(\mathbb{Z})$ ; the prefix  $MZ^g$  for  $\Gamma$  matches the label used by Cummins and Pauli [*Experiment. Math.* **12** (2003), 243–255]. The letter  $z$  is chosen so that the label  $MZ^g\text{-}Nz$  distinguishes  $G$  up to conjugacy in  $\mathrm{GL}_2(\hat{\mathbb{Z}})$ . In some of the tables, we also number the rows.

For each row of these tables, there is a positive integer  $N$  and a set of matrices  $S$  in  $\mathrm{GL}_2(\mathbb{Z}/N\mathbb{Z})$ ; the corresponding open subgroup  $G$  of  $\mathrm{GL}_2(\hat{\mathbb{Z}})$  consists of the matrices  $A \in \mathrm{GL}_2(\hat{\mathbb{Z}})$  whose image in  $\mathrm{GL}_2(\mathbb{Z}/N\mathbb{Z})$  lies in the subgroup generated by  $S$ . The integer  $i$  in each row is the index  $[\mathrm{GL}_2(\hat{\mathbb{Z}}) : G]$ .

For each row of a table, we also have a list of minimal supergroups up to conjugacy (given by labels or numberings of other rows); more precisely, the groups  $G'$  satisfying  $G \subsetneq G' \subseteq \mathrm{GL}_2(\hat{\mathbb{Z}})$ , up to conjugacy in  $\mathrm{GL}_2(\hat{\mathbb{Z}})$ , for which there are no subgroups strictly between  $G$  and  $G'$ .

If  $G$  has genus 0, then  $X_G$  is isomorphic to  $\mathbb{P}_{\mathbb{Q}}^1$ ; its function field is of the form  $\mathbb{Q}(t)$ . If  $G$  has genus 1, then the curve  $X_G$  is isomorphic to the elliptic curve in Table 5 given by the “curve” column of Table 4; its function field is of the form  $\mathbb{Q}(x, y)$ , where  $x$  and  $y$  are the given Weierstrass coordinates.

Now for simplicity, suppose that  $G$  has genus 0. The function  $F$  listed in the “map” column describes the morphism  $X_G \rightarrow X_{G'}$  for one of the corresponding minimal supergroups  $G'$  of  $G$  (or the first minimal supergroup listed if only one map is given). More precisely, after appropriately conjugating  $G'$  satisfying  $G \subsetneq G'$ , the morphism  $X_G \rightarrow X_{G'}$  is given by the rational function  $F(t) \in \mathbb{Q}(t)$ , i.e., the curves  $X_G$  and  $X_{G'}$  have function fields  $\mathbb{Q}(t)$  and  $\mathbb{Q}(u)$ , respectively, and are related by the equation  $u = F(t)$ . By composing these rational maps down to  $X_{\mathrm{GL}_2(\hat{\mathbb{Z}})} = \mathbb{P}_{\mathbb{Q}}^1$  (i.e., to the group labeled 1A<sup>0</sup>-1a), we obtain from  $G$  a rational function

$$J(t) \in \mathbb{Q}(t);$$

in Section 5, we showed that  $J(t)$  describes the morphism  $\pi_G$  from  $X_G$  to the  $j$ -line. If  $J'(t) \in \mathbb{Q}(t)$  is the rational function arising from  $G'$  in the same manner, then we will have  $J(t) = J'(F(t))$ . The function  $J(t)$  is independent of any choice of supergroups.

Similar remarks and conventions hold when  $G$  has genus 1.

#	label	$i$	$N$	generators	map	sup
0	1A <sup>0</sup> -1a	1	1			
1	3A <sup>0</sup> -3a	3	3	$(\frac{0}{2} \frac{1}{0}), (\frac{1}{1} \frac{1}{2}), (\frac{1}{0} \frac{0}{2})$	$t^3$	0
2	3B <sup>0</sup> -3a	4	3	$(\frac{0}{2} \frac{1}{1}), (\frac{1}{0} \frac{2}{2})$	$(t+3)^3(t+27)/t$	0
3	3C <sup>0</sup> -3a	6	3	$(\frac{0}{2} \frac{1}{0}), (\frac{1}{0} \frac{0}{2})$	$(t-9)(t+3)/t$	1
4	3D <sup>0</sup> -3a	12	3	$(\frac{0}{0} \frac{2}{2}), (\frac{1}{0} \frac{0}{2})$	$729/(t^3 - 27)$ $-27(t-3)/(t^2 + 3t + 9)$	2 3
5	9A <sup>0</sup> -9a	9	9	$(\frac{0}{4} \frac{2}{0}), (\frac{1}{4} \frac{1}{5}), (\frac{1}{0} \frac{0}{2})$	$t^3 + 9t - 6$	1
6	9B <sup>0</sup> -9a	12	9	$(\frac{1}{0} \frac{1}{1}), (\frac{2}{0} \frac{0}{5}), (\frac{1}{0} \frac{0}{2})$	$t(t^2 + 9t + 27)$	2
7	9C <sup>0</sup> -9a	12	9	$(\frac{2}{0} \frac{0}{5}), (\frac{4}{3} \frac{2}{4}), (\frac{1}{0} \frac{0}{2})$	$t^3$	2
8	9D <sup>0</sup> -9a	18	9	$(\frac{2}{0} \frac{0}{5}), (\frac{1}{3} \frac{3}{1}), (\frac{0}{4} \frac{2}{0}), (\frac{1}{0} \frac{0}{2})$	$-27/t^3$ $(t^2 - 3)/t$	3 5
9	9E <sup>0</sup> -9a	18	9	$(\frac{1}{0} \frac{3}{1}), (\frac{2}{1} \frac{1}{1}), (\frac{4}{0} \frac{2}{5})$	$-9(t^3 + 3t^2 - 9t - 3)/(8t^3)$	1
10	9F <sup>0</sup> -9a	27	9	$(\frac{0}{4} \frac{2}{1}), (\frac{4}{5} \frac{3}{4}), (\frac{4}{0} \frac{5}{5})$	$\frac{3^7(t^2-1)^3(t^6+3t^5+6t^4+t^3-3t^2+12t+16)^3(2t^3+3t^2-3t-5)}{(t^3-3t-1)^9}$	0
11	9G <sup>0</sup> -9a	27	9	$(\frac{0}{2} \frac{4}{3}), (\frac{5}{1} \frac{1}{4}), (\frac{5}{0} \frac{3}{4})$	$\frac{(t^3-9t-12)(9-3t^3)(5t^3+18t^2+18t+3)}{(t^3+3t^2-3)^3}$	1
12	9H <sup>0</sup> -9a	36	9	$(\frac{1}{0} \frac{3}{1}), (\frac{5}{3} \frac{0}{2}), (\frac{1}{2} \frac{0}{2})$	$3(t^3 + 9)/t^3$ $3t/(2t^2 - 3t + 6)$	4 9
13	9H <sup>0</sup> -9b	36	9	$(\frac{1}{0} \frac{3}{1}), (\frac{5}{3} \frac{0}{2}), (\frac{2}{0} \frac{1}{1})$	$3(t^3 + 9t^2 - 9t - 9)/(t^3 - 9t^2 - 9t + 9)$	4
14	9H <sup>0</sup> -9c	36	9	$(\frac{1}{0} \frac{3}{1}), (\frac{5}{3} \frac{0}{2}), (\frac{4}{0} \frac{2}{5})$	$-6(t^3 - 9t)/(t^3 + 9t^2 - 9t - 9)$ $-(t^2 + 3)/(t^2 + 8t + 3)$	4 10
15	9I <sup>0</sup> -9a	36	9	$(\frac{2}{0} \frac{1}{5}), (\frac{1}{3} \frac{2}{2})$	$-6(t^3 - 9t)/(t^3 - 3t^2 - 9t + 3)$	6
16	9I <sup>0</sup> -9b	36	9	$(\frac{2}{0} \frac{1}{5}), (\frac{4}{3} \frac{0}{0})$	$-3(t^3 + 9t^2 - 9t - 9)/(t^3 + 3t^2 - 9t - 3)$	6
17	9I <sup>0</sup> -9c	36	9	$(\frac{2}{0} \frac{2}{5}), (\frac{2}{3} \frac{2}{1})$	$(t^3 - 6t^2 + 3t + 1)/(t^2 - t)$	6
18	9J <sup>0</sup> -9a	36	9	$(\frac{1}{0} \frac{3}{1}), (\frac{2}{3} \frac{2}{8}), (\frac{1}{0} \frac{2}{2})$	$(t^3 - 3t + 1)/(t^2 - t)$	7
19	9J <sup>0</sup> -9b	36	9	$(\frac{1}{0} \frac{3}{1}), (\frac{2}{3} \frac{2}{8}), (\frac{2}{0} \frac{1}{1})$	$-18(t^2 - 1)/(t^3 - 3t^2 - 9t + 3)$	7
20	9J <sup>0</sup> -9c	36	9	$(\frac{1}{0} \frac{3}{1}), (\frac{5}{3} \frac{2}{5}), (\frac{4}{0} \frac{0}{5})$	$3(t^3 + 3t^2 - 9t - 3)/(t^3 - 3t^2 - 9t + 3)$	7
21	27A <sup>0</sup> -27a	36	27	$(\frac{1}{0} \frac{1}{1}), (\frac{2}{9} \frac{1}{5}), (\frac{1}{3} \frac{2}{2})$	$t^3$	6

TABLE 1. Genus zero groups of 3-power level.

label	$i$	$N$	generators	map	supergroup
1A <sup>0</sup> -1a	1	1			
5A <sup>0</sup> -5a	5	5	$(\begin{smallmatrix} 2 & 1 \\ 0 & 3 \end{smallmatrix}), (\begin{smallmatrix} 1 & 2 \\ 2 & 0 \end{smallmatrix}), (\begin{smallmatrix} 1 & 1 \\ 0 & 2 \end{smallmatrix})$	$t^3(t^2 + 5t + 40)$	1A <sup>0</sup> -1a
5B <sup>0</sup> -5a	6	5	$(\begin{smallmatrix} 2 & 0 \\ 0 & 3 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix})$	$(t^2 + 10t + 5)^3 / t$	1A <sup>0</sup> -1a
5C <sup>0</sup> -5a	10	5	$(\begin{smallmatrix} 3 & 1 \\ 0 & 2 \end{smallmatrix}), (\begin{smallmatrix} 1 & 2 \\ 2 & 0 \end{smallmatrix}), (\begin{smallmatrix} 2 & 2 \\ 2 & 1 \end{smallmatrix})$	$8000t^3(t+1)(t^2 - 5t + 10)^3 / (t^2 - 5)^5$	1A <sup>0</sup> -1a
5D <sup>0</sup> -5a	12	5	$(\begin{smallmatrix} 4 & 0 \\ 1 & 4 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix})$	$125t / (t^2 - 11t - 1)$	5B <sup>0</sup> -5a
5D <sup>0</sup> -5b	12	5	$(\begin{smallmatrix} 4 & 0 \\ 1 & 4 \end{smallmatrix}), (\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix})$	$(t^2 - 11t - 1) / t$	5B <sup>0</sup> -5a
5E <sup>0</sup> -5a	15	5	$(\begin{smallmatrix} 2 & 1 \\ 0 & 3 \end{smallmatrix}), (\begin{smallmatrix} 2 & 0 \\ 2 & 3 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 2 & 2 \end{smallmatrix})$	$(t+5)(t^2 - 5) / (t^2 + 5t + 5)$	5A <sup>0</sup> -5a
5G <sup>0</sup> -5a	30	5	$(\begin{smallmatrix} 3 & 1 \\ 0 & 2 \end{smallmatrix}), (\begin{smallmatrix} 2 & 1 \\ 0 & 1 \end{smallmatrix})$	$125 / (t(t^4 + 5t^3 + 15t^2 + 25t + 25))$ $(t^2 + 5) / t$	5E <sup>0</sup> -5a 5B <sup>0</sup> -5a
5G <sup>0</sup> -5b	30	5	$(\begin{smallmatrix} 3 & 1 \\ 0 & 2 \end{smallmatrix}), (\begin{smallmatrix} 2 & 1 \\ 3 & 3 \end{smallmatrix})$	$-t(t^2 + 5t + 10) / (t^3 + 5t^2 + 10t + 10)$ $-5(t^2 + 4t + 5) / (t^2 + 5t + 5)$	5C <sup>0</sup> -5a 5E <sup>0</sup> -5a
5H <sup>0</sup> -5a	60	5	$(\begin{smallmatrix} 4 & 0 \\ 0 & 4 \end{smallmatrix}), (\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix})$	$-1 / t^5$ $\frac{-(t^4 - 2t^3 + 4t^2 - 3t + 1)}{t(t^4 + 3t^3 + 4t^2 + 2t + 1)}$ $5t / (t^2 - t - 1)$	5D <sup>0</sup> -5a 5D <sup>0</sup> -5b 5G <sup>0</sup> -5a
25A <sup>0</sup> -25a	30	25	$(\begin{smallmatrix} 2 & 2 \\ 0 & 13 \end{smallmatrix}), (\begin{smallmatrix} 4 & 1 \\ 3 & 1 \end{smallmatrix}), (\begin{smallmatrix} 2 & 3 \\ 0 & 6 \end{smallmatrix})$	$(t-1)(t^4 + t^3 + 6t^2 + 6t + 11)$	5B <sup>0</sup> -5a
25B <sup>0</sup> -25a	60	25	$(\begin{smallmatrix} 9 & 10 \\ 0 & 14 \end{smallmatrix}), (\begin{smallmatrix} 0 & 7 \\ 7 & 2 \end{smallmatrix}), (\begin{smallmatrix} 2 & 8 \\ 0 & 1 \end{smallmatrix})$	$-t^5$	5D <sup>0</sup> -5b
25B <sup>0</sup> -25b	60	25	$(\begin{smallmatrix} 9 & 10 \\ 0 & 14 \end{smallmatrix}), (\begin{smallmatrix} 0 & 7 \\ 7 & 2 \end{smallmatrix}), (\begin{smallmatrix} 4 & 1 \\ 0 & 7 \end{smallmatrix})$	$(1 - t^2) / t$ $\frac{-(t^4 - 2t^3 + 4t^2 - 3t + 1)}{t(t^4 + 3t^3 + 4t^2 + 2t + 1)}$ $(t^2 + 4t - 1) / (t^2 - t - 1)$	25A <sup>0</sup> -25a 5D <sup>0</sup> -5a 25A <sup>0</sup> -25a
7B <sup>0</sup> -7a	8	7	$(\begin{smallmatrix} 2 & 0 \\ 0 & 4 \end{smallmatrix}), (\begin{smallmatrix} 3 & 0 \\ 1 & 5 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 3 \end{smallmatrix})$	$(t^2 + 5t + 1)^3(t^2 + 13t + 49) / t$	1A <sup>0</sup> -1a
7D <sup>0</sup> -7a	21	7	$(\begin{smallmatrix} 0 & 3 \\ 2 & 3 \end{smallmatrix}), (\begin{smallmatrix} 2 & 4 \\ 4 & 5 \end{smallmatrix}), (\begin{smallmatrix} 3 & 1 \\ 0 & 4 \end{smallmatrix})$	$\frac{(2t-1)^3(t^2-t+2)^3(2t^2+5t+4)^3(5t^2+2t-4)^3}{(t^3+2t^2-t-1)^7}$	1A <sup>0</sup> -1a
7E <sup>0</sup> -7a	24	7	$(\begin{smallmatrix} 6 & 0 \\ 1 & 6 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 3 \end{smallmatrix})$	$49(t^2 - t) / (t^3 - 8t^2 + 5t + 1)$	7B <sup>0</sup> -7a
7E <sup>0</sup> -7b	24	7	$(\begin{smallmatrix} 6 & 0 \\ 1 & 6 \end{smallmatrix}), (\begin{smallmatrix} 3 & 0 \\ 0 & 1 \end{smallmatrix})$	$(t^3 - 8t^2 + 5t + 1) / (t^2 - t)$	7B <sup>0</sup> -7a
7E <sup>0</sup> -7c	24	7	$(\begin{smallmatrix} 6 & 0 \\ 1 & 6 \end{smallmatrix}), (\begin{smallmatrix} 3 & 0 \\ 0 & 4 \end{smallmatrix})$	$-7(t^3 - 2t^2 - t + 1) / (t^3 - t^2 - 2t + 1)$	7B <sup>0</sup> -7a
7F <sup>0</sup> -7a	28	7	$(\begin{smallmatrix} 3 & 1 \\ 4 & 4 \end{smallmatrix}), (\begin{smallmatrix} 4 & 4 \\ 1 & 3 \end{smallmatrix}), (\begin{smallmatrix} 3 & 4 \\ 0 & 4 \end{smallmatrix})$	$\frac{t(t+1)^3(t^2-5t+1)^3(t^2-5t+8)^3(t^4-5t^3+8t^2-7t+7)^3}{(t^3-4t^2+3t+1)^7}$	1A <sup>0</sup> -1a
13A <sup>0</sup> -13a	14	13	$(\begin{smallmatrix} 2 & 0 \\ 0 & 7 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix})$	$(t^2 + 5t + 13)(t^4 + 7t^3 + 20t^2 + 19t + 1)^3 / t$	1A <sup>0</sup> -1a
13B <sup>0</sup> -13a	28	13	$(\begin{smallmatrix} 3 & 0 \\ 0 & 9 \end{smallmatrix}), (\begin{smallmatrix} 4 & 0 \\ 1 & 10 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix})$	$13t / (t^2 - 3t - 1)$	13A <sup>0</sup> -13a
13B <sup>0</sup> -13b	28	13	$(\begin{smallmatrix} 3 & 0 \\ 0 & 9 \end{smallmatrix}), (\begin{smallmatrix} 4 & 0 \\ 1 & 10 \end{smallmatrix}), (\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix})$	$(t^2 - 3t - 1) / t$	13A <sup>0</sup> -13a
13C <sup>0</sup> -13a	42	13	$(\begin{smallmatrix} 5 & 0 \\ 0 & 8 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix})$	$13(t^2 - t) / (t^3 - 4t^2 + t + 1)$	13A <sup>0</sup> -13a
13C <sup>0</sup> -13b	42	13	$(\begin{smallmatrix} 5 & 0 \\ 0 & 8 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}), (\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix})$	$(t^3 - 4t^2 + t + 1) / (t^2 - t)$	13A <sup>0</sup> -13a
13C <sup>0</sup> -13c	42	13	$(\begin{smallmatrix} 5 & 0 \\ 0 & 8 \end{smallmatrix}), (\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}), (\begin{smallmatrix} 2 & 0 \\ 0 & 3 \end{smallmatrix})$	$-(5t^3 - 7t^2 - 8t + 5) / (t^3 - 4t^2 + t + 1)$	13A <sup>0</sup> -13a

TABLE 2. Genus zero groups of  $\ell$ -power level for primes  $\ell > 3$ .



label	$i$	$N$	generators	map	supergroups
8B <sup>0</sup> -8a	12	8	$(\frac{3}{0} \ 0), (\frac{0}{3} \ 3), (\frac{1}{2} \ 2), (\frac{1}{0} \ 0), (\frac{1}{0} \ 5)$	$16t^2$	4C <sup>0</sup> -4a
8B <sup>0</sup> -8b	12	8	$(\frac{3}{0} \ 0), (\frac{0}{3} \ 3), (\frac{1}{2} \ 2), (\frac{1}{0} \ 0), (\frac{1}{0} \ 4)$	$32t^2$	4C <sup>0</sup> -4a
8B <sup>0</sup> -8c	12	8	$(\frac{3}{0} \ 0), (\frac{0}{3} \ 3), (\frac{1}{2} \ 2), (\frac{3}{0} \ 1), (\frac{1}{0} \ 0)$	$32t^2$	4C <sup>0</sup> -4b
8B <sup>0</sup> -8d	12	8	$(\frac{3}{0} \ 0), (\frac{0}{3} \ 3), (\frac{1}{2} \ 2), (\frac{3}{0} \ 1), (\frac{1}{0} \ 5)$	$16t^2$	4C <sup>0</sup> -4b
8C <sup>0</sup> -8a	12	8	$(\frac{3}{0} \ 0), (\frac{0}{5} \ 5), (\frac{0}{3} \ 3), (\frac{1}{0} \ 3), (\frac{1}{0} \ 5)$	$-8t^2$	4B <sup>0</sup> -4b
8C <sup>0</sup> -8b	12	8	$(\frac{3}{0} \ 0), (\frac{0}{5} \ 5), (\frac{0}{3} \ 3), (\frac{1}{0} \ 2), (\frac{1}{0} \ 4)$	$-4(t^2 + 4)$	4B <sup>0</sup> -4b
8C <sup>0</sup> -8c	12	8	$(\frac{3}{0} \ 0), (\frac{0}{5} \ 5), (\frac{0}{5} \ 2), (\frac{3}{0} \ 1), (\frac{1}{0} \ 5)$	$-8(t^2 + 2)$	4B <sup>0</sup> -4b
8C <sup>0</sup> -8d	12	8	$(\frac{3}{0} \ 0), (\frac{0}{5} \ 5), (\frac{0}{5} \ 2), (\frac{3}{2} \ 1), (\frac{1}{0} \ 5)$	$-t^2$	4B <sup>0</sup> -4b
8D <sup>0</sup> -8a	12	8	$(\frac{2}{3} \ 1), (\frac{0}{3} \ 3), (\frac{1}{0} \ 0), (\frac{1}{0} \ 5)$	$16/(t^2 - 2)$	4C <sup>0</sup> -4a
8D <sup>0</sup> -8b	12	8	$(\frac{2}{3} \ 1), (\frac{0}{3} \ 3), (\frac{1}{0} \ 0), (\frac{1}{0} \ 4)$	$32/(t^2 + 4)$	4C <sup>0</sup> -4a
8D <sup>0</sup> -8c	12	8	$(\frac{2}{3} \ 1), (\frac{0}{3} \ 3), (\frac{1}{4} \ 4), (\frac{1}{0} \ 0)$	$16/(t^2 + 2)$	4C <sup>0</sup> -4a
8D <sup>0</sup> -8d	12	8	$(\frac{2}{3} \ 1), (\frac{0}{3} \ 3), (\frac{1}{0} \ 4), (\frac{1}{0} \ 4)$	$32/(t^2 - 4)$	4C <sup>0</sup> -4a
8E <sup>0</sup> -16a	16	16	$(\frac{3}{0} \ 4), (\frac{2}{3} \ 3), (\frac{3}{0} \ 3), (\frac{0}{3} \ 1)$	$-4t/(t^2 + 2)$	4D <sup>0</sup> -8a
8E <sup>0</sup> -16b	16	16	$(\frac{3}{0} \ 4), (\frac{2}{3} \ 3), (\frac{3}{0} \ 3), (\frac{0}{1} \ 0)$	$-2(t^2 - 2t + 2)/(t^2 - 4t + 2)$	4D <sup>0</sup> -8a
8F <sup>0</sup> -8a	16	8	$(\frac{1}{1} \ 2), (\frac{0}{5} \ 0), (\frac{3}{0} \ 3), (\frac{2}{1} \ 1)$	$8(t^4 - 4t^2 - 8t - 4)/(t^2 - 2)^2$	4A <sup>0</sup> -4a
8G <sup>0</sup> -16a	24	16	$(\frac{1}{0} \ 2), (\frac{7}{0} \ 7), (\frac{3}{0} \ 11), (\frac{1}{8} \ 1), (\frac{3}{0} \ 1), (\frac{1}{2} \ 1)$	$(1 - t^2)/(2t)$	4E <sup>0</sup> -8e
8G <sup>0</sup> -8a	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{1}{0} \ 3), (\frac{1}{0} \ 5)$	$(t^2 - 1)/t$	8C <sup>0</sup> -8b, 8C <sup>0</sup> -8d, 4E <sup>0</sup> -4c
8G <sup>0</sup> -8b	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{1}{0} \ 3), (\frac{1}{4} \ 5)$	$(t^2 + 2)/(2t)$	8C <sup>0</sup> -8a, 8C <sup>0</sup> -8c, 4E <sup>0</sup> -4c
8G <sup>0</sup> -8c	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{1}{0} \ 3), (\frac{1}{0} \ 5)$	$(t^2 - 2)/(2t)$	4E <sup>0</sup> -8c, 8C <sup>0</sup> -8a, 8C <sup>0</sup> -8d
8G <sup>0</sup> -8d	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{1}{0} \ 5), (\frac{3}{2} \ 1)$	$t^2/2$	4E <sup>0</sup> -4b, 8D <sup>0</sup> -8d, 8D <sup>0</sup> -8b
8G <sup>0</sup> -8e	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{1}{2} \ 3), (\frac{3}{2} \ 1)$	$t^2$	4E <sup>0</sup> -4b, 8D <sup>0</sup> -8a, 8D <sup>0</sup> -8c
8G <sup>0</sup> -8f	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{1}{0} \ 3), (\frac{1}{0} \ 5)$	$(t^2 - 1)/(2t)$	4E <sup>0</sup> -4a, 8C <sup>0</sup> -8d, 8D <sup>0</sup> -8d
8G <sup>0</sup> -8g	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{1}{0} \ 3), (\frac{1}{4} \ 1)$	$(t^2 + 2)/(2t)$	4E <sup>0</sup> -8i, 8C <sup>0</sup> -8c, 8C <sup>0</sup> -8d
8G <sup>0</sup> -8h	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{1}{0} \ 3), (\frac{3}{2} \ 1)$	$2(t^2 + 1)/(t^2 + 2t - 1)$	4E <sup>0</sup> -8g, 8D <sup>0</sup> -8d, 8D <sup>0</sup> -8c
8G <sup>0</sup> -8i	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{1}{2} \ 3), (\frac{3}{2} \ 1)$	$(t^2 + 2)/(2t)$	4E <sup>0</sup> -8g, 8D <sup>0</sup> -8b, 8D <sup>0</sup> -8a
8G <sup>0</sup> -8j	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{3}{0} \ 5), (\frac{1}{4} \ 1)$	$2(t^2 + 1)/(t^2 - 2t - 1)$	4E <sup>0</sup> -8i, 8C <sup>0</sup> -8a, 8C <sup>0</sup> -8b
8G <sup>0</sup> -8k	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{3}{0} \ 5), (\frac{1}{4} \ 5)$	$2(t^2 - 1)/(t^2 + 1)$	8D <sup>0</sup> -8a, 8C <sup>0</sup> -8a, 4E <sup>0</sup> -4a
8G <sup>0</sup> -8l	24	8	$(\frac{1}{0} \ 1), (\frac{3}{0} \ 3), (\frac{5}{0} \ 0), (\frac{3}{1} \ 1), (\frac{3}{2} \ 1)$	$(t^2 - 2)/(2t)$	4E <sup>0</sup> -8a, 8D <sup>0</sup> -8d, 8D <sup>0</sup> -8a
8H <sup>0</sup> -8a	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 3), (\frac{1}{0} \ 0), (\frac{1}{0} \ 5)$	$2t/(t^2 - 2)$	8B <sup>0</sup> -8a, 8D <sup>0</sup> -8a, 4F <sup>0</sup> -4a
8H <sup>0</sup> -8b	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{0}{3} \ 3), (\frac{1}{2} \ 3)$	$4t/(t^2 - 2)$	4F <sup>0</sup> -8a, 8D <sup>0</sup> -8a, 8B <sup>0</sup> -8b
8H <sup>0</sup> -8c	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{1}{0} \ 3), (\frac{1}{0} \ 5)$	$t/(t^2 + 1)$	8B <sup>0</sup> -8b, 8D <sup>0</sup> -8b, 4F <sup>0</sup> -4a
8H <sup>0</sup> -8d	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{1}{0} \ 5), (\frac{1}{2} \ 2)$	$(t^2 + 1)/(t^2 + 2t - 1)$	8B <sup>0</sup> -8a, 8D <sup>0</sup> -8a, 4F <sup>0</sup> -4b
8H <sup>0</sup> -8e	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{1}{0} \ 3), (\frac{1}{0} \ 5)$	$2t/(t^2 + 2)$	8B <sup>0</sup> -8a, 8D <sup>0</sup> -8c, 4F <sup>0</sup> -4a
8H <sup>0</sup> -8f	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{1}{0} \ 3), (\frac{1}{2} \ 1)$	$(t^2 - 2)/(2t)$	4F <sup>0</sup> -8a, 8B <sup>0</sup> -8a, 8D <sup>0</sup> -8d
8H <sup>0</sup> -8g	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{1}{0} \ 4), (\frac{1}{0} \ 5)$	$t/(t^2 - 1)$	8B <sup>0</sup> -8b, 8D <sup>0</sup> -8d, 4F <sup>0</sup> -4a
8H <sup>0</sup> -8h	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{1}{0} \ 4), (\frac{2}{1} \ 1)$	$(t^2 - 1)/(2t)$	4F <sup>0</sup> -4b, 8B <sup>0</sup> -8b, 8D <sup>0</sup> -8d
8H <sup>0</sup> -8i	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{3}{0} \ 5), (\frac{1}{2} \ 1)$	$(t^2 + 2)/(2t)$	4F <sup>0</sup> -8b, 8B <sup>0</sup> -8a, 8D <sup>0</sup> -8d
8H <sup>0</sup> -8j	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{3}{0} \ 5), (\frac{2}{5} \ 2)$	$2(t^2 + 1)/(t^2 + 2t - 1)$	4F <sup>0</sup> -8b, 8D <sup>0</sup> -8a, 8B <sup>0</sup> -8b
8H <sup>0</sup> -8k	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{3}{4} \ 5), (\frac{1}{2} \ 2)$	$(t^2 + 2t - 1)/(t^2 + 1)$	4F <sup>0</sup> -8b, 8B <sup>0</sup> -8a, 8D <sup>0</sup> -8b
8H <sup>0</sup> -8l	24	8	$(\frac{3}{0} \ 0), (\frac{1}{4} \ 1), (\frac{0}{3} \ 0), (\frac{3}{4} \ 4), (\frac{1}{2} \ 2)$	$4t/(t^2 + 2)$	4F <sup>0</sup> -8b, 8D <sup>0</sup> -8c, 8B <sup>0</sup> -8b
8I <sup>0</sup> -8a	24	8	$(\frac{7}{0} \ 0), (\frac{0}{5} \ 2), (\frac{1}{0} \ 4), (\frac{1}{0} \ 6)$	$4t^2/(t^2 - 2)$	8C <sup>0</sup> -8d
8I <sup>0</sup> -8b	24	8	$(\frac{7}{0} \ 0), (\frac{0}{5} \ 2), (\frac{3}{0} \ 2), (\frac{1}{0} \ 4)$	$4t^2/(t^2 + 2)$	8C <sup>0</sup> -8d
8I <sup>0</sup> -8c	24	8	$(\frac{7}{0} \ 0), (\frac{0}{5} \ 2), (\frac{3}{0} \ 1), (\frac{5}{0} \ 4)$	$4/(t^2 - 1)$	8C <sup>0</sup> -8d
8I <sup>0</sup> -8d	24	8	$(\frac{7}{0} \ 0), (\frac{0}{5} \ 2), (\frac{5}{0} \ 3), (\frac{5}{0} \ 4)$	$4/(t^2 + 1)$	8C <sup>0</sup> -8d



TABLE 3. Genus 0 groups of 2-power level.

group	$i$	$N$	generators	curve	map	supergroups
16C <sup>1</sup> -16c	24	16	$(\frac{2}{3} \frac{1}{2}), (\frac{0}{5} \frac{3}{8}), (\frac{1}{0} \frac{0}{5}), (\frac{1}{0} \frac{3}{8})$	<a href="#">256a2</a>	$(x - 1)/2$	8D <sup>0</sup> -8a
16C <sup>1</sup> -16d	24	16	$(\frac{2}{3} \frac{1}{2}), (\frac{0}{5} \frac{3}{8}), (\frac{3}{0} \frac{0}{5}), (\frac{1}{0} \frac{8}{3})$	<a href="#">256a1</a>	$x + 1$	8D <sup>0</sup> -8a
16B <sup>1</sup> -16a	24	16	$(\frac{3}{0} \frac{0}{11}), (\frac{0}{5} \frac{3}{0}), (\frac{2}{9} \frac{3}{6}), (\frac{1}{0} \frac{3}{3}), (\frac{1}{0} \frac{5}{5})$	<a href="#">256b2</a>	$x/4$	8B <sup>0</sup> -8a
16B <sup>1</sup> -16c	24	16	$(\frac{3}{0} \frac{0}{11}), (\frac{0}{5} \frac{3}{0}), (\frac{2}{9} \frac{3}{6}), (\frac{1}{0} \frac{4}{3}), (\frac{1}{0} \frac{0}{5})$	<a href="#">256b1</a>	$x/2$	8B <sup>0</sup> -8a
16I <sup>1</sup> -16d	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{0}{5} \frac{3}{8}), (\frac{1}{12} \frac{4}{1}), (\frac{1}{0} \frac{0}{5}), (\frac{1}{0} \frac{8}{3})$	<a href="#">256a1</a>	$x - 1$	8H <sup>0</sup> -8a, 16E <sup>0</sup> -16c, 16C <sup>1</sup> -16c
16I <sup>1</sup> -16f	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{0}{5} \frac{3}{8}), (\frac{1}{12} \frac{4}{1}), (\frac{1}{0} \frac{8}{3}), (\frac{2}{5} \frac{3}{2})$	<a href="#">256a2</a>	$\frac{4x-4y+12}{x^2-2x-15}$	8H <sup>0</sup> -8b, 16E <sup>0</sup> -16c, 16C <sup>1</sup> -16d
16I <sup>1</sup> -16g	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{0}{5} \frac{3}{8}), (\frac{1}{12} \frac{4}{1}), (\frac{1}{0} \frac{8}{5}), (\frac{1}{10} \frac{2}{3})$	<a href="#">256a2</a>	$\frac{2(y-x-1)}{x^2-2x-11}$	8H <sup>0</sup> -8d, 16E <sup>0</sup> -16b, 16C <sup>1</sup> -16d
16I <sup>1</sup> -16h	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{0}{5} \frac{3}{8}), (\frac{1}{12} \frac{4}{1}), (\frac{1}{0} \frac{7}{7}), (\frac{1}{10} \frac{7}{2})$	<a href="#">256a1</a>	$1/x$	8H <sup>0</sup> -8j, 16E <sup>0</sup> -16b, 16C <sup>1</sup> -16c
16I <sup>1</sup> -16j	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{0}{5} \frac{3}{8}), (\frac{1}{12} \frac{4}{1}), (\frac{3}{0} \frac{0}{5}), (\frac{1}{0} \frac{8}{3})$	<a href="#">256a2</a>	$(x + 3)/2$	8H <sup>0</sup> -8a, 16E <sup>0</sup> -16d, 16C <sup>1</sup> -16d
16I <sup>1</sup> -16k	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{0}{5} \frac{3}{8}), (\frac{1}{12} \frac{4}{1}), (\frac{3}{0} \frac{0}{5}), (\frac{2}{5} \frac{3}{2})$	<a href="#">256a2</a>	$\frac{-x+1}{x+3}$	8H <sup>0</sup> -8j, 16E <sup>0</sup> -16a, 16C <sup>1</sup> -16d
8H <sup>1</sup> -16b	48	16	$(\frac{7}{0} \frac{0}{7}), (\frac{1}{0} \frac{8}{1}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{3}{0} \frac{0}{5}), (\frac{1}{2} \frac{2}{9})$	<a href="#">256a2</a>	$(x + 3)/2$	8H <sup>0</sup> -8i, 8K <sup>0</sup> -16c, 8D <sup>1</sup> -16b
8H <sup>1</sup> -16c	48	16	$(\frac{7}{0} \frac{0}{7}), (\frac{1}{0} \frac{8}{1}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{3}{0} \frac{0}{5}), (\frac{1}{6} \frac{2}{9})$	<a href="#">256a2</a>	$\frac{x-1}{x+3}$	8H <sup>0</sup> -8j, 8K <sup>0</sup> -16a, 8D <sup>1</sup> -16b
8H <sup>1</sup> -16e	48	16	$(\frac{7}{0} \frac{0}{7}), (\frac{1}{0} \frac{8}{1}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{7}{0} \frac{0}{9}), (\frac{1}{2} \frac{1}{2})$	<a href="#">256a1</a>	$x - 1$	8H <sup>0</sup> -8i, 8K <sup>0</sup> -16b, 8D <sup>1</sup> -16c
8H <sup>1</sup> -16g	48	16	$(\frac{7}{0} \frac{0}{7}), (\frac{1}{0} \frac{8}{1}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{7}{0} \frac{0}{9}), (\frac{2}{5} \frac{3}{2})$	<a href="#">256a1</a>	$-1/x$	8H <sup>0</sup> -8j, 8K <sup>0</sup> -16d, 8D <sup>1</sup> -16c
16D <sup>1</sup> -16d	24	16	$(\frac{3}{0} \frac{8}{11}), (\frac{0}{5} \frac{3}{0}), (\frac{5}{2} \frac{2}{1}), (\frac{3}{0} \frac{2}{5}), (\frac{5}{0} \frac{4}{1})$	<a href="#">128a1</a>	$(x + 1)/2$	8B <sup>0</sup> -8d
8H <sup>1</sup> -16j	48	16	$(\frac{7}{0} \frac{0}{7}), (\frac{1}{0} \frac{8}{1}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{7}{0} \frac{4}{9}), (\frac{1}{2} \frac{1}{2})$	<a href="#">256a2</a>	$\frac{2(x+y+1)}{x^2-2x-11}$	8H <sup>0</sup> -8k, 8K <sup>0</sup> -16d, 8D <sup>1</sup> -16b
8D <sup>1</sup> -16b	24	16	$(\frac{7}{0} \frac{0}{7}), (\frac{3}{0} \frac{4}{11}), (\frac{0}{5} \frac{3}{0}), (\frac{3}{0} \frac{5}{0}), (\frac{1}{2} \frac{2}{9})$	<a href="#">256a1</a>	$x + 1$	4F <sup>0</sup> -8b
8H <sup>1</sup> -16k	48	16	$(\frac{7}{0} \frac{0}{7}), (\frac{1}{0} \frac{8}{1}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{7}{0} \frac{4}{9}), (\frac{1}{6} \frac{2}{9})$	<a href="#">256a2</a>	$\frac{4(x-y+3)}{x^2-2x-15}$	8H <sup>0</sup> -8l, 8K <sup>0</sup> -16b, 8D <sup>1</sup> -16b
8D <sup>1</sup> -16c	24	16	$(\frac{7}{0} \frac{0}{7}), (\frac{3}{0} \frac{4}{11}), (\frac{0}{5} \frac{3}{0}), (\frac{3}{0} \frac{4}{5}), (\frac{1}{2} \frac{1}{2})$	<a href="#">256a2</a>	$(x - 1)/2$	4F <sup>0</sup> -8b
16J <sup>1</sup> -16e	48	16	$(\frac{7}{0} \frac{0}{7}), (\frac{0}{5} \frac{3}{0}), (\frac{5}{2} \frac{2}{1}), (\frac{1}{0} \frac{6}{7}), (\frac{7}{0} \frac{3}{3})$	<a href="#">128a2</a>	$\frac{x^2+2x-7}{8x-8}$	8B <sup>0</sup> -8d, 8L <sup>0</sup> -8a, 16B <sup>0</sup> -16b, 16D <sup>1</sup> -16d
16J <sup>1</sup> -16g	48	16	$(\frac{7}{0} \frac{0}{7}), (\frac{0}{5} \frac{3}{0}), (\frac{5}{2} \frac{2}{1}), (\frac{5}{0} \frac{4}{1}), (\frac{3}{0} \frac{6}{1})$	<a href="#">128a2</a>	$(1 - x)/2$	8L <sup>0</sup> -8b, 16B <sup>0</sup> -16d, 16D <sup>1</sup> -16d
16F <sup>1</sup> -16a	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{1}{0} \frac{3}{3}), (\frac{1}{0} \frac{5}{0})$	<a href="#">256b1</a>	$-2/x$	8H <sup>0</sup> -8a, 16B <sup>0</sup> -16a, 16B <sup>1</sup> -16a
16F <sup>1</sup> -16c	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{1}{0} \frac{5}{0}), (\frac{1}{2} \frac{2}{3})$	<a href="#">256b2</a>	$\frac{x^2+2y-8}{x^2-8x+8}$	16B <sup>0</sup> -16c, 8H <sup>0</sup> -8d, 16B <sup>1</sup> -16c
16F <sup>1</sup> -16d	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{1}{0} \frac{3}{3}), (\frac{1}{0} \frac{0}{5})$	<a href="#">256b2</a>	$4/x$	8H <sup>0</sup> -8e, 16B <sup>0</sup> -16c, 16B <sup>1</sup> -16c
16F <sup>1</sup> -16h	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{3}{0} \frac{0}{5}), (\frac{1}{2} \frac{1}{2})$	<a href="#">256b1</a>	$x$	8H <sup>0</sup> -8i, 16B <sup>0</sup> -16c, 16B <sup>1</sup> -16a
16F <sup>1</sup> -16j	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{3}{0} \frac{4}{1}), (\frac{1}{2} \frac{1}{2})$	<a href="#">256b2</a>	$x/2$	8H <sup>0</sup> -8f, 16B <sup>0</sup> -16a, 16B <sup>1</sup> -16c
16F <sup>1</sup> -16k	48	16	$(\frac{3}{0} \frac{0}{11}), (\frac{1}{4} \frac{4}{1}), (\frac{0}{5} \frac{3}{0}), (\frac{3}{0} \frac{4}{5}), (\frac{1}{2} \frac{1}{2})$	<a href="#">256b2</a>	$\frac{x^2+2y-8}{x^2+4x+8}$	16B <sup>0</sup> -16a, 8H <sup>0</sup> -8k, 16B <sup>1</sup> -16c
11C <sup>1</sup> -11a	55	11	$(\frac{3}{4} \frac{4}{2}), (\frac{3}{1} \frac{1}{8}), (\frac{6}{0} \frac{4}{5})$	<a href="#">121b1</a>	$J(x, y)$	1A <sup>0</sup> -1a

TABLE 4. Genus 1 groups  $G$  of prime power level for which  $X_G(\mathbb{Q})$  is infinite.

label	Weierstrass model
<a href="#">121b1</a>	$y^2 + y = x^3 - x^2 - 7x + 10$
<a href="#">128a1</a>	$y^2 = x^3 + x^2 + x + 1$
<a href="#">128a2</a>	$y^2 = x^3 + x^2 - 9x + 7$
<a href="#">256a1</a>	$y^2 = x^3 + x^2 - 3x + 1$
<a href="#">256a2</a>	$y^2 = x^3 + x^2 - 13x - 21$
<a href="#">256b1</a>	$y^2 = x^3 - 2x$
<a href="#">256b2</a>	$y^2 = x^3 + 8x$

TABLE 5. Some elliptic curves.

$$J(x, y) := \frac{(f_1 f_2 f_3 f_4)^3}{f_5^2 f_6^{11}},$$

$$f_1 = x^2 + 3x - 6, \quad f_2 = 11(x^2 - 5)y + (2x^4 + 23x^3 - 72x^2 - 28x + 127),$$

$$f_3 = 6y + 11x - 19, \quad f_4 = 22(x - 2)y + (5x^3 + 17x^2 - 112x + 120),$$

$$f_5 = 11y + (2x^2 + 17x - 34), \quad f_6 = (x - 4)y - (5x - 9).$$