Correction to ‘New topologically slice knots’

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In Figure 1.5 of [1] we gave an incorrect example for Theorem 1.3. In this note we present a correct example.

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We first recall the satellite construction for knots. Let K, C be knots. Let $A \subset S^3 \setminus K$ be a curve, unknotted in S^3. Then $S^3 \setminus \nu A$ is a solid torus. Now let $\psi : \partial(\overline{\nu A}) \to \partial(\overline{\nu C})$ be a diffeomorphism which sends a meridian of A to a longitude of C, and a longitude of A to a meridian of C. The space

$$(S^3 \setminus \nu A) \cup_\psi (S^3 \setminus \nu C)$$

is a 3-sphere and the image of K is denoted by $S = S(K, C, A)$. We say S is the satellite knot with companion C, orbit K and axis A. Note that by doing this construction we replaced a tubular neighborhood of C by a knot in a solid torus, namely $K \subset S^3 \setminus \nu A$.

We now consider the knot K in Figure 1. Note that K is the knot 6_1. K clearly bounds an immersed band; pushing this band into D^4 we can resolve the singularities to get a smooth slice disk D for K.

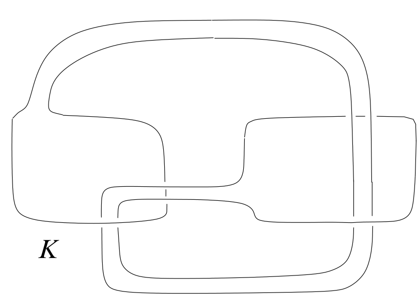

Figure 1: The knot K

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It follows from [1, Proposition 7.4] that if $A \subset S^3 \setminus K$ is a curve such that $[A]$ represents an element in $\ker\{\pi_1(S^3 \setminus K) \to \pi_1(D^4 \setminus D)\}$, then $S(K, C, A)$ is topologically slice. We recall that $\pi_1(D^4 \setminus D)$ is isomorphic to the semi–direct product

$$\langle a, c \mid aca^{-1} = c^2 \rangle \cong \mathbb{Z} \ltimes \mathbb{Z}[1/2].$$

Here the generator a of $\mathbb{Z}$ acts on the normal subgroup $\mathbb{Z}[1/2]$ via multiplication by 2. In [1, Figure 1.5] we proposed a curve A and claimed that it represents the trivial element in $\pi_1(D^4 \setminus D) \cong \mathbb{Z} \ltimes \mathbb{Z}[1/2]$. Unfortunately we miscalculated the image of A in $\mathbb{Z} \ltimes \mathbb{Z}[1/2]$. In fact this A represents a non–trivial element in $\pi_1(D^4 \setminus D)$. Hence the curve A of [1, Figure 1.5] does not give an example for [1, Proposition 7.4]. We now present a correct example.

Perhaps the first example of a pair K, A which satisfies the above conditions which comes to mind is to take K, A which form a slice link $K \cup A$. But it is easy to see that the null–concordance from $K \cup A$ to a trivial link $K' \cup A'$ induces a concordance of $S(K, C, A)$ to $S(K', C, A')$. But clearly $S(K', C, A')$ is the trivial link. This shows that in this case $S(K, C, A)$ is slice. We therefore have to find examples of K, A such that $K \cup A$ is not slice.

Now let A be the simple closed curve of Figure 2. Since $D \cap S^3 = K$ we can resolve

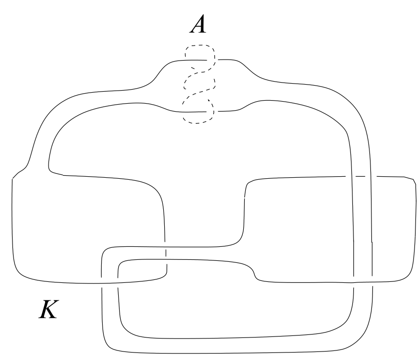

Figure 2: The knot K and the curve A

the crossings of A using a homotopy in $S^3 \setminus K \subset D^4 \setminus D$. We get a curve without crossings which is a meridian for the band. Now we push this curve into D^4 'beyond D' and then we can contract this curve. This shows that A is null–homotopic in $D^4 \setminus D$. A straightforward calculation shows that the Alexander polynomial of the link $K \cup A$ is non–trivial, hence the link $K \cup A$ is not slice by Kawauchi [2].

Finally we point out that by untwisting A (and therefore twisting K) as in Figure 3 we get a diagram of K in a 'planar' torus. Wrapping this torus around a knot C gives immediately a diagram for $S(K, C, A)$. For example if we take C to be the figure-8

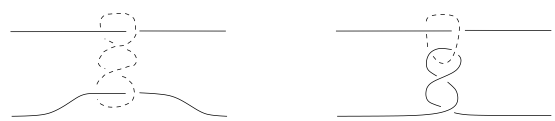

Figure 3: Untwisting A.

knot we get the diagram in Figure 4 with 26 crossings.

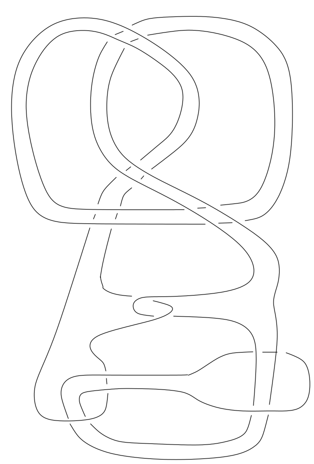

Figure 4: Satellite knot of the figure-8 knot.

We point out that in general if C has a diagram with crossing number c and writhe w, then $S(K, C, A)$ has clearly a diagram of crossing number $4c + 2|w| + 10$. This is significantly lower than the crossing number for the (incorrect) example of A given in [1, Figure 1.5] and will hopefully put our examples within reach of Rasmussen's s–invariant.

References

[1] **S Friedl**, **P Teichner**, *New topologically slice knots*, Geom. Topol. 9 (2005) 2129–2158 MR2209368

[2] **A Kawauchi**, *On the Alexander polynomials of cobordant links*, Osaka J. Math. 15 (1978) 151–159 MR0488022

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