Erratum to "Geometry of contact transformations and domains: orderability versus squeezing"

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The purpose of this erratum is to correct a number of inconsistencies in our paper [1]. These are related to the grading of generalized Floer homology and do not affect formulations and proofs of the main results of the paper. The source of the mistakes in the grading is as follows. Consider the framed Hamiltonian structure

$$(V = M \times S^1, \Theta = -\alpha + H_t dt, \lambda = dt)$$

associated to a Hamiltonian function $H : M \times S^1 \to \mathbb{R}$ on an exact symplectic manifold $(M, \omega = d\alpha)$, as in Example 4.2.2. The symplectic form $\Omega = d\Theta$ on the cut of this Hamiltonian structure equals the form $-\omega$ (mind the minus sign). In the proof of Proposition 4.37 we claim that "the change of the sign of the symplectic form does not affect Conley–Zehnder indices", which is wrong for the definition of Conley–Zehnder indices as presented in Section 4.4.1. In order to correct this, the following changes should be made in paper:

LIST OF CORRECTIONS

Page 1677, line 13: replace $2k$ by $-2k$.

Page 1677, line -9: replace $g_t R_*^{tT} g_0^{-1}$ by $(g_t R_*^{tT} g_0^{-1})^{-1}$.

Page 1698, line 10: Add the following text: Let us emphasize that in the usual Floer homology the CZ–index of the orbit is defined through the linearization of the Hamiltonian flow along the orbit. Furthermore, the action of an integer $k \in \mathbb{Z}$ on the set of coherent symplectic trivializations corresponds to the twisting by a loop of symplectic matrices with the Maslov index $+2k$. Thus, informally speaking, the gradings of closed orbits in the generalized Floer homology for stable Hamiltonian structures and for the usual Hamiltonian Floer homology are defined in the "opposite way".

Page 1699, line -11: Replace "since the change of the sign of the symplectic form does not affect Conley–Zehnder indices" to "due to our conventions on the Conley–Zehnder index for generalized and usual Floer homologies".

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With these corrections, the rest of the paper remains unchanged.

Let us emphasize also that in Theorem 5.7 the isomorphism L_r between symplectic homology of a tube in T^*X and cohomology of the corresponding sublevel set of the energy functional on the loop space $\mathcal{L}X$ does not preserve grading. It follows from Salamon and Weber [3] that if the manifold X is orientable and the degrees of non-contractible orbits are determined by the vertical Lagrangian subbundle of $T(T^*X)$, the isomorphism L_r sends $\mathsf{SH}_k^{(-\infty,-1)}(U_r)$ to $H^{n-k}(\mathcal{L}^{\frac{1}{2r^2}}X)$ where $n = \dim X$.

Let us also point out that our definition of the Conley–Zehnder index for paths of symplectic matrices in $\mathbb{R}^{2n}$ differs from the one defined by Robbin and Salamon in [2]: the sum of two indices equals n.

References

[1] **Y Eliashberg**, **S S Kim**, **L Polterovich**, *Geometry of contact transformations and domains: orderability versus squeezing*, Geom. Topol. 10 (2006) 1635–1747 MR2284048

[2] **J Robbin**, **D Salamon**, *The Maslov index for paths*, Topology 32 (1993) 827–844 MR1241874

[3] **D A Salamon**, **J Weber**, *Floer homology and the heat flow*, Geom. Funct. Anal. 16 (2006) 1050–1138 MR2276534

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