Erratum for the paper 'On the chain-level intersection pairing for PL manifolds'

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Greg Friedman has pointed out that there are sign errors in [3], and in particular Lemma 10.5(b) (which is a key step in the proof of the main theorem) is not correct as stated.

The purpose of this note is to provide a correction.

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In Section 5 of [3], the umkehr map

$$H_p(A,B) \to H_{p+n-m}(A',B')$$

should have a sign $(-1)^{(m-p)(n-m)}$ as in [1, pages 314–315] (see [2] for an explanation of where this sign comes from). Also, it's convenient to let the symbol $f_!$ stand for the desuspension

$$\Sigma^{-m} H_*(A,B) \to \Sigma^{-n} H_*(A',B').$$

Note that this map preserves degrees.

With these changes, Lemma 5.1 says that the diagram

$$\begin{tikzcd} \Sigma^{-m} H_*(A,B) \arrow[r, "f_!"] \arrow[d, "\partial"'] & \Sigma^{-n} H_*(A',B') \arrow[d, "\partial"] \\ \Sigma^{-m} H_*(B) \arrow[r, "f_!"] & \Sigma^{-n} H_*(B') \end{tikzcd}$$

commutes.

In Section 8, observe that if C_* and D_* are chain complexes and $m,n \in \mathbb{Z}$ then $(\Sigma^{-m} C_*) \otimes (\Sigma^{-n} D_*)$ and $\Sigma^{-(m+n)}(C_* \otimes D_*)$ will be *different* chain complexes when n is odd: they are the same as graded abelian groups but have different differentials. However, there is an isomorphism

$$\Theta : (\Sigma^{-m} C_*) \otimes (\Sigma^{-n} D_*) \to \Sigma^{-(m+n)}(C_* \otimes D_*)$$

which takes $\Sigma^{-m} x \otimes \Sigma^{-n} y$ to $(-1)^{n|x|} \Sigma^{-(m+n)}(x \otimes y)$, and similarly for any number of tensor factors.

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Given manifolds M_1 and M_2 of dimensions m_1, m_2, define

$$\overline{\varepsilon}\colon (\Sigma^{-m_1} C_* M_1) \otimes (\Sigma^{-m_2} C_* M_2) \to \Sigma^{-(m_1+m_2)} C_*(M_1 \times M_2)$$

to be

$$(-1)^{m_1 m_2} (\Sigma^{-(m_1+m_2)} \varepsilon) \circ \Theta.$$

The motivation for this is that $\overline{\varepsilon}$ is Poincaré dual to the exterior product in cohomology (see [2] for details). Similarly, given $M_1, \dots, M_k$ define

$$\overline{\varepsilon}\colon \bigotimes \Sigma^{-m_i} C_* M_i \to \Sigma^{-\sum m_i} C_*(\textstyle\prod M_i)$$

to be

$$(-1)^{e_2(m_1,\dots,m_k)} (\Sigma^{-\sum m_i} \varepsilon) \circ \Theta$$

where e_2 is the second elementary symmetric function (so that, for example, $e_2(m_1, m_2, m_3) = m_1 m_2 + m_1 m_3 + m_2 m_3$).

Now define

$$G_2 \subset (\Sigma^{-m} C_* M) \otimes (\Sigma^{-m} C_* M)$$

to be $\overline{\varepsilon}^{-1}(\Sigma^{-2m} C_*^{\Delta}(M \times M))$ and define μ_2 to be $\Delta_! \circ \overline{\varepsilon}$.

With these changes, Remark 8.1 becomes correct (it wasn't before); see [2] for the proof.

In Section 10, Definition 10.1 should be restated: define G_k to be the subcomplex of $(\Sigma^{-m} C_* M)^{\otimes k}$ consisting of elements x for which both $\Sigma^{mk} \overline{\varepsilon}_k(x)$ and $\Sigma^{mk} \overline{\varepsilon}_k(\partial x)$ are in general position with respect to all generalized diagonal maps.

The diagram in Lemma 10.5(b) should be replaced by

$$\begin{tikzcd}
\Sigma^{-m_1} C_*^f M_1 \otimes \Sigma^{-m_2} C_*^f M_2 \arrow[r, "\overline{\varepsilon}"] \arrow[d, "f_! \otimes g_!"'] & \Sigma^{-(m_1+m+2)} C_*^{f \times g}(M_1 \times M_2) \arrow[d, "(f \times g)_!"] \\
\Sigma^{-n_1} C_*^f N_1 \otimes \Sigma^{-n_2} C_*^f N_2 \arrow[r, "\overline{\varepsilon}"] & \Sigma^{-(n_1+n+2)} C_*^{f \times g}(N_1 \times N_2)
\end{tikzcd}$$

At the end of Section 10, the definition of G_R should be

$$\overline{\varepsilon_{k'}}^{-1} \circ (R^*)_! \circ \overline{\varepsilon}_k.$$

In Section 11, Lemma 11.1 should say that the inclusion

$$G_{k+l} \hookrightarrow (\Sigma^{-m} C_* M)^{k+l} \cong (\Sigma^{-m} C_* M)^k \otimes (\Sigma^{-m} C_* M)^l$$

has its image in $G_k \otimes G_l$. Now define

$$\xi_{k,l} : G_{k+l} \to G_k \otimes G_l$$

to be the inclusion provided by Lemma 11.1.

References

[1] **A Dold**, *Lectures on algebraic topology*, Grundlehrenseries 200, Springer, New York (1972) MR0415602

[2] **G Friedman**, *On the chain-level intersection pairing for PL manifolds*, to appear arXiv:0808.1749

[3] **J E McClure**, *On the chain-level intersection pairing for PL manifolds*, Geom. Topol. 10 (2006) 1391–1424 MR2255502

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