



Geometry & Topology

Volume 28 (2024)

**Correction to the article
Bimodules in bordered Heegaard Floer homology**

ROBERT LIPSHITZ
PETER OZSVÁTH
DYLAN P THURSTON

Correction to the article Bimodules in bordered Heegaard Floer homology

ROBERT LIPSHITZ
PETER OZSVÁTH
DYLAN P THURSTON

We correct some errors in our earlier paper (Geom. Topol. 19 (2015) 525–724).

57K18, 57R58; 53D40

Grading refinement data In the first sentence of the proof of [2, Proposition 3.7] (this is Proposition 3.10 in the arXiv version), the definition of the $(G(\mathcal{Z}), G(\mathcal{Z}))$ -set T is wrong and, in particular, the elements $\psi(s) \cdot \psi'(s)^{-1}$ do not lie in T .

To correct this, for each $i = 0, \dots, 2k$, fix an idempotent $s_i \subset [2k]$ with $|s_i| = i$ and let T_i be the orbit of $\psi(s_i) \cdot \psi'(s_i)^{-1} \in G'(\mathcal{Z})$ under the left action of $G(\mathcal{Z})$, ie

$$T_i = G(\mathcal{Z}) \cdot (\psi(s_i)\psi'(s_i)^{-1}).$$

We claim that T_i is closed under the right action of $G(\mathcal{Z})$ and that the elements $\psi(s) \cdot \psi'(s)^{-1}$ (for $s \subset [2k]$ with $|s| = i$) all lie in T_i .

For the first claim, given $g \in G$ we have

$$M_*((\psi(s)\psi'(s)^{-1})g(\psi(s)\psi'(s)^{-1})^{-1}) = 0,$$

so $((\psi(s)\psi'(s)^{-1})g(\psi(s)\psi'(s)^{-1})^{-1}) \in G(\mathcal{Z})$ and hence $(\psi(s)\psi'(s)^{-1})g \in G(\mathcal{Z}) \cdot (\psi(s)\psi'(s)^{-1})$.

The second claim follows from the fact that

$$M_*(\psi(s)\psi'(s)(\psi(s_i)\psi'(s_i))^{-1}) = 0,$$

so $\psi(s)\psi'(s)(\psi(s_i)\psi'(s_i))^{-1} \in G$.

These claims imply that grading a generator $I(s) \in {}^{\mathcal{A}}[\mathbb{I}]_{\mathcal{A}}$ by $\psi(s) \cdot \psi'(s)^{-1}$ defines a grading on the summand of ${}^{\mathcal{A}}[\mathbb{I}]_{\mathcal{A}}$ with strands grading i by T_i . The rest of the proof of the proposition then goes through unchanged, except working one strands grading i at a time and using T_i in place of T .

The mapping class group action on the category of graded modules Theorem 15 asserts that the bimodules $\widehat{CFDA}(\phi)$ induce an action of the mapping class group on $H_*(\text{Mod}_{\mathcal{A}(\mathcal{Z})})$, which is true, and that this action preserves the subcategories $\text{H}(\widehat{\text{Mod}}_{\mathcal{A}(\mathcal{Z})})$, which is false. There are two problems with the

second statement:

- (1) The grading sets for the bimodules $\widehat{CFDA}(\phi)$ associated to mapping classes are graded by $G(\mathcal{Z})$ -sets S_ϕ , so that S_ϕ is isomorphic to $G(\mathcal{Z})$ as a left $G(\mathcal{Z})$ -set and as a right $G(\mathcal{Z})$ -set, but not as a set with an action of $G(\mathcal{Z}) \times G(\mathcal{Z})^{\text{op}}$. This point is studied further in our later paper [1, Section 5]. Probably this issue could be handled by restricting to the Torelli subgroup, or perhaps a further subgroup.
- (2) Even when S_ϕ is isomorphic to $G(\mathcal{Z})$ as a $G(\mathcal{Z}) \times G(\mathcal{Z})^{\text{op}}$ -set, the isomorphism is not canonical. So, if M is a module graded by $G(\mathcal{Z})$, then $M \boxtimes_{\mathcal{A}(\mathcal{Z})} \widehat{CFDA}(\phi)$ does not have a canonical grading by $G(\mathcal{Z})$, but rather by a $G(\mathcal{Z})$ -set isomorphic to $G(\mathcal{Z})$. This is equivalent to a *relative* grading by $G(\mathcal{Z})$, not an absolute grading by $G(\mathcal{Z})$.

Perhaps it is possible to define canonical absolute gradings on the modules $\widehat{CFDA}(\phi)$ by $G(\mathcal{Z})$ for ϕ in the Torelli group or, perhaps, a nontrivial subgroup of it. Alternatively, one could look for an action of a central extension of the Torelli group. Investigating this would be an interesting future project.

To summarize, the correct statement is the following:

Theorem 15' *The bimodules $\widehat{CFDA}(\phi)$ induce a weak action of the genus- k mapping class groupoid $\text{MCG}_0(k)$ on $\{H_*(\text{Mod}_{\mathcal{A}(\mathcal{Z})}) \mid \text{genus}(F(\mathcal{Z})) = k\}$.*

The proof of Theorem 15' is identical to the proof of Theorem 15, except that the last paragraph (which was incorrect) is no longer needed.

We thank Andy Manion and Raphael Rouquier for pointing out these mistakes. We also thank the referee for further comments.

Other corrections In the first paragraph of [2, Section 10.1], the notation for the algebras is confused. The algebras \mathcal{A} and \mathcal{B} are, respectively,

$$\mathcal{A} = \left(\begin{array}{ccc} & \sigma_1 & \\ \curvearrowright & \sigma_2 & \curvearrowleft \\ \sigma_3 & & \end{array} \begin{array}{c} \iota_0 \\ \iota_1 \end{array} \right) / \left(\begin{array}{c} \sigma_3 \sigma_2 \\ \sigma_2 \sigma_1 \end{array} \right), \quad \mathcal{B} = \left(\begin{array}{ccc} & \rho_1 & \\ \curvearrowright & \rho_2 & \curvearrowleft \\ \rho_3 & & \end{array} \begin{array}{c} j_0 \\ j_1 \end{array} \right) / \left(\begin{array}{c} \rho_3 \rho_2 \\ \rho_2 \rho_1 \end{array} \right)$$

with our usual convention that $\rho_1 \rho_2$ is read left to right, ie this means the arrow ρ_1 followed by the arrow ρ_2 . The element σ_{12} is shorthand for $\sigma_1 \sigma_2$ and so on.

We thank Jesse Cohen for pointing out this mistake.

References

[1] **R Lipshitz, P S Ozsváth, D P Thurston**, *Computing \widehat{HF} by factoring mapping classes*, *Geom. Topol.* 18 (2014) 2547–2681 MR Zbl

- [2] **R Lipshitz, P S Ozsváth, D P Thurston**, *Bimodules in bordered Heegaard Floer homology*, *Geom. Topol.* 19 (2015) 525–724 MR Zbl

*Department of Mathematics, University of Oregon
Eugene, OR, United States*

*Department of Mathematics, Princeton University
Princeton, NJ, United States*

*Department of Mathematics, Indiana University
Bloomington, IN, United States*

lipshitz@uoregon.edu, petero@math.princeton.edu, dpthurst@indiana.edu

Received: 4 February 2022 Revised: 19 July 2022

GEOMETRY & TOPOLOGY

msp.org/gt

MANAGING EDITOR

András I Stipsicz Alfréd Rényi Institute of Mathematics
stipsicz@renyi.hu

BOARD OF EDITORS

Mohammed Abouzaid	Stanford University abouzaid@stanford.edu	Mark Gross	University of Cambridge mgross@dpmms.cam.ac.uk
Dan Abramovich	Brown University dan_abramovich@brown.edu	Rob Kirby	University of California, Berkeley kirby@math.berkeley.edu
Ian Agol	University of California, Berkeley ianagol@math.berkeley.edu	Bruce Kleiner	NYU, Courant Institute bkleiner@cims.nyu.edu
Arend Bayer	University of Edinburgh arend.bayer@ed.ac.uk	Sándor Kovács	University of Washington skovacs@uw.edu
Mark Behrens	University of Notre Dame mbehren1@nd.edu	Urs Lang	ETH Zürich urs.lang@math.ethz.ch
Mladen Bestvina	University of Utah bestvina@math.utah.edu	Marc Levine	Universität Duisburg-Essen marc.levine@uni-due.de
Martin R Bridson	University of Oxford bridson@maths.ox.ac.uk	Ciprian Manolescu	University of California, Los Angeles cm@math.ucla.edu
Jim Bryan	University of British Columbia jbryan@math.ubc.ca	Haynes Miller	Massachusetts Institute of Technology hrm@math.mit.edu
Dmitri Burago	Pennsylvania State University burago@math.psu.edu	Tomasz Mrowka	Massachusetts Institute of Technology mrowka@math.mit.edu
Tobias H Colding	Massachusetts Institute of Technology colding@math.mit.edu	Aaron Naber	Northwestern University anaber@math.northwestern.edu
Simon Donaldson	Imperial College, London s.donaldson@ic.ac.uk	Peter Ozsváth	Princeton University petero@math.princeton.edu
Yasha Eliashberg	Stanford University eliash-gt@math.stanford.edu	Leonid Polterovich	Tel Aviv University polterov@post.tau.ac.il
Benson Farb	University of Chicago farb@math.uchicago.edu	Colin Rourke	University of Warwick gt@maths.warwick.ac.uk
David M Fisher	Rice University davidfisher@rice.edu	Roman Sauer	Karlsruhe Institute of Technology roman.sauer@kit.edu
Mike Freedman	Microsoft Research michaelf@microsoft.com	Stefan Schwede	Universität Bonn schwede@math.uni-bonn.de
David Gabai	Princeton University gabai@princeton.edu	Natasa Sesum	Rutgers University natasas@math.rutgers.edu
Stavros Garoufalidis	Southern U. of Sci. and Tech., China stavros@mpim-bonn.mpg.de	Gang Tian	Massachusetts Institute of Technology tian@math.mit.edu
Cameron Gordon	University of Texas gordon@math.utexas.edu	Ulrike Tillmann	Oxford University tillmann@maths.ox.ac.uk
Jesper Grodal	University of Copenhagen jg@math.ku.dk	Nathalie Wahl	University of Copenhagen wahl@math.ku.dk
Misha Gromov	IHÉS and NYU, Courant Institute gromov@ihes.fr	Anna Wienhard	Universität Heidelberg wienhard@mathi.uni-heidelberg.de

See inside back cover or msp.org/gt for submission instructions.

The subscription price for 2024 is US \$805/year for the electronic version, and \$1135/year (+\$70, if shipping outside the US) for print and electronic. Subscriptions, requests for back issues and changes of subscriber address should be sent to MSP. Geometry & Topology is indexed by Mathematical Reviews, Zentralblatt MATH, Current Mathematical Publications and the Science Citation Index.

Geometry & Topology (ISSN 1465-3060 printed, 1364-0380 electronic) is published 9 times per year and continuously online, by Mathematical Sciences Publishers, c/o Department of Mathematics, University of California, 798 Evans Hall #3840, Berkeley, CA 94720-3840. Periodical rate postage paid at Oakland, CA 94615-9651, and additional mailing offices. POSTMASTER: send address changes to Mathematical Sciences Publishers, c/o Department of Mathematics, University of California, 798 Evans Hall #3840, Berkeley, CA 94720-3840.

GT peer review and production are managed by EditFLOW[®] from MSP.

PUBLISHED BY

 **mathematical sciences publishers**
nonprofit scientific publishing
<http://msp.org/>

© 2024 Mathematical Sciences Publishers

GEOMETRY & TOPOLOGY

Volume 28 Issue 2 (pages 497–1003) 2024

On the top-weight rational cohomology of \mathcal{A}_g	497
MADELINE BRANDT, JULIETTE BRUCE, MELODY CHAN, MARGARIDA MELO, GWYNETH MORELAND and COREY WOLFE	
Algebraic uniqueness of Kähler–Ricci flow limits and optimal degenerations of Fano varieties	539
JIYUAN HAN and CHI LI	
Valuations on the character variety: Newton polytopes and residual Poisson bracket	593
JULIEN MARCHÉ and CHRISTOPHER-LLOYD SIMON	
The local (co)homology theorems for equivariant bordism	627
MARCO LA VECCHIA	
Configuration spaces of disks in a strip, twisted algebras, persistence, and other stories	641
HANNAH ALPERT and FEDOR MANIN	
Closed geodesics with prescribed intersection numbers	701
YANN CHAUBET	
On endomorphisms of the de Rham cohomology functor	759
SHIZHANG LI and SHUBHODIP MONDAL	
The nonabelian Brill–Noether divisor on $\overline{\mathcal{M}}_{13}$ and the Kodaira dimension of $\overline{\mathcal{R}}_{13}$	803
GAVRIL FARKAS, DAVID JENSEN and SAM PAYNE	
Orbit equivalences of \mathbb{R} –covered Anosov flows and hyperbolic-like actions on the line	867
THOMAS BARTHELMÉ and KATHRYN MANN	
Microlocal theory of Legendrian links and cluster algebras	901
ROGER CASALS and DAPING WENG	
Correction to the article Bimodules in bordered Heegaard Floer homology	1001
ROBERT LIPSHITZ, PETER OZSVÁTH and DYLAN P THURSTON	