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Correction to the article Bimodules in bordered Heegaard Floer homology

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We correct some errors in our earlier paper (Geom. Topol. 19 (2015) 525–724).

57K18, 57R58; 53D40

Grading refinement data In the first sentence of the proof of [2, Proposition 3.7] (this is Proposition 3.10 in the arXiv version), the definition of the $(G(\mathcal{Z}), G(\mathcal{Z}))$–set T is wrong and, in particular, the elements $\psi(\boldsymbol{s}) \cdot \psi'(\boldsymbol{s})^{-1}$ do not lie in T.

To correct this, for each $i = 0, \dots, 2k$, fix an idempotent $\boldsymbol{s}_i \subset [2k]$ with $|\boldsymbol{s}_i| = i$ and let T_i be the orbit of $\psi(\boldsymbol{s}_i) \cdot \psi'(\boldsymbol{s}_i)^{-1} \in G'(\mathcal{Z})$ under the left action of $G(\mathcal{Z})$, ie

$$T_i = G(\mathcal{Z}) \cdot (\psi(\boldsymbol{s}_i)\psi'(\boldsymbol{s}_i)^{-1}).$$

We claim that T_i is closed under the right action of $G(\mathcal{Z})$ and that the elements $\psi(\boldsymbol{s}) \cdot \psi'(\boldsymbol{s})^{-1}$ (for $\boldsymbol{s} \subset [2k]$ with $|\boldsymbol{s}| = i$) all lie in T_i.

For the first claim, given $g \in G$ we have

$$M_*\big((\psi(\boldsymbol{s})\psi'(\boldsymbol{s})^{-1})g(\psi(\boldsymbol{s})\psi'(\boldsymbol{s})^{-1})^{-1}\big) = 0,$$

so $\big((\psi(\boldsymbol{s})\psi'(\boldsymbol{s})^{-1})g(\psi(\boldsymbol{s})\psi'(\boldsymbol{s})^{-1})^{-1}\big) \in G(\mathcal{Z})$ and hence $(\psi(\boldsymbol{s})\psi'(\boldsymbol{s})^{-1})g \in G(\mathcal{Z}) \cdot (\psi(\boldsymbol{s})\psi'(\boldsymbol{s})^{-1})$.

The second claim follows from the fact that

$$M_*\big(\psi(\boldsymbol{s})\psi'(\boldsymbol{s})(\psi(\boldsymbol{s}_i)\psi'(\boldsymbol{s}_i))^{-1}\big) = 0,$$

so $\psi(\boldsymbol{s})\psi'(\boldsymbol{s})(\psi(\boldsymbol{s}_i)\psi'(\boldsymbol{s}_i))^{-1} \in G$.

These claims imply that grading a generator $I(\boldsymbol{s}) \in {}^{\mathcal{A}}[\mathbb{I}]_{\mathcal{A}}$ by $\psi(\boldsymbol{s}) \cdot \psi'(\boldsymbol{s})^{-1}$ defines a grading on the summand of ${}^{\mathcal{A}}[\mathbb{I}]_{\mathcal{A}}$ with strands grading i by T_i. The rest of the proof of the proposition then goes through unchanged, except working one strands grading i at a time and using T_i in place of T.

The mapping class group action on the category of graded modules Theorem 15 asserts that the bimodules $\widehat{CFDA}(\phi)$ induce an action of the mapping class group on $H_*(\mathrm{Mod}_{\mathcal{A}(\mathcal{Z})})$, which is true, and that this action preserves the subcategories $\mathsf{H}(\widetilde{\mathrm{Mod}}_{\mathcal{A}(\mathcal{Z})})$, which is false. There are two problems with the second statement:

(1) The grading sets for the bimodules $\widehat{CFDA}(\phi)$ associated to mapping classes are graded by $G(\mathcal{Z})$–sets S_ϕ, so that S_ϕ is isomorphic to $G(\mathcal{Z})$ as a left $G(\mathcal{Z})$–set and as a right $G(\mathcal{Z})$–set, but not as a set with an action of $G(\mathcal{Z}) \times G(\mathcal{Z})^{\mathrm{op}}$. This point is studied further in our later paper [1, Section 5]. Probably this issue could be handled by restricting to the Torelli subgroup, or perhaps a further subgroup.

(2) Even when S_ϕ is isomorphic to $G(\mathcal{Z})$ as a $G(\mathcal{Z}) \times G(\mathcal{Z})^{\mathrm{op}}$–set, the isomorphism is not canonical. So, if M is a module graded by $G(\mathcal{Z})$, then $M \boxtimes_{\mathcal{A}(\mathcal{Z})} \widehat{CFDA}(\phi)$ does not have a canonical grading by $G(\mathcal{Z})$, but rather by a $G(\mathcal{Z})$–set isomorphic to $G(\mathcal{Z})$. This is equivalent to a *relative* grading by $G(\mathcal{Z})$, not an absolute grading by $G(\mathcal{Z})$.

Perhaps it is possible to define canonical absolute gradings on the modules $\widehat{CFDA}(\phi)$ by $G(\mathcal{Z})$ for ϕ in the Torelli group or, perhaps, a nontrivial subgroup of it. Alternatively, one could look for an action of a central extension of the Torelli group. Investigating this would be an interesting future project.

To summarize, the correct statement is the following:

Theorem 15′ *The bimodules $\widehat{CFDA}(\phi)$ induce a weak action of the genus-k mapping class groupoid $\mathrm{MCG}_0(k)$ on $\{H_*(\mathrm{Mod}_{\mathcal{A}(\mathcal{Z})}) \mid \mathrm{genus}(F(\mathcal{Z})) = k\}$.*

The proof of Theorem 15′ is identical to the proof of Theorem 15, except that the last paragraph (which was incorrect) is no longer needed.

We thank Andy Manion and Raphael Rouquier for pointing out these mistakes. We also thank the referee for further comments.

Other corrections In the first paragraph of [2, Section 10.1], the notation for the algebras is confused. The algebras $\mathcal{A}$ and $\mathcal{B}$ are, respectively,

$$\mathcal{A} = \left(\iota_0 \; \begin{array}{c} \xrightarrow{\sigma_1} \\ \xleftarrow{\sigma_2} \\ \xrightarrow[\sigma_3]{} \end{array} \; \iota_1 \right) \Big/ \begin{pmatrix} \sigma_3\sigma_2, \\ \sigma_2\sigma_1 \end{pmatrix}, \quad \mathcal{B} = \left(j_0 \; \begin{array}{c} \xrightarrow{\rho_1} \\ \xleftarrow{\rho_2} \\ \xrightarrow[\rho_3]{} \end{array} \; j_1 \right) \Big/ \begin{pmatrix} \rho_3\rho_2, \\ \rho_2\rho_1 \end{pmatrix}$$

with our usual convention that $\rho_1\rho_2$ is read left to right, ie this means the arrow ρ_1 followed by the arrow ρ_2. The element σ_{12} is shorthand for $\sigma_1\sigma_2$ and so on.

We thank Jesse Cohen for pointing out this mistake.

References

[1] **R Lipshitz**, **P S Ozsváth**, **D P Thurston**, *Computing $\widehat{HF}$ by factoring mapping classes*, Geom. Topol. 18 (2014) 2547–2681 MR Zbl

[2] **R Lipshitz**, **P S Ozsváth**, **D P Thurston**, *Bimodules in bordered Heegaard Floer homology*, Geom. Topol. 19 (2015) 525–724 MR Zbl

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