



# *Geometry & Topology*

Volume 28 (2024)

**Correction to the article  
Bimodules in bordered Heegaard Floer homology**

ROBERT LIPSHITZ  
PETER OZSVÁTH  
DYLAN P THURSTON

## Correction to the article Bimodules in bordered Heegaard Floer homology

ROBERT LIPSHITZ  
PETER OZSVÁTH  
DYLAN P THURSTON

We correct some errors in our earlier paper ([Geom. Topol. 19 \(2015\) 525–724](#)).

[57K18](#), [57R58](#); [53D40](#)

**Grading refinement data** In the first sentence of the proof of [2, Proposition 3.7] (this is Proposition 3.10 in the [arXiv](#) version), the definition of the  $(G(\mathcal{Z}), G(\mathcal{Z}))$ -set  $T$  is wrong and, in particular, the elements  $\psi(s) \cdot \psi'(s)^{-1}$  do not lie in  $T$ .

To correct this, for each  $i = 0, \dots, 2k$ , fix an idempotent  $s_i \in [2k]$  with  $|s_i| = i$  and let  $T_i$  be the orbit of  $\psi(s_i) \cdot \psi'(s_i)^{-1} \in G'(\mathcal{Z})$  under the left action of  $G(\mathcal{Z})$ , ie

$$T_i = G(\mathcal{Z}) \cdot (\psi(s_i)\psi'(s_i)^{-1}).$$

We claim that  $T_i$  is closed under the right action of  $G(\mathcal{Z})$  and that the elements  $\psi(s) \cdot \psi'(s)^{-1}$  (for  $s \in [2k]$  with  $|s| = i$ ) all lie in  $T_i$ .

For the first claim, given  $g \in G$  we have

$$M_*((\psi(s)\psi'(s)^{-1})g(\psi(s)\psi'(s)^{-1})^{-1}) = 0,$$

so  $((\psi(s)\psi'(s)^{-1})g(\psi(s)\psi'(s)^{-1})^{-1}) \in G(\mathcal{Z})$  and hence  $(\psi(s)\psi'(s)^{-1})g \in G(\mathcal{Z}) \cdot (\psi(s)\psi'(s)^{-1})$ .

The second claim follows from the fact that

$$M_*((\psi(s)\psi'(s)(\psi(s_i)\psi'(s_i))^{-1})^{-1}) = 0,$$

so  $\psi(s)\psi'(s)(\psi(s_i)\psi'(s_i))^{-1} \in G$ .

These claims imply that grading a generator  $I(s) \in {}^{\mathcal{A}}[\mathbb{I}]_{\mathcal{A}}$  by  $\psi(s) \cdot \psi'(s)^{-1}$  defines a grading on the summand of  ${}^{\mathcal{A}}[\mathbb{I}]_{\mathcal{A}}$  with strands grading  $i$  by  $T_i$ . The rest of the proof of the proposition then goes through unchanged, except working one strands grading  $i$  at a time and using  $T_i$  in place of  $T$ .

**The mapping class group action on the category of graded modules** Theorem 15 asserts that the bimodules  $\widehat{CFDA}(\phi)$  induce an action of the mapping class group on  $H_*(\text{Mod}_{\mathcal{A}(\mathcal{Z})})$ , which is true, and that this action preserves the subcategories  $\text{H}(\widehat{\text{Mod}}_{\mathcal{A}(\mathcal{Z})})$ , which is false. There are two problems with the

second statement:

- (1) The grading sets for the bimodules  $\widehat{CFDA}(\phi)$  associated to mapping classes are graded by  $G(\mathcal{Z})$ -sets  $S_\phi$ , so that  $S_\phi$  is isomorphic to  $G(\mathcal{Z})$  as a left  $G(\mathcal{Z})$ -set and as a right  $G(\mathcal{Z})$ -set, but not as a set with an action of  $G(\mathcal{Z}) \times G(\mathcal{Z})^{\text{op}}$ . This point is studied further in our later paper [1, Section 5]. Probably this issue could be handled by restricting to the Torelli subgroup, or perhaps a further subgroup.
- (2) Even when  $S_\phi$  is isomorphic to  $G(\mathcal{Z})$  as a  $G(\mathcal{Z}) \times G(\mathcal{Z})^{\text{op}}$ -set, the isomorphism is not canonical. So, if  $M$  is a module graded by  $G(\mathcal{Z})$ , then  $M \boxtimes_{\mathcal{A}(\mathcal{Z})} \widehat{CFDA}(\phi)$  does not have a canonical grading by  $G(\mathcal{Z})$ , but rather by a  $G(\mathcal{Z})$ -set isomorphic to  $G(\mathcal{Z})$ . This is equivalent to a *relative* grading by  $G(\mathcal{Z})$ , not an absolute grading by  $G(\mathcal{Z})$ .

Perhaps it is possible to define canonical absolute gradings on the modules  $\widehat{CFDA}(\phi)$  by  $G(\mathcal{Z})$  for  $\phi$  in the Torelli group or, perhaps, a nontrivial subgroup of it. Alternatively, one could look for an action of a central extension of the Torelli group. Investigating this would be an interesting future project.

To summarize, the correct statement is the following:

**Theorem 15'** *The bimodules  $\widehat{CFDA}(\phi)$  induce a weak action of the genus- $k$  mapping class groupoid  $\text{MCG}_0(k)$  on  $\{H_*(\text{Mod}_{\mathcal{A}(\mathcal{Z})}) \mid \text{genus}(F(\mathcal{Z})) = k\}$ .*

The proof of Theorem 15' is identical to the proof of Theorem 15, except that the last paragraph (which was incorrect) is no longer needed.

We thank Andy Manion and Raphael Rouquier for pointing out these mistakes. We also thank the referee for further comments.

**Other corrections** In the first paragraph of [2, Section 10.1], the notation for the algebras is confused. The algebras  $\mathcal{A}$  and  $\mathcal{B}$  are, respectively,

$$\mathcal{A} = \left( \begin{array}{ccc} & \xrightarrow{\sigma_1} & \\ \sigma_2 \swarrow & & \searrow \sigma_3 \\ l_0 & & l_1 \end{array} \right) / \left( \begin{array}{c} \sigma_3 \sigma_2, \\ \sigma_2 \sigma_1 \end{array} \right), \quad \mathcal{B} = \left( \begin{array}{ccc} & \xrightarrow{\rho_1} & \\ \rho_2 \swarrow & & \searrow \rho_3 \\ j_0 & & j_1 \end{array} \right) / \left( \begin{array}{c} \rho_3 \rho_2, \\ \rho_2 \rho_1 \end{array} \right)$$

with our usual convention that  $\rho_1 \rho_2$  is read left to right, ie this means the arrow  $\rho_1$  followed by the arrow  $\rho_2$ . The element  $\sigma_{12}$  is shorthand for  $\sigma_1 \sigma_2$  and so on.

We thank Jesse Cohen for pointing out this mistake.

## References

[1] **R Lipshitz, PS Ozsváth, DP Thurston**, *Computing  $\widehat{HF}$  by factoring mapping classes*, *Geom. Topol.* 18 (2014) 2547–2681 [MR](#) [Zbl](#)

- [2] **R Lipshitz, P S Ozsváth, D P Thurston**, *Bimodules in bordered Heegaard Floer homology*, *Geom. Topol.* 19 (2015) 525–724 [MR](#) [Zbl](#)

*Department of Mathematics, University of Oregon  
Eugene, OR, United States*

*Department of Mathematics, Princeton University  
Princeton, NJ, United States*

*Department of Mathematics, Indiana University  
Bloomington, IN, United States*

[lipshitz@uoregon.edu](mailto:lipshitz@uoregon.edu), [petero@math.princeton.edu](mailto:petero@math.princeton.edu), [dpthurst@indiana.edu](mailto:dpthurst@indiana.edu)

Received: 4 February 2022      Revised: 19 July 2022

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Geometry & Topology (ISSN 1465-3060 printed, 1364-0380 electronic) is published 9 times per year and continuously online, by Mathematical Sciences Publishers, c/o Department of Mathematics, University of California, 798 Evans Hall #3840, Berkeley, CA 94720-3840. Periodical rate postage paid at Oakland, CA 94615-9651, and additional mailing offices. POSTMASTER: send address changes to Mathematical Sciences Publishers, c/o Department of Mathematics, University of California, 798 Evans Hall #3840, Berkeley, CA 94720-3840.

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GT peer review and production are managed by EditFLOW<sup>®</sup> from MSP.

PUBLISHED BY

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# GEOMETRY & TOPOLOGY

Volume 28    Issue 2 (pages 497–1003)    2024

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