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Studying the impacts of changing climate on the Finger Lakes  
wine industry

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# Studying the impacts of changing climate on the Finger Lakes wine industry

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We report the results of a project with Six Mile Creek Winery in Ithaca, NY, in which we investigate possible climate impacts on the area wine industry. Specifically, we examine winter minimum temperatures in Ithaca since temperatures below  $-15^{\circ}\text{F}$  damage buds of French-American hybrid grapes and temperatures below  $-5^{\circ}\text{F}$  affect Vinifera grape varieties. We used the generalized extreme value distribution to model the winter minimum and adjusted this model based on climate simulation data.

## 1. Introduction

Climate change is a serious issue facing society and it is now feasible to address local questions concerning climate. Simulations are now being run using a  $50\text{ km}^2$  grid; meaning there are data streams for each  $50 \times 50\text{ km}$  box covering the United States that can be analyzed to address specific local questions about future climate. These data streams provide data every three hours consisting of surface specific humidity, precipitation, surface pressure, surface downwelling shortwave radiation, surface air temperature, zonal surface wind speed, and meridional surface wind speed for the periods from 1968 to 2000 and 2038 to 2070. The reason for the two periods is that computer simulations do not necessarily predict future climate well (note that climate is considered to be a distribution resulting from approximately 30 years of weather) but they are consistent in that the changes in variables from the recent scenario 1968–2000 to the future scenario 2038–2070 represent an estimate of the change in these variables over those two periods. For example, the sample average winter minimum temperature in the computer simulation for 1968–2000 is  $-19.25^{\circ}\text{F}$ , which is a few degrees off from the sample average winter minimum temperature for that period of  $-16.09^{\circ}\text{F}$ . The sample average winter minimum from the computer simulation for the future scenario of 2038–2070 is  $-12.92^{\circ}\text{F}$ .

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What this suggests to us is that the mean winter minimum temperature is likely to increase  $6.33^\circ\text{F}$  to around  $-9.76^\circ\text{F}$ .

We chose to study changes in the minimum winter temperature in our area (the Finger Lakes region is a major wine producing region) because the French-American hybrid grapes will begin to have bud damage when the temperature falls below  $-15^\circ\text{F}$ , whereas *Vinifera* grape varieties (such as Riesling, Cabernet Franc, and Chardonnay) will start to have bud damage when temperatures fall below  $-5^\circ\text{F}$ . We note that all of these grapes are currently being grown despite cold winter temperatures, with the Riesling grape being a premier product, due to microclimates near the Finger Lakes and various techniques to keep the vines warmer in the winter. Our goal is to estimate the probability that the winter minimum temperature in Ithaca, NY, will fall below  $-15^\circ\text{F}$  and will fall below  $-5^\circ\text{F}$  by mid-century. To do this we need to estimate the current distribution of winter minimum temperatures and use the simulations to estimate how this distribution may change in the future.

## 2. Winter minimum distributions

Since winter minimum temperatures are extreme events, an appropriate distribution to use to model the data is the generalized extreme value (GEV) family for minima given by

$$G(z) = 1 - \exp\left(-\left(1 - \xi \frac{z - \mu_t}{\sigma}\right)^{-1/\xi}\right) \quad (1)$$

defined on  $\{z : 1 - \xi(z - \mu_t)/\sigma > 0\}$ , where  $-\infty < \mu_t, \xi < \infty$  and  $\sigma > 0$ . Here  $\mu_t$  is the location parameter,  $\sigma$  is the scale parameter, and  $\xi$  is the shape parameter. For this paper we used the Extremes Toolkit [Katz et al. 2009] for the statistical software R to fit the GEV model to our data sets. The Extremes Toolkit is maximizing the log-likelihood function

$$l(\mu_t, \sigma, \xi) = -m \log \sigma - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^m \log \left(1 + \xi \frac{z_i - \mu_t}{\sigma}\right) - \sum_{i=1}^m \left(1 + \xi \frac{z_i - \mu_t}{\sigma}\right)^{-1/\xi},$$

under the conditions that  $\xi \neq 0$  and  $1 + \xi \frac{z_i - \mu_t}{\sigma} > 0$  for all  $i$ . When  $\xi = 0$ , the log-likelihood function to be maximized is

$$l(\mu_t, \sigma) = -m \log \sigma - \sum_{i=1}^m \frac{z_i - \mu_t}{\sigma} - \sum_{i=1}^m \exp\left(-\frac{z_i - \mu_t}{\sigma}\right).$$

In either case this yields the maximum likelihood estimate for the parameter vector  $(\mu_t, \sigma, \xi)$ . The Extremes Toolkit also allows for any of the three variables to be time varying along with indicator variables, and we will use a location parameter of the form  $\mu_t = \mu_0 + \alpha t + \beta 1_{t>t_0}$  ( $1_{t>t_0} = 1$  if  $t > t_0$  and 0 if  $t \leq t_0$ ). Coles

1893	-14	1914	-9	1943	-10	1964	-23	1986	-18
1894	-9	1915	-8	1944	-14	1965	-14	1987	-14
1895	-18	1916	-11	1945	-8	1966	-14	1988	-19
1896	-6	1917	-22	1946	-9	1967	-23	1989	-15
1897	-4	1926	-11	1947	-20	1968	-7	1990	-7
1898	-16	1927	-2	1948	-5	1969	-17	1991	-7
1899	-2	1928	-7	1949	-8	1970	-11	1992	-17
1900	-8	1929	-8	1950	-7	1971	-11	1993	-24
1901	-2	1930	-1	1951	-10	1972	-17	1994	-12
1902	-5	1931	3	1952	2	1973	-12	1995	-16
1903	-20	1932	-1	1953	-14	1974	-12	1996	-11
1904	-6	1933	-24	1954	-13	1975	-21	1998	-13
1905	-11	1934	-11	1955	-7	1976	-17	1999	-11
1906	-10	1935	-6	1956	-25	1978	-23	2000	-5
1907	-13	1936	-5	1957	-14	1979	-13	2001	6
1908	-8	1937	-9	1958	-10	1980	-21	2002	-14
1909	-6	1938	-2	1959	-10	1981	-23	2003	-17
1910	-1	1939	-3	1960	-25	1982	-15	2004	-22
1911	-16	1940	-3	1961	-17	1983	-22	2005	-11
1912	-1	1941	-8	1962	-18	1984	-11	2006	-9
1913	-15	1942	-14	1963	-11	1985	-8	2007	-5

**Table 1.** Observed minimum winter temperature for the years 1893 to 2007, in °F. Data in all tables and figures refer to Ithaca, NY.

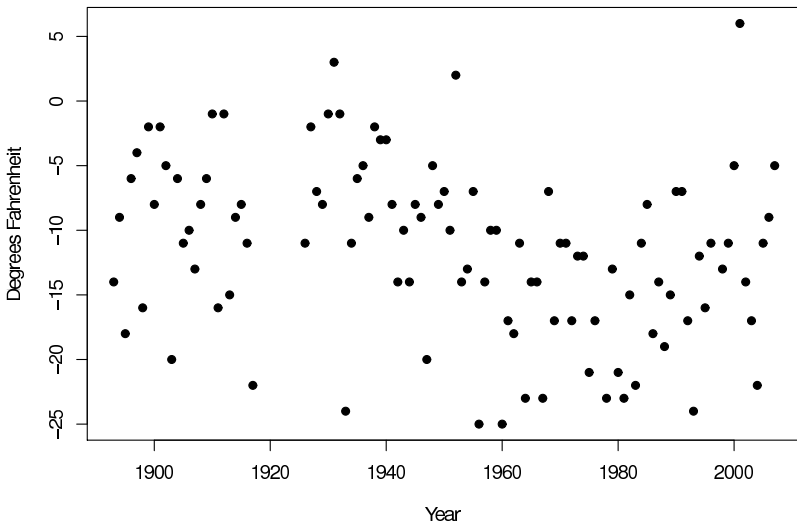
[2001] discusses the GEV distribution and modeling details. We obtained the winter minimum temperatures for Ithaca from the NOAA (National Oceanic and Atmospheric Administration) National Data Center [NNDC 2009] for the years 1893 through 2008, but winter minimums for 1977 and 1997 were missing. The observed data is listed in Table 1.

### 3. Results

The parameters for the GEV fitted to the observed data are in Table 2, for the model  $\mu_t = \mu_0 + \alpha t$  of the location parameter, where  $t$  is scaled so that  $t = 0$  for 1893 and  $t = 1$  for 2007. The time scale was significant ( $p = 0.004$ ) with a coefficient of  $-6.26649^\circ\text{F}$ . This seems rather large and we conjecture that this is due to the colder temperatures in the 1960s and the fact that we have more data before 1960 than after. A scatter plot of the data, Figure 1, does not show any clear changepoints, although the winter minimum temperatures appear lower from roughly 1960–1980.

$\mu_0$	-6.07284	(1.28365)
$\alpha$	-6.26649	(2.12176)
$\sigma$	6.47464	(0.48768)
$\xi$	-0.31045	(0.06412)

**Table 2.** Expected values and standard errors of parameters for observed Ithaca winter minimum temperatures for the winters 1893–2007 with the location parameter  $\mu_t = \mu_0 + \alpha t$ ; here  $t$  is scaled so that  $t = 0$  for 1893 and  $t = 1$  for 2007.



**Figure 1.** Observed winter minimum temperatures.

There are minor weather station changes in 1969 and 1987, and it may be that since the station is associated with Cornell University that the station is particularly stable. In 1969 the station was raised 10 feet and the longitude changed from  $-76.466670^\circ$  to  $-76.45000^\circ$ . In 1987 there was a change in equipment. Despite these seemingly minor changes there appear to be some changepoint issues. We first used a model with

$$\mu_t = \begin{cases} \mu_0 + \alpha t & \text{for } t < 1969, \\ \mu_0 + \alpha t + \beta_1 & \text{for } 1969 \leq t \leq 1987, \\ \mu_0 + \alpha t + \beta_2 & \text{for } t > 1987, \end{cases} \quad (2)$$

to test all changepoints simultaneously. A likelihood ratio test with the Extremes Toolkit was used to check for significant differences between the model with and without changepoints. The test uses the deviance statistic  $D = 2\{\ell_1(\mathcal{M}_1) - \ell_0(\mathcal{M}_0)\}$ ,

$\mu_0$	-4.91412	(1.39459)
$\alpha$	-10.10339	(2.76638)
$\beta$	4.48365	(2.09895)
$\sigma$	6.27504	(0.47723)
$\xi$	-0.29469	(0.06717)

**Table 3.** Expected values and standard errors (in parentheses) of parameters for observed winter minimum temperatures for the winters 1893–2007 with the location parameter  $\mu_t$  given by (4) and  $t$  scaled so that  $t = 0$  for 1893 and  $t = 1$  for 2007.

where  $\mathcal{M}_0$  is a submodel of  $\mathcal{M}_1$  and  $\ell_0(\mathcal{M}_0)$  and  $\ell_1(\mathcal{M}_1)$  are the maximized values of the log-likelihood for their respective models [Coles 2001, p. 35]. The test was not significant ( $p = 0.0514$ ) and so the addition of the changepoints does not improve the model. We also tested the changepoints individually by using the following expressions for  $\mu_t$ :

$$\mu_t = \begin{cases} \mu_0 + \alpha t & \text{for } t < 1969, \\ \mu_0 + \alpha t + \beta & \text{for } t \geq 1969, \end{cases} \quad (3)$$

$$\mu_t = \begin{cases} \mu_0 + \alpha t & \text{for } t \leq 1987, \\ \mu_0 + \alpha t + \beta & \text{for } t > 1987. \end{cases} \quad (4)$$

In testing each of these against the model without any changepoints the model with (3) was not significant ( $p = 0.6951$ ) and the model with (4) was significant ( $p = 0.0341$ ). We will use the model with  $\mu_t$  given by (4). The values of the parameters for the observed model we will use with  $\mu_t$  given by (4) are given in Table 3.

The climate simulations used are the GFDL RCM3 data found at [NARCCAP 2009]. The future scenario simulations assume the IPCC A2 scenario for greenhouse gas emissions. In brief, the A2 scenario is an estimate of our future greenhouse gas emissions based on socioeconomic factors. It assumes our emissions will continue to rise, but at a decreasing rate. The scenario predicts these emissions based on the idea that the global economy will become more regional and each region will become more self-reliant. These various regions are less involved internationally. Similarly global environmental concerns are weak, and regional attempts at controlling pollution are only enough to maintain their environmental amenities [IPCC 2001].

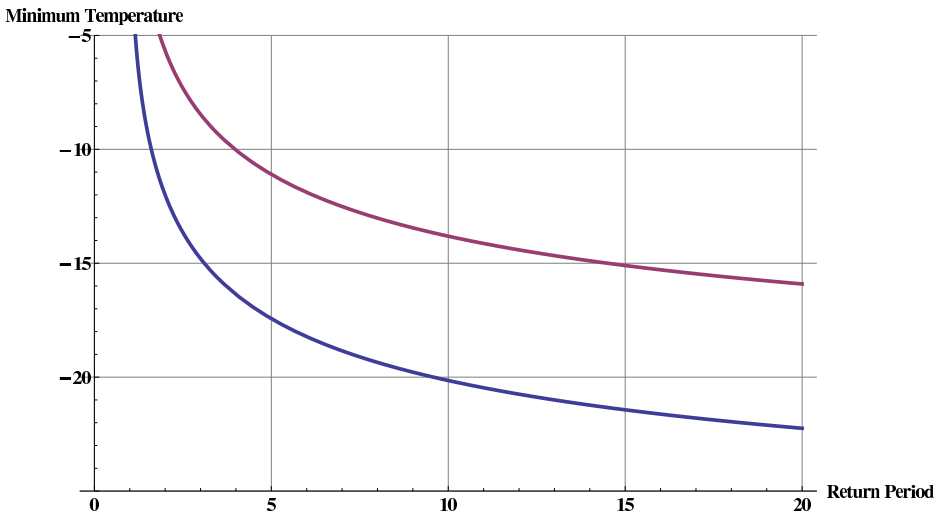
The simulation data we used is listed in Table 4. The simulation data is for the winter periods of 1968–1999 and 2038–2069. Here we take the approach that each period has its own GEV distribution, since climate is considered to be 30 years of weather, and we will test to see if there are any statistically significant changes in

1968	-21.6	1984	-13.6	2038	-19.7	2054	-5.0
1969	-19.1	1985	-25.8	2039	-14.3	2055	-11.5
1970	-12.8	1986	-22.8	2040	-19.9	2056	-15.4
1971	-24.8	1987	-15.0	2041	-6.1	2057	-7.2
1972	-24.0	1988	-11.8	2042	-14.4	2058	0.3
1973	-18.6	1989	-15.3	2043	-18.9	2059	-10.8
1974	-12.0	1990	-6.7	2044	-17.6	2060	-4.8
1975	-12.7	1991	-13.7	2045	-8.8	2061	-16.9
1976	-23.0	1992	-20.6	2046	-19.1	2062	-3.8
1977	-14.9	1993	-11.9	2047	-8.4	2063	-15.4
1978	-10.9	1994	-14.9	2048	-5.6	2064	-5.1
1979	-20.3	1995	-16.6	2049	-4.8	2065	-3.3
1980	-18.2	1996	-20.9	2050	-10.0	2066	-9.9
1981	-27.6	1997	-19.2	2051	-14.7	2067	-22.4
1982	-22.3	1998	-21.0	2052	-3.4	2068	-10.4
1983	-15.0	1999	-14.3	2053	-20.0	2069	-9.4

**Table 4.** Simulated minimum winter temperatures, using the IPCC A2 scenario for greenhouse gas emissions, obtained from the GFDL RCM3 simulations found at [\[NARCCAP 2009\]](#).

those parameters. We assume that the simulated data will provide a good estimate of any changes in the parameters under the A2 scenario, even if the simulations do not match observation well, as noted in the introduction. If there is a significant change in any parameter, then we will apply that change to the observed parameter, yielding a future distribution. To detect significant differences in the parameters between the simulated winter minimums for the winters of 1968–1999 and 2038–2069 we fit a GEV distribution to all the simulated data with indicator functions for each parameter. For example, the location parameter is of the form  $\mu_t = \mu_0 + \gamma 1_{t>2000}$  ( $1_{t>2000} = 1$  if  $t > 2000$  and 0 if  $t \leq 2000$ ) and if the addition of the indicator variable is significant using the likelihood ratio test, then the coefficient of the indicator provides an estimate of how much to adjust the observed parameter. The only significant indicator variable is for the location parameter and has a coefficient of 6.33435°F ( $p < 0.001$ ). In other words, we expect a shift of 6.33435°F in the location parameter from the period 1968–1999 to 2038–2069. We do not expect a change in the scale or shape parameters. We will shift over the same time period and hence we will take our observed location parameter to be

$$\mu = -4.91412 - 10.10339(0.929825) + 4.48365(1) = -9.82485^\circ\text{F}.$$



**Figure 2.** Return level curves for the observed winter minimum ( $t = 0.9298$ , winter of 1999), bottom curve, and the predicted mid-century winter minimum.

(Note the 1999 winter minimum corresponds to  $t = 0.929825$  since  $t$  was scaled to start in 1893,  $t = 0$ , and end in 2007,  $t = 1$ ). Hence our predicted location parameter for mid-century is

$$\mu = -9.82485 + 6.33435 = -3.4905^{\circ}\text{F}.$$

We will use the same scale and shape parameters from our observed data. In summary, the value of the simulation data is that it estimates a shift in the location parameter for winter minimum temperatures under an IPCC A2 scenario of  $6.33435^{\circ}\text{F}$  from the period 1968–1999 to 2038–2069. Now, while our observed location parameter does have a time variable extrapolating this parameter to mid-century would be too far of an extrapolation, and would not necessarily take into consideration changing greenhouse gasses as the simulation data does. In fact, we noted earlier that the coefficient of the time parameter may just reflect a decade of cooler temperatures. We should also mention that there are statistical downscaling methods to build models that combine the observed and simulated data to make projections [Wilby et al. 2004].

Two return level curves, observed and predicted, are plotted in Figure 2. In these graphs we see, for example, that the observed data tells us that about once every 10 years the winter minimum temperature will fall below  $-20^{\circ}\text{F}$ . On the other hand, we predict that by mid-century the winter minimum temperature will fall below  $-14^{\circ}\text{F}$  only once every 10 years.



Temperature	Probability below temperature	
	Observed data	Predicted climate
-25°F	1.44%	0.00%
-20°F	10.45%	0.63%
-15°F	32.21%	6.90%
-10°F	62.18%	25.17%
-5°F	86.46%	54.12%
0°F	97.33%	83.25%

**Table 5.** The probabilities that the winter minimum temperature will fall below the given values for the observed winter minimum and predicted winter minimum.

Table 5 gives more detailed information about the probability that the winter minimum temperature will fall below a given temperature. Focusing on the  $-15^{\circ}\text{F}$  threshold, we see that currently there is a 32% chance the temperature will drop below  $-15^{\circ}\text{F}$  but by mid-century this drops to only a 7% chance. Similarly, the chance of dropping below  $-5^{\circ}\text{F}$  changes from 86% to 54%.

#### 4. Conclusion and discussion

The changes in the ability of the Finger Lakes wine industry to grow French-American hybrid grapes and *Vinifera* grapes is still unclear. For one, we still need to consider area microclimates. The weather station from which we collected data is within the region, but it is not located on the shore of any of the Finger Lakes. Land that is adjacent to a lake, in particular Cayuga and Seneca, have warmer winter temperatures. For instance, suppose your winery location was typically  $5^{\circ}\text{F}$  warmer in the winter than the weather station location due to being in a lake microclimate. By looking at the  $-20^{\circ}\text{F}$  row in Table 5, the chances of a minimum below  $-15^{\circ}\text{F}$  falls from about 1 in 10 years to about 1 in 100 years. We also point out that we are using a general  $-15^{\circ}\text{F}$  and  $-5^{\circ}\text{F}$  cutoffs when, in fact, there are varietal differences.

We see this study as the beginning of aiding the Finger Lakes wine industry in dealing with climate instability. While this study may seem to suggest a positive effect for the region, there are other possible impacts that are likely to be negative that still need to be studied. For example, in the winter of 2004 the industry experienced a single freeze event in January that reduced the *Vitis vinifera* crop by almost half [Zabada et al. 2007]. In general, grapevines are susceptible to rapid temperature drops during the acclimation and deacclimation periods of dormancy. Also, increasing rain or humidity would increase mildew problems, which would

increase the need for mildew control. Finally, the region is known for its Riesling wines but the riesling grapes require an adequate number of cool nights at the end of the growing season. If there were some change in the cool night numbers that reduced the quality of the Riesling wine, then the region would have to rebrand itself with some other wine. On the other hand, it is also worth studying how much the summer growing season may change as longer warmer summers would help the industry.

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