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ONE-DIMENSIONAL THERMOELASTIC WAVES IN ELASTIC HALF-SPACE WITH DUAL PHASE-LAG EFFECTS

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The theory of thermoelasticity with dual phase-lag effects is employed to study the problem of one-dimensional disturbances in an elastic half-space with its plane boundary subjected to (i) a constant step input of temperature and zero stress, and (ii) a constant step input of stress and zero temperature. The Laplace transform method is used to solve the problem. Expressions for displacement, temperature and stress fields are obtained for small values of time. It is found that the solutions consist of two coupled waves both of which propagate with finite speeds and attenuation, influenced by the two delay times and thermoelastic coupling constant. The discontinuities that occur at the wave fronts are obtained. The characteristic features of the underlying theory are analyzed by comparing the results of the present analysis with their counterparts in coupled thermoelasticity theory (CTE) and in other generalized thermoelasticity theories ETE, TRDTE and TEWOED.

1. Introduction

Thermoelasticity theories which involve finite speed of thermal signals (second sound) have created much interest during the last three decades. The conventional coupled dynamic thermoelasticity theory (CTE), based on the mixed parabolic-hyperbolic governing equations of [Biot 1956; Chadwick 1960], predicts an infinite speed of propagation of thermoelastic disturbances. To remove the paradox of infinite speed for propagation of thermoelastic disturbances, several generalized thermoelasticity theories have been developed, which involve hyperbolic governing equations. Among these generalized theories, the extended thermoelasticity theory (ETE) proposed by Lord and Shulman [1967] involving one relaxation time (called single-phase-lag model) and the temperature-rate-dependent theory of thermoelasticity (TRDTE) proposed by Green and Lindsay [1972] involving two relaxation times are two important models of generalized theory of thermoelasticity. Experimental studies [Kaminski 1990; Mitra et al. 1995; Tzou 1995a; 1995b] indicate that the relaxation times can be of relevance in the cases involving a rapidly propagating crack tip, a localized moving heat source with high intensity, shock wave propagation, laser technique etc. Because of the experimental evidence in support of finiteness of heat propagation speed, the generalized thermoelasticity theories are considered to be more realistic than the conventional theory in dealing with practical problems involving very large heat fluxes at short intervals like those occurring in laser units and energy channels. For a review of the relevant literature, see [Chandrasekharaiah 1986; Ignaczak 1989].

Green and Naghdi [1977; 1992; 1993] formulated three different models of thermoelasticity among which, in one of these models, there is no dissipation of thermoelastic energy. This model is referred to as

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the G–N model of thermoelasticity without energy dissipation (TEWOED). Problems concerning generalized thermoelasticity theories and G–N theory have been studied by many authors [RoyChoudhuri and Debnath 1983; RoyChoudhuri 1984; 1985; 1987; Dhaliwal and Rokne 1988; 1989; RoyChoudhuri 1990; Chandrasekharaiah and Murthy 1993; Chandrasekhariah and Srinath 1996; RoyChoudhuri and Banerjee 2004; RoyChoudhuri and Bandyopadhyay 2005; RoyChoudhuri and Dutta 2005; 2005]. Tzou [1995a; 1995b] and Ozisik and Tzou [1994] have developed a new model called dual phase-lag model for heat transport mechanism in which Fourier's law is replaced by an approximation to a modification of Fourier's law with two different time translations for the heat flux and the temperature gradient. According to this model, classical Fourier's law $\vec{q} = -k\vec{\nabla}T$ has been generalized as $\vec{q}(P, t + \tau_q) = -k\vec{\nabla}T(P, t + \tau_T)$ where the temperature gradient $\vec{\nabla}T$ at a point P of the material at time $t + \tau_T$ corresponds to the heat flux vector $\vec{q}$ at the same point at time $t + \tau_q$. Here k is the thermal conductivity of the material. The delay time τ_T is interpreted as that caused by the microstructural interactions (small-scale heat transport mechanisms occurring in microscale) and is called the phase-lag of the temperature gradient. The other delay time is τ_q interpreted as the relaxation time due to the fast transient effects of thermal inertia (small-scale effects of heat transport in time) and is called the phase-lag of the heat flux. If $\tau_q = \tau$, $\tau_T = 0$, Tzou [1995a; 1995b] refers to the model as the single phase-lag model. The case $\tau_q \neq \tau_T (\neq 0)$ corresponds to the dual phase-lag model of the constitutive equation connecting the heat flux vector and the temperature gradient. The case $\tau_q = \tau_T (\neq 0)$ becomes identical with the classical Fourier's law. Further for materials with $\tau_q > \tau_T$, the heat flux vector is the result of a temperature gradient and for materials with $\tau_T > \tau_q$, the temperature gradient is the result of a heat flux vector. For a review of the relevant literature, see [Chandrasekharaiah 1998]. A hyperbolic thermoelastic model was developed in this same reference, taking into account the phase-lag of both temperature gradient and heat flux vector and also the second order term in τ_q in Taylor's expansion of heat flux vector and the first order term in τ_T in Taylor's expansion of the temperature gradient in the generalization of classical Fourier's law. It may be pointed out that ETE was formulated by taking into account the thermal relaxation time, which is in fact the phase-lag of the heat flux vector (single phase-lag model).

The purpose of the present paper is to consider thermoelastic interaction in an elastic half-space in the context of the thermoelasticity theory based on the Tzou model [1995a; 1995b] of heat transport mechanism with dual phase-lag effects. The plane boundary is subjected to (i) a constant step input of temperature and zero stress and (ii) a constant step input of stress and zero temperature. Laplace transform is used as a mathematical tool. The expressions for displacement, temperature and stress in the half-space are derived for small times. The solution for displacement, temperature and stress consist of two waves – one, the predominantly elastic wave (E-wave) and the other, the predominantly thermal wave (T-wave) in nature, both propagating with finite speeds modified by the nondimensional delay times τ_q^* and τ_T^*. It is observed that the displacement is continuous at both the wave fronts while both the temperature and stress fields suffer finite jumps at these locations. Further the waves suffer exponential attenuation at both the wave fronts as in ETE and TRDTE. Similar problems have been studied in [Dhaliwal and Rokne 1988; 1989] in the context of ETE and TRDTE, and in [Chandrasekhariah and Srinath 1996] in the context of TEWOED. The results of the present analysis are compared with those derived in the context of ETE, TRDTE, TEWOED and CTE. The present investigation has brought to light some similarities and differences for the theories ETE, TRDTE, TEWOED and CTE.

2. Formulation of the problem: basic equations

An isotropic elastic homogeneous half-space is considered. The plane boundary is subjected to a constant step input of temperature and zero stress. We study the disturbances produced in the half-space. The solid is subjected to one-dimensional deformation so that all the field variables are functions of the spatial coordinate x and time t. If θ is the temperature increase at time t above the uniform reference temperature θ_0 and $\vec{u}$ the displacement vector, the heat transport equation which includes dual phase-lag effects (see [Ozisik and Tzou 1994; Tzou 1995a; 1995b]) is

$$k\Big(1+\tau_T\frac{\partial}{\partial t}\Big)\nabla^2\theta = \Big(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\Big)(\rho c_v\dot{\theta}+\beta\theta_0\dot{\Delta}-\rho R), \tag{1}$$

where τ_T and τ_q are the phase-lag of the temperature gradient and of the heat flux respectively, often referred to as the delay times, k is the thermal conductivity of the solid, and R is the heat source term. In addition, $\Delta = \operatorname{div}\bar{u}$ and $\beta = (3\lambda+2\mu)\alpha_t$. The displacement equation of motion is

$$\mu\nabla^2\vec{u}+(\lambda+\mu)\operatorname{grad}\Delta-\beta\operatorname{grad}\theta+\rho\vec{F}=\rho\ddot{\vec{u}}. \tag{2}$$

Here λ and μ are Lamé constants, ρ is the constant mass density of the solid, α_t is the coefficient of linear thermal expansion of the material, k is the thermal conductivity, c_v is the specific of the solid.

For one-dimensional deformation $\vec{u}=(u(x,t),0,0)$ and $\theta=\theta(x,t)$. In absence of heat source and body forces, the Equations (1) and (2), in case of one- dimensional disturbances, reduce to

$$k\Big(1+\tau_T\frac{\partial}{\partial t}\Big)\frac{\partial^2\theta}{\partial x^2} = \Big(1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2}\Big)\Big(\rho c_v\dot{\theta}+\beta\theta_0\frac{\partial^2 u}{\partial x\partial t}\Big) \tag{3}$$

and

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}-\beta\frac{\partial\theta}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}. \tag{4}$$

We introduce the following nondimensional variables

$$\xi=\frac{c_1x}{\kappa},\qquad \eta=\frac{c_1^2t}{\kappa},\qquad \Theta=\frac{\theta}{\theta_0},\qquad U=\frac{c_1(\lambda+2\mu)u}{\kappa\beta\theta_0},$$

where $\kappa = k/\rho c_v$ is the thermal diffusivity. The Equations (3)–(4) reduce to the following nondimensional forms

$$\Big(1+\tau_T^*\frac{\partial}{\partial\eta}\Big)\frac{\partial^2\Theta}{\partial\xi^2} = \Big(1+\tau_q^*\frac{\partial}{\partial\eta}+\frac{1}{2}\tau_q^{*^2}\frac{\partial^2}{\partial\eta^2}\Big)\Big(\frac{\partial\Theta}{\partial\eta}+\varepsilon\frac{\partial^2 U}{\partial\xi\partial\eta}\Big) \tag{5}$$

and

$$\frac{\partial^2 U}{\partial\xi^2}-\frac{\partial\Theta}{\partial\xi}=\frac{\partial^2 U}{\partial\eta^2}, \tag{6}$$

where

$$\tau_q^*=\frac{\tau_q c_1^2}{\kappa},\qquad \tau_T^*=\frac{\tau_T c_1^2}{\kappa},\qquad \varepsilon=\frac{\beta^2\theta_0}{\rho^2 c_v c_1^2},\quad \tau=\frac{\sigma_{xx}}{\beta\theta_0}=\frac{\partial U}{\partial\xi}-\Theta$$

are respectively the nondimensional delay times, the thermoelasticity coupling, and the nondimensional stress.

If τ_q^{*2} is neglected and $\tau_T^*=0$, on setting $\tau_q^*=\tau=$ thermal relaxation parameter, the equation (5) and (6) reduce to L-S theory.

Further if τ_q^{*2} is neglected and $\tau_T^* \neq 0$, Equation (5) reduces to

$$\left(1+\tau_T^*\frac{\partial}{\partial\eta}\right)\frac{\partial^2\Theta}{\partial\xi^2}=\left(1+\tau_q^*\frac{\partial}{\partial\eta}\right)\left(\frac{\partial\Theta}{\partial\eta}+\varepsilon\frac{\partial^2 U}{\partial\xi\partial\eta}\right). \tag{7}$$

This equation with the equation of motion (6) then constitutes a coupled system of field equations of a thermoelasticity theory with non-Fourier heat transport equation (7).

3. Solution of the problem in the Laplace transform domain

We now proceed to study one-dimensional thermoelastic disturbances in the half-space $\xi \geq 0$ on the basis of Equations (5)–(6). We define the Laplace transforms of the functions U(ξ, η) and $\Theta(\xi, \eta)$ by

$$\left\{\overline{U}(\xi, s), \overline{\Theta}(\xi, s)\right\}=\int_0^\infty \{U(\xi,\eta), \Theta(\xi,\eta)\}e^{-s\eta}d\eta,$$

where $\mathrm{Re}(s) > 0$, s is the Laplace transform parameter. We assume that the medium is at rest at $\eta = 0$ and has its temperature, temperature-velocity and temperature acceleration equal to zero at $\eta = 0$. This means that

$$U=\frac{\partial U}{\partial\eta}=\frac{\partial^2 U}{\partial\eta^2}=0 \qquad \text{and} \qquad \Theta=\frac{\partial\Theta}{\partial\eta}=\frac{\partial^2\Theta}{\partial\eta^2}=0, \qquad \text{for } \eta=0, \xi\geq 0. \tag{8}$$

If the disturbances are caused by the sudden application of a constant step in temperature on the boundary which is stress-free at time $\eta > 0$ (Danilovskaya's problem [1950]), then this leads to the boundary conditions

$$\Theta(0,\eta)=\Theta_0 H(\eta), \quad \tau(0,\eta)=0, \quad \eta>0,$$

where Θ_0 is a positive constant, and $H(\eta)$ is the Heaviside unit step function, taking the value 1 if $\eta > 0$ and 0 if $\eta \leq 0$.

On using stress-strain-temperature relations, the conditions become

$$\theta(0,\eta)=\Theta_0 H(\eta), \qquad \frac{\partial U}{\partial\xi}(0,\eta)=\Theta_0 H(\eta), \qquad \text{for } \eta>0. \tag{9}$$

Alternatively, if the thermoelastic interactions are caused by a uniform step in the stress applied to the boundary of the half-space, which is held at reference temperature θ_0, then the following boundary conditions hold:

$$\tau(0,\eta)=-\tau_0 H(\eta), \qquad \Theta(0,\eta)=0, \qquad \eta>0,$$

where τ_0 is a positive constant, or

$$\frac{\partial U}{\partial\xi}(0,\eta)=-\tau_0 H(\eta), \qquad \Theta(0,\eta)=0, \qquad \text{for } \eta>0. \tag{10}$$

Now the equations (5) and (6), on taking Laplace transform, reduce to

$$(ND^2-M)\overline{\Theta}=\varepsilon MD\overline{U}, \tag{11}$$

$$(D^2-s^2)\overline{U}=D\overline{\Theta}, \tag{12}$$

where

$$D = \frac{d}{d\xi}, \qquad M = s\Big(1 + \tau_q^* s + \frac{1}{2}\tau_q^{*2} s^2\Big), \qquad N = 1 + \tau_T^* s.$$

This leads to the following equation satisfied by $\overline{U}$ and $\overline{\Theta}$

$$\big[ND^4 - (Ns^2 + M + M\varepsilon)D^2 + Ms^2\big](\overline{U}, \overline{\Theta}) = 0. \tag{13}$$

The solutions of equation (13), vanishing as $\xi \to \infty$, are assumed to take the form

$$\overline{U} = c_1 e^{-m_1\xi} + c_2 e^{-m_2\xi}, \qquad \overline{\Theta} = c_1^1 e^{-m_1\xi} + c_2^1 e^{-m_2\xi}, \tag{14}$$

where $m_{1,2}$ are the roots with positive real part of the equation

$$Nm^4 - (Ns^2 + M + M\varepsilon)m^2 + Ms^2 = 0. \tag{15}$$

Again, on taking the Laplace transform of the boundary conditions (9) we have

$$\overline{\Theta} = \frac{\Theta_0}{s} \qquad \text{and} \qquad \frac{d\overline{U}}{d\xi} = \frac{\Theta_0}{s}, \qquad \text{on } \xi = 0. \tag{16}$$

Substituting the solutions (14) into (12) and equating the coefficients of like exponentials, we obtain

$$c_1^1 = \frac{c_1(s^2 - m_1^2)}{m_1}, \qquad c_2^1 = \frac{c_2(s^2 - m_2^2)}{m_2}$$

Using the conditions (16) and solving for c_1, c_2, we arrive at the following solutions in the Laplace transform domain:

Case (i):

$$\begin{aligned}
\overline{U} &= \frac{\Theta_0}{s}\,\frac{1}{(m_2^2 - m_1^2)}\big[m_1 e^{-m_1\xi} - m_2 e^{-m_2\xi}\big],\\
\overline{\Theta} &= \frac{\Theta_0}{s}\,\frac{1}{(m_2^2 - m_1^2)}\big[(s^2 - m_1^2)\,e^{-m_1\xi} - (s^2 - m_2^2)\,e^{-m_2\xi}\big],\\
\overline{\tau} &= \frac{\Theta_0 s}{(m_2^2 - m_1^2)}\big[e^{-m_2\xi} - e^{-m_1\xi}\big], \qquad \text{for } \xi > 0.
\end{aligned} \tag{17}$$

Case (ii):

$$\begin{aligned}
\overline{U} &= \frac{\tau_0}{s^3(m_1^2 - m_2^2)}\big[m_1(s^2 - m_2^2)\,e^{-m_1\xi} - m_2(s^2 - m_1^2)\,e^{-m_2\xi}\big],\\
\overline{\Theta} &= \frac{\tau_0(s^2 - m_1^2)(s^2 - m_2^2)}{s^3(m_1^2 - m_2^2)}\big[e^{-m_1\xi} - e^{-m_2\xi}\big],\\
\overline{\tau} &= \frac{\tau_0}{s(m_1^2 - m_2^2)}\big[-(s^2 - m_2^2)\,e^{-m_1\xi} + (s^2 - m_1^2)\,e^{-m_2\xi}\big], \quad \text{for } \xi > 0.
\end{aligned} \tag{18}$$

The roots of the biquadratic equation (15) are given by

$$m_{1,2}^2 = \frac{1}{2N}\Big[Ns^2 + M(1+\varepsilon) \pm \big\{(Ns^2 + M(1+\varepsilon))^2 - 4MNs^2\big\}^{1/2}\Big]. \tag{19}$$

Clearly the roots given by (19) are real if s is real, since

$$(Ns^2+M(1+\varepsilon))^2-4MNs^2=(M-Ns^2)^2+M^2\varepsilon^2+2M\varepsilon(M+Ns^2)>0.$$

The inverse Laplace transforms of (17)–(18) then determine U, Θ, τ. Since $m_{1,2}$ involve the Laplace parameter s, determination of U, Θ, τ is difficult. Since the second sound effects are short-lived, it is sufficient to derive and analyze the solutions for small η. This is done by taking Laplace parameter s to be large.

Taking the sign $+$ in (19), we have for large s,

$$m_1 \cong \frac{s}{v_1}+\frac{1}{2}\frac{\lambda_2}{\lambda_1}\frac{1}{v_1}+\frac{1}{2v_1}\left(\frac{\lambda_3}{\lambda_1}-\frac{1}{4}\frac{\lambda_3^2}{\lambda_1^2}\right)\frac{1}{s}. \tag{20}$$

Taking the sign $-$ in (19), we have for large s

$$m_2 \cong \frac{s}{v_2}+\frac{1}{2}\frac{\mu_2}{\mu_1}\frac{1}{v_2}+\frac{1}{2v_2}\left(\frac{\mu_3}{\mu_1}-\frac{1}{4}\frac{\mu_3^2}{\mu_1^2}\right)\frac{1}{s}, \tag{21}$$

where $\lambda_1=A+\sqrt{A^2-4F}$, $\lambda_2=B+\frac{L_1}{2}\sqrt{A^2-4F}-\frac{A+\sqrt{A^2-4F}}{\tau_T^*}$,

$$v_1=\left(\frac{2\tau_T^*}{\lambda_1}\right)^{1/2}, \qquad v_2=\left(\frac{2\tau_T^*}{\mu_1}\right)^{1/2},$$

$$\lambda_3=C+\frac{\sqrt{A^2-4F}}{8}(4L_2^2-L_1^2)+\frac{1}{\tau_T^{*2}}(A+\sqrt{A^2-4F})-\frac{1}{\tau_T^*}(B+\frac{L_1}{2}\sqrt{A^2-4F}), \tag{22}$$

$$L_1=\frac{2AB-4D}{A^2-4F}, \qquad L_2=\frac{B^2+2CA-4E}{A^2-4F}, \qquad A=\tau_T^*+\frac{1}{2}(1+\varepsilon)\tau_q^{*2},$$

$$B=1+(1+\varepsilon)\tau_q^*, \qquad C=1+\varepsilon, \qquad D=\frac{1}{2}\tau_q^{*2}+\tau_T^*\tau_q^*,$$

$$E=\tau_T^*+\tau_q^*, \qquad F=\frac{1}{2}\tau_T^*\tau_q^{*2},$$

$$\mu_1=A-\sqrt{A^2-4F},$$

$$\mu_2=B-\frac{L_1}{2}\sqrt{A^2-4F}-\frac{1}{\tau_T^*}\big(A-\sqrt{A^2-4F}\big),$$

$$\mu_3=C-\frac{\sqrt{A^2-4F}}{8}(4L_2^2-L_1^2)+\frac{1}{\tau_T^{*2}}\big(A-\sqrt{A^2-4F}\big)-\frac{1}{\tau_T^*}\Big(B-\frac{L_1}{2}\sqrt{A^2-4F}\Big).$$

We note that

$$A^2-4F=\frac{1}{4}\big[\big(2\tau_T^*-\tau_q^*\big)^2+\varepsilon^2\tau_q^{*4}+4\tau_T^*\tau_q^{*2}\varepsilon\big]>0$$

and $A>0$.

This indicates that $v_{1,2}$ are both real.

Clearly $\lambda_1, \mu_1>0$ since $A>\sqrt{A^2-4F}$ and $F>0$. Further $\lambda_1>\mu_1$ implies $v_2>v_1$.

Now we are to prove that, under suitable restrictions on material constants, λ_2 and μ_2 are positive. We have

$$\lambda_2 = B + \frac{L_1}{2}\sqrt{A^2-4F} - \frac{A+\sqrt{A^2-4F}}{\tau_T^*}, \qquad \mu_2 = B - \frac{L_1}{2}\sqrt{A^2-4F} - \frac{A-\sqrt{A^2-4F}}{\tau_T^*}.$$

Now

$$\lambda_2 > 0, \text{ if } B + \frac{L_1}{2}\sqrt{A^2-4F} > \frac{A+\sqrt{A^2-4F}}{\tau_T^*}.$$

That is, if

$$B > \frac{A}{\tau_T^*} + \frac{\sqrt{A^2-4F}}{\tau_T^*}\left(1 - \frac{L_1\tau_T^*}{2}\right).$$

Similarly,

$$\mu_2 > 0, \text{ if } B > \frac{A}{\tau_T^*} - \frac{\sqrt{A^2-4F}}{\tau_T^*}\left(1 - \frac{L_1\tau_T^*}{2}\right).$$

We impose the restriction on material parameters such that $1 > \frac{L_1}{2}\tau_T^*$. Then since $A, \tau_T^* > 0$, B must be positive under the restriction $1 > \frac{L_1}{2}\tau_T^*$. The required restriction, on substitution for L_1, yields $A^2 - 4F > \tau_T^*(AB - 2D)$. This leads to the inequality

$$(\tau_q^* - 2\tau_T^*)[(1+\varepsilon)\tau_q^{*^2} - 2\tau_T^*(1-\varepsilon)] > 0.$$

This is satisfied if $\tau_q^* > 2\tau_T^*$ and

$$(1+\varepsilon)\tau_q^{*^2} - 2\tau_T^*(1-\varepsilon) > (1+\varepsilon)4\tau_T^{*^2} - 2\tau_T^*(1-\varepsilon) = 2\tau_T^*\{2(1+\varepsilon)\tau_T^* + \varepsilon - 1\} > 0 \quad \text{or} \quad 2(1+\varepsilon)\tau_T^* > 1-\varepsilon.$$

Thus the two conditions are $2(1+\varepsilon)\tau_T^* > 1-\varepsilon$, that is,

$$\tau_T* > \frac{1-\varepsilon}{2(1+\varepsilon)} \qquad \text{and} \qquad \tau_q^* > \sqrt{\frac{2\tau_T^*(1-\varepsilon)}{(1+\varepsilon)}}.$$

Further since $\tau_q^* > 2\tau_T^*$, we must have

$$\tau_q^* > \frac{1-\varepsilon}{1+\varepsilon} =: \varepsilon_0.$$

The required restrictions on material constants for $\lambda_2 > 0$ then reduce to

$$\tau_q > \frac{k\varepsilon_0}{\rho c_v c_1^2} \qquad \text{and} \qquad \tau_T > \frac{k}{2\rho c_v c_1^2}\varepsilon_0.$$

Since $\lambda_2 > 0$ implies $\mu_2 > 0$, the inequalities

$$\tau_q > \frac{k\varepsilon_0}{\rho c_v c_1^2} \qquad \text{and} \qquad \tau_T > \frac{k\varepsilon_0}{2\rho c_v c_1^2}$$

imply that $\mu_2 > 0$. Using the results

$$m_1 \cong \frac{s}{v_1} + \frac{1}{2}\frac{\lambda_2}{\lambda_1}\frac{1}{v_1} \qquad \text{and} \qquad m_2 \cong \frac{s}{v_2} + \frac{1}{2}\frac{\mu_2}{\mu_1}\frac{1}{v_2}$$

for large s, we obtain the following results after simplification:

$$\frac{m_1}{s(m_2^2-m_1^2)} \cong \frac{1}{L_0 v_1}\frac{1}{s^2} - \frac{1}{L_0}\left(\frac{M_0}{L_0}+\frac{\lambda_2}{2\lambda_1}\right)\frac{1}{v_1}\frac{1}{s^3},$$

$$\frac{m_2}{s(m_2^2-m_1^2)} \cong \frac{1}{L_0 v_2}\frac{1}{s^2} - \frac{1}{L_0}\left(\frac{M_0}{L_0}+\frac{\mu_2}{2\mu_1}\right)\frac{1}{v_2}\frac{1}{s^3},$$

$$\frac{s^2-m_2^2}{s(m_2^2-m_1^2)} \cong \frac{1}{L_0}\left(\frac{v_2^2-1}{v_2^2}\frac{1}{s} - \left(\frac{M_0}{L_0}\frac{v_2^2-1}{v_2^2}+\frac{\mu_2}{\mu_1 v_2^2}\right)\frac{1}{s^2} + \left(\frac{\lambda_2}{\lambda_1 v_1^2}\frac{M_0}{L_0}+\frac{v_1^2-1}{v_1^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\right)\frac{1}{s^3}\right),$$

$$\frac{s^2-m_1^2}{s(m_2^2-m_1^2)} \cong \frac{1}{L_0}\left(\frac{v_1^2-1}{v_1^2}\frac{1}{s} - \left(\frac{M_0}{L_0}\frac{v_1^2-1}{v_1^2}+\frac{\lambda_2}{\lambda_1 v_1^2}\right)\frac{1}{s^2} + \left(\frac{\mu_2}{\mu_1 v_2^2}\frac{M_0}{L_0}+\frac{v_2^2-1}{v_2^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\right)\frac{1}{s^3}\right),$$

$$\frac{(s^2-m_1^2)(s^2-m_2^2)}{s^3(m_1^2-m_2^2)} \cong -\frac{1}{L_0}\left(\frac{(v_1^2-1)(v_2^2-1)}{v_1^2 v_2^2}\frac{1}{s} - \left(\frac{(v_1^2-1)\mu_2}{v_1^2\cdot\mu_1 v_2^2}+\frac{(v_2^2-1)}{v_2^2}\alpha_1\right)\frac{1}{s^2} + \left(\frac{\mu_2}{\mu_1 v_2^2}\alpha_1+\frac{(v_2^2-1)}{v_2^2}\alpha_2\right)\frac{1}{s^3}\right),$$

where

$$L_0=\frac{1}{v_2^2}-\frac{1}{v_1^2},\qquad M_0=\frac{\mu_2}{\mu_1 v_2^2}-\frac{\lambda_2}{\lambda_1 v_1^2},\qquad N_0=\frac{1}{4}\left(\frac{\mu_2^2}{\mu_1^2 v_2^2}-\frac{\lambda_2^2}{\lambda_1^2 v_1^2}\right),$$

$$\alpha_1=\frac{M_0}{L_0}\frac{(v_1^2-1)}{v_1^2}+\frac{\lambda_2}{\lambda_1 v_1^2},\ \alpha_2=\frac{\lambda_2}{\lambda_1 v_1^2}\frac{M_0}{L_0}+\frac{(v_1^2-1)}{v_1^2}\frac{(M_0^2-N_0L_0)}{L_0^2}.$$

Finally we obtain the following solutions for displacement, temperature and stress fields in the Laplace transform domain for large s:

Case (i):

$$\overline{U}(\xi,s) \cong \Theta_0\Bigg[\left\{\frac{1}{L_0 v_1}\frac{1}{s^2}-\frac{1}{L_0}\left(\frac{M_0}{L_0}+\frac{\lambda_2}{2\lambda_1}\right)\frac{1}{v_1}\frac{1}{s^3}\right\}e^{-\left(\frac{s}{v_1}+\frac{\lambda_2}{2\lambda_1 v_1}\right)\xi} - \left\{\frac{1}{L_0 v_2}\frac{1}{s^2}-\frac{1}{L_0}\left(\frac{M_0}{L_0}+\frac{\mu_2}{2\mu_1}\right)\frac{1}{v_2}\frac{1}{s^3}\right\}e^{-\left(\frac{s}{v_2}+\frac{\mu_2}{2\mu_1 v_2}\right)\xi}\Bigg],$$

$$\overline{\Theta}(\xi,s) \cong \frac{\Theta_0}{L_0}\Bigg[\left\{\frac{(v_1^2-1)}{v_1^2}\frac{1}{s}-\left(\frac{M_0}{L_0}\frac{v_1^2-1}{v_1^2}+\frac{\lambda_2}{\lambda_1 v_1^2}\right)\frac{1}{s^2} + \left(\frac{\lambda_2}{\lambda_1 v_1^2}\frac{M_0}{L_0}+\frac{(v_1^2-1)}{v_1^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\right)\frac{1}{s^3}\right\}e^{-\left(\frac{s}{v_1}+\frac{\lambda_2}{2\lambda_1 v_1}\right)\xi} - \left\{\frac{(v_2^2-1)}{v_2^2}\frac{1}{s}-\left(\frac{M_0}{L_0}\frac{(v_2^2-1)}{v_2^2}+\frac{\mu_2}{\mu_1 v_2^2}\right)\frac{1}{s^2} + \left(\frac{\mu_2}{\mu_1 v_2^2}\frac{M_0}{L_0}+\frac{(v_2^2-1)}{v_2^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\right)\frac{1}{s^3}\right\}e^{-\left(\frac{s}{v_2}+\frac{\mu_2}{2\mu_1 v_2}\right)\xi}\Bigg],$$

$$\overline{\tau}(\xi,s)\cong\frac{\Theta_0}{L_0}\Bigg[-\left\{\frac{1}{s}-\frac{M_0}{L_0}\frac{1}{s^2}+\frac{M_0^2-L_0N_0}{L_0^2}\frac{1}{s^3}\right\}e^{-\left(\frac{s}{v_1}+\frac{\lambda_2}{2\lambda_1 v_1}\right)\xi}+\left\{\frac{1}{s}-\frac{M_0}{L_0}\frac{1}{s^2}+\frac{M_0^2-L_0N_0}{L_0^2}\frac{1}{s^3}\right\}e^{-\left(\frac{s}{v_2}+\frac{\mu_2}{2\mu_1 v_2}\right)\cdot\xi}\Bigg].$$

Case (ii):

$$\begin{aligned}\overline{U}(\xi,s)\cong\frac{\tau_0}{L_0}\Bigg[&-\frac{1}{v_1}\left\{\frac{(v_2^2-1)}{v_2^2}\frac{1}{s^2}-\left(\frac{M_0}{L_0}\frac{v_2^2-1}{v_2^2}+\frac{\mu_2}{\mu_1 v_2^2}\right)\frac{1}{s^3}\right.\\&\left.+\left(\frac{\mu_2}{\mu_1 v_2^2}\frac{M_0}{L_0}+\frac{(v_2^2-1)}{v_2^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\right)\frac{1}{s^4}\right\}e^{-\left(\frac{s}{v_1}+\frac{\lambda_2}{2\lambda_1 v_1}\right)\xi}\\&+\frac{1}{v_2}\left\{\frac{(v_1^2-1)}{v_1^2}\frac{1}{s^2}-\left(\frac{M_0}{L_0}\frac{(v_1^2-1)}{v_1^2}+\frac{\lambda_2}{\lambda_1}\frac{1}{v_1^2}\right)\frac{1}{s^3}\right.\\&\left.+\left(\frac{\lambda_2}{\lambda_1 v_1^2}\frac{M_0}{L_0}+\frac{(v_1^2-1)}{v_1^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\right)\frac{1}{s^4}\right\}e^{-\left(\frac{s}{v_2}+\frac{\mu_2}{2\mu_1 v_2}\right)\xi}\Bigg],\end{aligned}$$

$$\begin{aligned}\overline{\Theta}(\xi,s)\cong-\frac{\tau_0}{L_0}\Bigg[&\frac{(v_1^2-1)(v_2^2-1)}{v_1^2v_2^2}\frac{1}{s}-\left\{\frac{(v_1^2-1)\mu_2}{v_1^2v_2^2\mu_1}+\frac{(v_2^2-1)}{v_2^2}\left(\frac{M_0}{L_0}\frac{(v_1^2-1)}{v_1^2}+\frac{\lambda_2}{\lambda_1 v_1^2}\right)\right\}\cdot\frac{1}{s^2}\\&+\left\{\frac{\mu_2}{\mu_1 v_2^2}\alpha_1+\frac{v_2^2-1}{v_2^2}\alpha_2\right\}\frac{1}{s^3}\Bigg]\times\left\{e^{-\left(\frac{s}{v_1}+\frac{\lambda_2}{2\lambda_1 v_1}\right)\xi}-e^{-\left(\frac{s}{v_2}+\frac{\mu_2}{2\mu_1 v_2}\right)\xi}\right\},\end{aligned}$$

$$\begin{aligned}\overline{\tau}(\xi,s)\cong\frac{\tau_0}{L_0}\Bigg[&\left\{\frac{(v_2^2-1)}{v_2^2}\frac{1}{s}-\left(\frac{M_0}{L_0}\frac{(v_2^2-1)}{v_2^2}+\frac{\mu_2}{\mu_1 v_2^2}\right)\frac{1}{s^2}\right.\\&\left.+\left(\frac{\mu_2}{\mu_1 v_2^2}\frac{M_0}{L_0}+\frac{(v_2^2-1)}{v_2^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\right)\frac{1}{s^3}\right\}e^{-\left(\frac{s}{v_1}+\frac{\lambda_2}{2\lambda_1 v_1}\right)\xi}\\&-\left\{\frac{(v_1^2-1)}{v_1^2}\frac{1}{s}-\left(\frac{M_0}{L_0}\frac{(v_1^2-1)}{v_1^2}+\frac{\lambda_2}{\lambda_1 v_1^2}\right)\frac{1}{s^2}\right.\\&\left.+\left(\frac{\lambda_2}{\lambda_1 v_1^2}\frac{M_0}{L_0}+\frac{(v_1^2-1)(M_0^2-L_0N_0)}{v_1^2L_0^2}\right)\frac{1}{s^3}\right\}e^{-\left(\frac{s}{v_2}+\frac{\mu_2}{2\mu_1 v_2}\right)\xi}\Bigg].\end{aligned}$$

4. Derivation of small-time solutions

Inverse Laplace transforms of the expressions yield the following small-time solutions for displacement, temperature and stress fields.

Case (i):

$$U(\xi,\eta)\cong\Theta_0\Bigg[e^{-\frac{\lambda_2}{2\lambda_1 v_1}\xi}\Bigg\{\frac{1}{L_0 v_1}\Big(\eta-\frac{\xi}{v_1}\Big)$$
$$-\frac{1}{L_0}\Big(\frac{M_0}{L_0}+\frac{\lambda_2}{2\lambda_1}\Big)\frac{1}{v_1}\frac{1}{2}\Big(\eta-\frac{\xi}{v_1}\Big)^2\Bigg\}H\Big(\eta-\frac{\xi}{v_1}\Big)-e^{-\frac{\mu_2}{2\mu_1 v_2}\xi}\Bigg\{\frac{1}{L_0 v_2}\Big(\eta-\frac{\xi}{v_2}\Big),$$
$$-\frac{1}{L_0}\Big(\frac{M_0}{L_0}+\frac{\mu_2}{2\mu_1}\Big)\frac{1}{v_1}\frac{1}{2}\Big(\eta-\frac{\xi}{v_2}\Big)^2\Bigg\}H\Big(\eta-\frac{\xi}{v_2}\Big)\Bigg], \quad (23)$$

$$\Theta(\xi,\eta)\cong\frac{\Theta_0}{L_0}\Bigg[e^{-\frac{\lambda_2}{2\lambda_1 v_1}\xi}\Bigg\{\frac{(v_1^2-1)}{v_1^2}-\Big(\frac{M_0}{L_0}\frac{(v_1^2-1)^2}{v_1^2}+\frac{\lambda_2}{\lambda_1 v_1^2}\Big)\Big(\eta-\frac{\xi}{v_1}\Big)$$
$$+\Big(\frac{\lambda_2}{\lambda_1 v_1^2}\frac{M_0}{L_0}+\frac{(v_1^2-1)}{v_1^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\Big)\frac{1}{2}\Big(\eta-\frac{\xi}{v_1}\Big)^2\Bigg\}H\Big(\eta-\frac{\xi}{v_1}\Big)$$
$$-e^{-\frac{\mu_2}{2\mu_1 v_2}\xi}\Bigg\{\frac{(v_2^2-1)}{v_2^2}-\Big(\frac{M_0}{L_0}\frac{(v_2^2-1)}{v_2^2}+\frac{\mu_2}{\mu_1 v_2^2}\Big)\Big(\eta-\frac{\xi}{v_2}\Big)$$
$$+\Big(\frac{\mu_2}{\mu_1 v_2^2}\frac{M_0}{L_0}+\frac{(v_2^2-1)}{v_2^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\Big)\frac{1}{2}\Big(\eta-\frac{\xi}{v_2}\Big)^2\Bigg\}H\Big(\eta-\frac{\xi}{v_2}\Big)\Bigg], \quad (24)$$

$$\tau(\xi,\eta)\cong\frac{\Theta_0}{L_0}\Bigg[-e^{-\frac{\lambda_2}{2\lambda_1 v_1}\xi}\Bigg\{1-\frac{M_0}{L_0}\Big(\eta-\frac{\xi}{v_1}\Big)+\frac{M_0^2-L_0N_0}{L_0^2}\frac{1}{2}\Big(\eta-\frac{\xi}{v_2}\Big)^2\Bigg\}H\Big(\eta-\frac{\xi}{v_1}\Big)$$
$$+e^{-\frac{\mu_2}{2\mu_1 v_2}\xi}\Bigg\{1-\frac{M_0}{L_0}\Big(\eta-\frac{\xi}{v_2}\Big)+\frac{(M_0^2-L_0N_0)}{L_0^2}\frac{1}{2}\Big(\eta-\frac{\xi}{v_2}\Big)^2\Bigg\}H\Big(\eta-\frac{\xi}{v_2}\Big)\Bigg]. \quad (25)$$

Case (ii):

$$U(\xi,\eta)\cong\frac{\tau_0}{L_0}\Bigg[-\frac{e}{v_1}^{-\frac{\lambda_2}{2\lambda_1 v_1}\xi}\Bigg\{\frac{(v_2^2-1)}{v_2^2}\Big(\eta-\frac{\xi}{v_1}\Big)-\Big(\frac{M_0}{L_0}\frac{v_2^2-1}{v_2^2}+\frac{\mu_2}{\mu_1 v_2^2}\Big)\frac{1}{2}\Big(\eta-\frac{\xi}{v_1}\Big)^2$$
$$+\Big(\frac{\mu_2}{\mu_1 v_2^2}\frac{M_0}{L_0}+\frac{(v_2^2-1)}{v_2^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\Big)\frac{1}{6}\Big(\eta-\frac{\xi}{v_1}\Big)^3\Bigg\}H\Big(\eta-\frac{\xi}{v_1}\Big)$$
$$+\frac{e^{-\frac{\mu_2}{2\mu_1 v_2}\xi}}{v_2}\Bigg\{\frac{(v_1^2-1)}{v_1^2}\Big(\eta-\frac{\xi}{v_2}\Big)-\Big(\frac{M_0}{L_0}\frac{v_1^2-1}{v_1^2}+\frac{\lambda_2}{\lambda_1}\frac{1}{v_1^2}\Big)\frac{1}{2}\Big(\eta-\frac{\xi}{v_2}\Big)^2$$
$$+\Big(\frac{\lambda_2}{\lambda_1 v_1^2}\frac{M_0}{L_0}+\frac{(v_1^2-1)}{v_1^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\Big)\frac{1}{6}\Big(\eta-\frac{\xi}{v_2}\Big)^3\Bigg\}H\Big(\eta-\frac{\xi}{v_2}\Big)\Bigg], \quad (26)$$

$$\Theta(\xi,\eta) \cong -\frac{\tau_0}{L_0}\left[\phi\Big(\eta-\frac{\xi}{v_1}\Big)H\Big(\eta-\frac{\xi}{v_1}\Big)e^{-\frac{\lambda_2}{2\lambda_1 v_1}\xi} - \varphi\Big(\eta-\frac{\xi}{v_2}\Big)H\Big(\eta-\frac{\xi}{v_2}\Big)e^{-\frac{\mu_2}{2\mu_1 v_2}\xi}\right], \tag{27}$$

$$\begin{aligned}\tau(\xi,\eta) \cong \frac{\tau_0}{L_0}\Bigg[& e^{-\frac{\lambda_2}{2\lambda_1 v_1}\xi}\left\{\frac{v_2^2-1}{v_2^2}H\Big(\eta-\frac{\xi}{v_1}\Big)-\Big(\frac{M_0}{L_0}\frac{v_2^2-1}{v_2^2}+\frac{\mu_2}{\mu_1 v_2^2}\Big)\Big(\eta-\frac{\xi}{v_1}\Big)H\Big(\eta-\frac{\xi}{v_1}\Big)\right.\\ & \left.+\Big(\frac{\mu_2}{\mu_1 v_2^2}\frac{M_0}{L_0}+\frac{(v_2^2-1)}{v_2^2}\frac{(M_0^2-N_0L_0)}{L_0^2}\Big)\frac{1}{2}\Big(\eta-\frac{\xi}{v_1}\Big)^2 H\Big(\eta-\frac{\xi}{v_1}\Big)\right\}\\ & -e^{-\frac{\mu_2}{2\mu_1 v_2}\xi}\left\{\frac{v_1^2-1}{v_1^2}H\Big(\eta-\frac{\xi}{v_2}\Big)-\Big(\frac{M_0}{L_0}\frac{(v_1^2-1)}{v_1^2}+\frac{\lambda_2}{\lambda_1 v_1^2}\Big)\Big(\eta-\frac{\xi}{v_2}\Big)H\Big(\eta-\frac{\xi}{v_2}\Big)\right.\\ & \left.+\Big(\frac{\lambda_2}{\lambda_1 v_1^2}\frac{M_0}{L_0}+\frac{(v_1^2-1)(M_0^2-N_0L_0)}{v_1^2L_0^2}\Big)\frac{1}{2}\Big(\eta-\frac{\xi}{v_2}\Big)^2 H\Big(\eta-\frac{\xi}{v_2}\Big)\right\}\Bigg],\end{aligned} \tag{28}$$

where

$$\varphi(\eta)=\frac{(v_1^2-1)(v_2^2-1)}{v_1^2v_2^2}-\left\{\frac{(v_1^2-1)\mu_2}{v_1^2v_2^2\mu_1}+\frac{(v_2^2-1)}{v_2^2}\Big(\frac{M_0}{L_0}\frac{(v_1^2-1)}{v_1^2}+\frac{\lambda_2}{\lambda_1 v_1^2}\Big)\right\}\eta+ \left\{\frac{\mu_2}{\mu_1 v_2^2}\alpha_1+\frac{v_2^2-1}{v_2^2}\alpha_2\right\}\frac{1}{2}\eta^2.$$

5. Analysis of the solutions

The short-time solutions (23)–(28) for displacement, temperature, and stress fields reveal the existence of two waves. Each of U, Θ and τ is composed of two parts and that each part corresponds to a wave propagating with a finite speed. The speed of the wave corresponding to the first part is v_1 and that corresponding to the second part is v_2. The faster wave has its speed equal to v_2 and the slower wave has its speed equal to v_1.

One cannot classify (5) independently of (6) (as hyperbolic or parabolic), since a type of the coupled system (5)–(6) must be determined.

For $\tau_q^{*2}=0$, we obtain from (22),

$$\begin{aligned} A&=\tau_T^*, & B&=1+(1+\varepsilon)\tau_q^*,\\ C&=1+\varepsilon, & D&=\tau_T^*\tau_q^*,\\ E&=\tau_T^*+\tau_q^*, & F&=0,\\ \lambda_1&=2\tau_T^*, & \lambda_2&=2\varepsilon\tau_q^*,\\ \mu_1&=0, & \mu_2&=2\tau_q^*,\end{aligned}$$

$$L_1 = \frac{2 + 2\tau_q^*(\varepsilon - 1)}{\tau_T^*},$$

$$L_2 = \left\{(1 - \tau_q^*)^2 + \varepsilon^2 \tau_q^{*2} + 2\varepsilon\tau_q^*(1 + \tau_q^*) + 2\tau_T^*(\varepsilon - 1)\right\}(\tau_T^*)^2.$$

These give

$$v_2 = \infty, \qquad v_1 \longrightarrow 1, \qquad L_0 \longrightarrow -1, \qquad M_0 = -\varepsilon\tau_q^*/\tau_T^*, \qquad N_0 = -\frac{\varepsilon^2 \tau_q^{*2}}{4\tau_T^{*2}}.$$

Therefore, for $\tau_q^{*2} = 0$, the system of equations (5)–(6) is of the mixed parabolic-hyperbolic type.

For $\tau_q^{*2} \neq 0$, and $\tau_T^* \neq 0$, v_2 corresponds to the modified speed of thermal signals and v_1 corresponds to the modified elastic dilatational wave speed, both modified by delay times in the thermoelastic solid with dual phase-lag effects. Since $v_1 < v_2$, the faster wave is the predominantly modified Tzou wave (T-wave) and the slower is a predominantly modified elastic wave (E-wave). The first term of the solutions (23)–(28) represents the contribution of the E-wave near its wave front $\xi = v_1\eta$ and the second term represents the contribution of the T-wave near its wave front $\xi = v_2\eta$. We observe also that both the waves experience decay exponentially with the distance (attenuation), which is also the case in CTE, ETE and TRDTE, but not the case in TEWOED where the waves do not experience attenuation (see [Dhaliwal and Rokne 1988;1989; Chandrasekhariah and Srinath 1996;1997]. From (23)–(28), we further note that all of U, Θ, τ are identically zero for $\xi > v_2\eta$. This implies that at a given instant of time $\eta*$, the points of the solid $\xi > 0$ that are beyond the faster wave front $\xi = v_2\eta*$ do not experience any disturbances. This observation confirms that, like ETE, TRDTE, TEWOED the thermoelasticity theory with dual phase-lag effects is also a wave thermoelasticity theory. Moreover it is interesting to record that at a given instant, the region $0 < \xi < v_2\eta*$ is the domain of influence of the disturbance, contrary to the result that this domain extends and the effects occur instantaneously everywhere in the solid as predicted by CTE, see [Boley and Tolins 1962].

By direct inspection of the solutions (23)–(28), we find that in both cases U is continuous whereas both Θ and τ are discontinuous at both the wave fronts. The finite jumps experienced by temperature and stress at the wave fronts are as follows:

Case (i):

$$[\Theta]_{\xi = v_1\eta} = -\frac{\Theta_0}{L_0}\frac{v_1^2 - 1}{v_1^2} e^{-\frac{\lambda_2}{2\lambda_1}\eta},$$

$$[\Theta]_{\xi = v_2\eta} = \frac{\Theta_0}{L_0}\frac{v_2^2 - 1}{v_2^2} e^{-\frac{\mu_2}{2\mu_1}\eta},$$

$$[\tau]_{\xi = v_1\eta} = \frac{\Theta_0}{L_0} e^{-\frac{\lambda_2}{2\lambda_1}\eta},$$

$$[\tau]_{\xi = v_2\eta} = -\frac{\Theta_0}{L_0} e^{-\frac{\mu_2}{2\mu_1}\eta}.$$

Case (ii):

$$
\begin{aligned}
[\Theta]_{\xi=v_1\eta} &= \frac{\tau_0}{L_0}\frac{(v_1^2-1)(v_2^2-1)}{v_1^2v_2^2}e^{-\frac{\lambda_2}{2\lambda_1}\cdot\eta},\\
[\Theta]_{\xi=v_2\eta} &= -\frac{\tau_0}{L_0}\frac{(v_1^2-1)(v_2^2-1)}{v_1^2v_2^2}e^{-\frac{\mu_2}{2\mu_1}\cdot\eta},\\
[\tau]_{\xi=v_1\eta} &= -\frac{\tau_0}{L_0}\frac{v_2^2-1}{v_2^2}e^{-\frac{\lambda_2}{2\lambda_1}\cdot\eta},\\
[\tau]_{\xi=v_2\eta} &= +\frac{\tau_0}{L_0}\frac{v_1^2-1}{v_1^2}e^{-\frac{\mu_2}{2\mu_1}\cdot\eta}.
\end{aligned}
\tag{29}
$$

Here $[f]$ denotes the jump of the function f across a wave front. The finite jumps are not constants but they decay exponentially with distance from the boundary. The same results are observed to occur in ETE, TRDTE [Dhaliwal and Rokne 1988;1989], but not in TEWOED where the jumps are all constants [Chandrasekhariah and Srinath 1996;1997]. However the discontinuity in temperature and stress at both the wave fronts is a situation common in the context of ETE, TRDTE and TEWOED [Norwood and Warren 1969; Sherief and Dhaliwal 1981; Dhaliwal and Rokne 1988;1989; Chandrasekhariah and Srinath 1996]. Clearly the finite jumps of Θ and τ in the present analysis depend on the delay times τ_T^*, τ_q^* and the thermo elastic coupling ε. The magnitudes of these jumps are exact, valid for short times and hold for all possible values of ε, τ_T^* and τ_q^*. The expressions for jumps for case (ii) reveal another interesting phenomenon. The temperature is discontinuous at both the wave fronts in spite of the fact that the boundary load is purely mechanical in nature. This implies that the application of a discontinuous mechanical load on the boundary does generate discontinuities in temperature. This phenomenon is present in ETE as well, see [Sharma 1987] but absent in TRDTE [Chandrasekharaiah and Keshavan 1992]. According to TRDTE, the temperature is continuous when the applied load is purely mechanical in nature [Chandrasekharaiah and Keshavan 1992]. A similar observation has been made in the half-space problem in the context of TEWOED [Chandrasekharaiah 1996; Chandrasekharaiah and Srinath 1997].

If the effect of τ_q^{*2} is neglected with $\tau_T^* \neq 0$, $v_1 \to 1$, and $v_2 \to \infty$, then there exists only one wave front $\xi = \eta$ (E-wave front) and the T-wave propagates with infinite speed as expected from Equations (5)–(6) with $\tau_q^{*2} = 0$. In this case the jumps of Θ and τ at the elastic wave front $\xi = \eta$ are given as follows:

Case (i):

$$
[\Theta]_{\xi=\eta} = 0, \qquad [\tau]_{\xi=\eta} = -\Theta_0 e^{-\frac{1}{2}\varepsilon\frac{\tau_q^*}{\tau_T^*}\xi}.
$$

Case (ii):

$$
[\Theta]_{\xi=\eta} = 0, \qquad [\tau]_{\xi=\eta} = \tau_0 \cdot e^{-\frac{1}{2}\varepsilon\cdot\frac{\tau_q^*}{\tau_T^*}\xi}. \tag{30}
$$

The jumps of temperature disappear at the E-wave front whereas that of stress τ exists and depends on ε, τ_T^*, τ_q^*. These results are in complete agreement (for $\tau_T^* = \tau_q^*$) with the corresponding results obtained in the context of CTE with classical Fourier's law, see [Boley and Tolins 1962]. Moreover, it is interesting to record from solutions (25) and (28) that because of delay times, the position of jumps of stress shifts from $\xi = \eta$ in CTE to $\xi = v_1\eta$ in the present analysis. Also the dual phase-lag model introduced by [Tzou 1995a;1995b] brings to light the jumps of temperature Θ at the E-wave front too, which are not encountered in CTE.

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