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DENSITY MEASUREMENTS IN A SUPERSONIC JET

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We use a nonintrusive optical technique for heterodyne detection of the light scattered elastically by the molecules of a moving transparent gas, a phenomenon known as Rayleigh scattering. It can be shown that the signal that comes out of the photodetector is proportional to the spatial Fourier transform as a function of time of the density fluctuations, for a wave vector given by the optical set-up. This is the only technique we are aware of that can study density fluctuations *inside* a flow.

In this paper we present results obtained from a supersonic axisymmetric air jet. The signal that comes out of the photodetector is processed, and the power spectrum calculated. In the spectrum, density fluctuations of two different origins can be identified: acoustic, that is, those that propagate at the speed of sound and are related to pressure variations, and entropic, those that have constant pressure and are convected by the flow. At certain locations we have found an additional peak related to the interaction between the flow and the shock structure. Furthermore, Rayleigh scattering can be used to visualize the shock structure of the flow. We provide supporting images for our results.

1. Introduction

The original objectives of this work were to localize sound sources in a supersonic jet, relate the production of sound with flow phenomena and determine the acoustic radiation pattern inside and outside the flow. We use a Rayleigh scattering technique that can measure density fluctuations inside the flow for a given wavevector. Our jet is very turbulent and sound sources are not localized. However, we have been able to visualize the shock structure, allowing us to relate the spectrum at each location and at each angle to the compression and expansion waves in the supersonic region of the jet. We have been able to determine the direction of propagation of sound waves inside and outside the jet including the mixing layer. We have also found an unexpected peak in the spectrum that is related to the interaction between the flow and the shock structure.

2. Background

2.1. ***Acoustic emission.*** There are several theories that try to explain acoustic emission in a jet. Some of them are based on the interactions between large-scale structures in the flow: pairing, cut and connect, and annihilation. In general, it is accepted that large-scale structure interactions produce sound that propagates at large angles, while small structures produce sound that propagates at small angles. There are other possible sources of emission such as the interaction of the flow with the shock waves and the feedback of acoustic waves that reenter the flow.

Keywords: aeroacoustics, Rayleigh scattering, supersonic flow.
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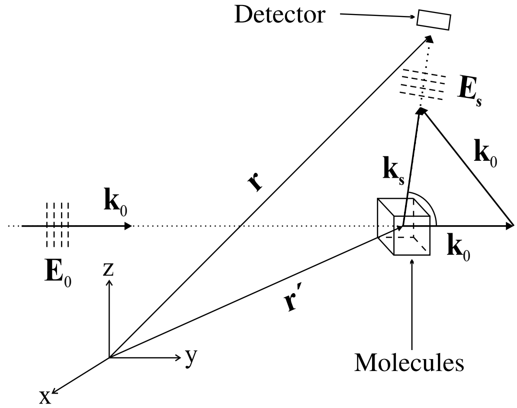

Figure 1. Molecules scatter light in all directions. Selecting detector orientation, we specify scattering angle and size of fluctuations to be studied.

Traditionally, experimental studies on acoustic waves have been performed by placing many microphones in the far field and correlating these measurements with events measured inside the flow. Not only is there a problem trying to determine sources from far field measurements as the solution of the inverse problem is not unique, but this method fails to take into account certain phenomena such as the diffraction of the acoustic waves by the mixing layer.

2.2. ***Rayleigh scattering.*** The elastic scattering of an electromagnetic wave of wavelength λ_0 by a neutral particle of dimensions smaller than the wavelength is known as Rayleigh scattering. In a static transparent gas, light is scattered homogeneously, and the scattered field is constant. If the gas is in motion or with strong density variations, the characteristics of the scattered light reflect the characteristics of the structure and motion of the gas. In the far field, the light scattered by one molecule is given by

$$\vec{E}_{S0} = r_0^R \frac{e^{ik_0 r}}{r} \{\vec{r} \times \vec{E}_0(\vec{r'}) \times \vec{r}\},$$

where r_0^R is the Rayleigh scattering cross section. Figure 1 shows the wave vectors of the incident and scattered light. The total scattered field can be obtained from the integral

$$\vec{E}_S = \vec{E}_{S0}(\vec{r}, t) \int_{V_S} d^3r' n(\vec{r'}, t) e^{i\vec{k_\Delta}\cdot\vec{r}} = \vec{E}_{S0}(\vec{r}, t) n(\vec{k}_\Delta, t),$$

where $n(\vec{r'}, t)$ is the distribution of molecules in the scattering volume V_s, $n(\vec{k}_\Delta, t)$ is the spatial Fourier transform of the density function and $\vec{E}_{S0}(\vec{r}, t)$ is the field scattered by one molecule. The scattered field has information about the motions of the molecules in the scattering volume through the spatial Fourier transform of the density.

2.3. ***Density fluctuations.*** When the speed of the flow is close to Mach1, compressibility becomes important and the equations that describe the flow are more complicated than for the incompressible case. However, if we consider small oscillations about the equilibrium, the equations can be linearized. Monin and Yaglom [1987] have shown that if we write the equations of motion in terms of the vorticity Ω, the divergence D of the velocity, the entropy S and the pressure P, all possible motions can be described by three noninteracting modes:

$$\frac{d\Omega(t)}{dt}=0, \qquad \frac{dS(t)}{dt}=0, \qquad \frac{d^2D(t)}{dt^2}+a_0^2k^2D(t)=0, \qquad \frac{d^2P(t)}{dt^2}+a_0^2k^2P(t)=0.$$

The incompressible vorticity mode and the entropy mode are stationary or move at constant speed. The acoustic or potential mode is related to pressure fluctuations that propagate at the speed of sound as we can see from the wave equation. We can expect to measure the two modes related to the compressible part of the flow: entropic and acoustic.

2.4. ***Structure of a supersonic jet.*** Figure 2 shows the structure of a supersonic jet. The discontinuity at the edge of the nozzle produces a perturbation that propagates at the speed of sound. Each new perturbation catches on the previous one because the speed of the flow is supersonic. The addition of these perturbations creates a shock, that is, a conic region of very high density. Starting with an expansion, a stationary pattern of shocks is formed in the supersonic region of the jet. As the speed decays, the flow becomes subsonic and the shocks disappear.

3. Experimental setup and techniques

3.1. ***Visualization.*** Figure 3 shows the experimental setup used for visualization. All the light scattered at small angles is collected by a lens and sent to a screen where the flow pattern can be visualized [Azpeitia 2004].

3.2. ***Heterodyne detection of the scattered light.*** The amplitude of the scattered light is extremely small and cannot be measured by a common diode. To solve this problem we mix, on the surface of the photodetector, the scattered light with a well known beam of light called the local oscillator. The frequency of the local oscillator is different from the frequency of the incident beam. This technique is known as heterodyning. Figure 4 shows the experimental setup.

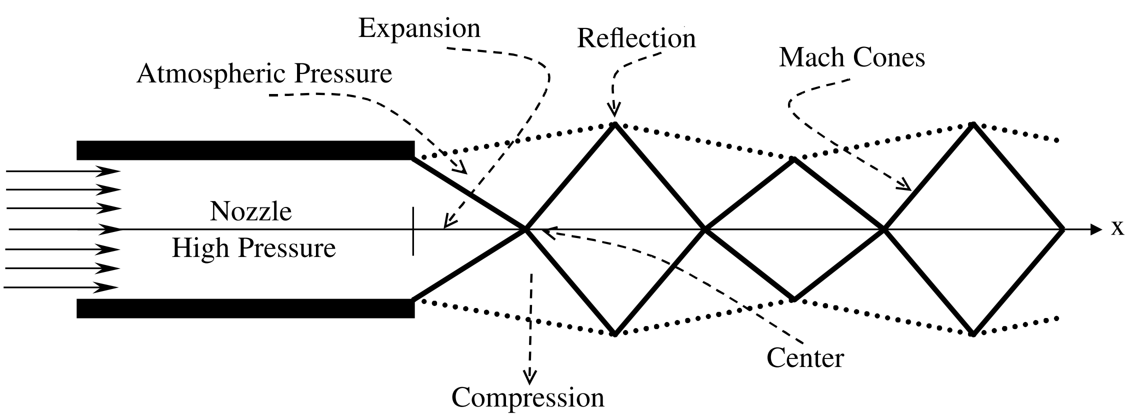

Figure 2. Discontinuity at nozzle end creates perturbation that gives a stationary shock pattern for supersonic flow.

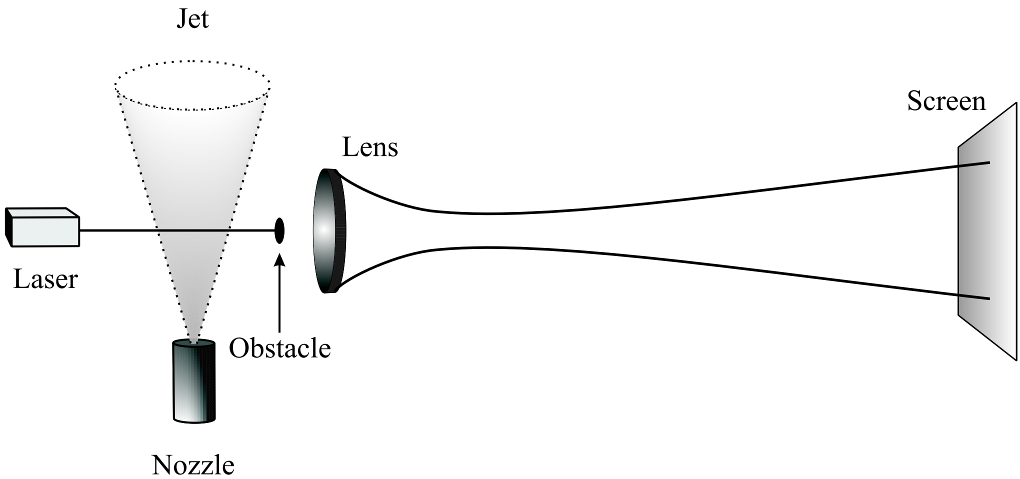

Figure 3. Laser light is sent through the jet. Central part of beam is blocked, while small angle scattering is collected on the screen.

The beam that comes out of the laser is sent into an acoustic modulator. The modulator acts as a Bragg cell and several orders of diffraction come out. Order zero goes through without being deviated; we refer to this beam as incident or primary. Order one is diffracted at a particular angle, is less intense and is displaced in frequency by 110 MHz. We will refer to this beam as the local oscillator. Both beams are manipulated so that they cross at angle θ in the volume to be studied. The local oscillator is sent directly to a photodetector, while the main beam is blocked just after the scattering. The photodetector then sees the part of the incident field scattered at the angle q and the local oscillator. The scattering angle determines the wavenumber of the fluctuations through $k_\Delta = 2k_0 \sin(\theta/2)$, where $\vec{k}_0$ is the wavevector

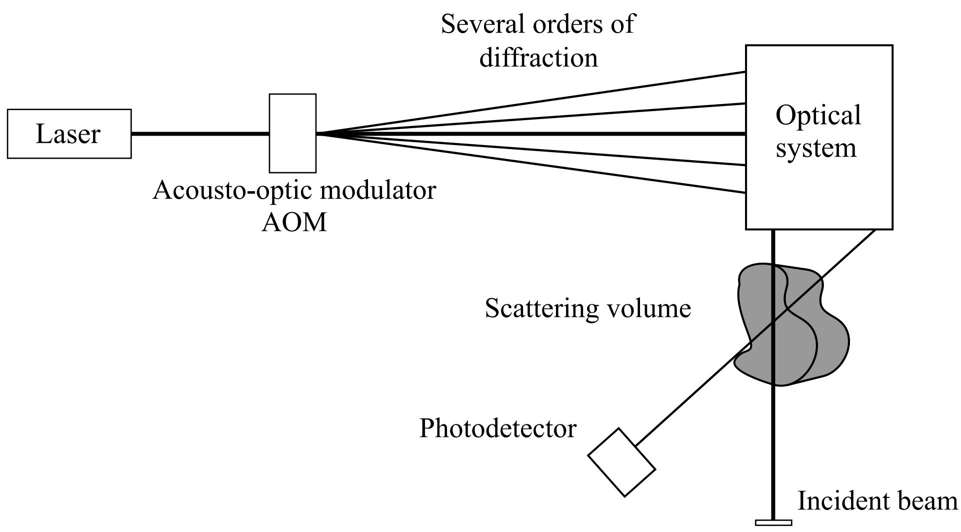

Figure 4. Light sent to acousto-optic modulator comes out as several beams displaced in frequency. One is used as local oscillator and mixed with scattered light at detector.

of the incident field, $\vec{k}_\Delta$ is the wavevector of the density fluctuations, and k_0, k_Δ are their respective magnitudes.

The photodetector is sensitive to the intensity of the incident light, so the current it produces is proportional to the square of the electric field incident on its surface. The current of the photodiode is then proportional to

$$(\vec{E}_S + \vec{E}_{OL})^2 = |\vec{E}_S|^2 + |\vec{E}_{OL}|^2 + 2\vec{E}_S \cdot \vec{E}_{OL}.$$

The first two terms are constant. In particular, the first is too small and the second is of no interest. The third term gives a current that oscillates at the frequency that is the difference of the frequencies of the two electric fields. It contains the information we are interested in, and is modulated by the amplitude of the local oscillator. The current proportional to this term is known as the heterodyne current.

It can be shown [Stern and Grésillon 1983; Aguilar 2003] that the spectral density of the heterodyne current $I(\omega)$ produced by all the scatterers is of the form

$$I(\omega) = \frac{1}{8\pi k_0^2}\Big(\frac{\eta e}{\hbar\omega_0}\Big)^2 n_0(r_0^R)^2 \frac{\epsilon_0}{\mu_0}\big(\vec{E}_S \cdot \vec{E}_{OL}\big)^2 \int d^3k |W(\vec{k}_\Delta - \vec{k})|^2 \times \big[S(\vec{k}, \omega - \omega_\Delta) + S(\vec{k}, \omega + \omega_\Delta)\big],$$

where η is the efficiency of the detector, n_0 the mean density, W is related to the Gaussian profiles of the beams and $S(\vec{k}_\Delta, \omega)$ is the form factor defined by

$$S(\vec{k}_\Delta, \omega) = \frac{|n(\vec{k}_\Delta, \omega)|^2}{n_0 V}.$$

To obtain the spectral density, the signal from the photodetector is either sent directly to a spectrum analyzer or acquired with a computer and treated with periodgrams that have a higher spectral resolution than the analyzer and the traditional Fourier transform.

The diameter of our nozzle is 0.8 mm. It is mounted on a rotating and translating traverse in such a way that density fluctuations can be studied inside and outside the jet and in all directions. The technique described is sensitive to the wavevector of the fluctuations. Therefore, we can determine the direction of propagation of acoustic waves inside the flow, including the mixing layer.

4. Results

4.1. ***Visualization.*** When we visualize the near region of the flow we can see the shocks created by the discontinuity at the nozzle exit. Figure 5(a) shows the first shock after the nozzle.

If we use a cylindrical lens, we can create a sheet of light and observe a series of shocks along the jet. Figure 5(b) is a superposition of four images showing four jets with different exit velocities. The exit pressure is increasing from top to bottom. It can be observed that as the pressure increases, the crossover region where the expansion and the compression meet becomes flat.

4.2. ***Heterodyne detection.*** Figure 6 shows the spectral density obtained with the spectrum analyzer for fluctuations that propagate perpendicular to the flow at a point in the centerline of the jet. The peak is so broad that it is hard to differentiate between entropic and acoustic fluctuations. Figure 7 shows the spectrum for fluctuations perpendicular to the flow outside the flow. As expected, only acoustic fluctuations are detected in this region. The frequency of the center of the peak divided by k_Δ corresponds to the speed of sound.

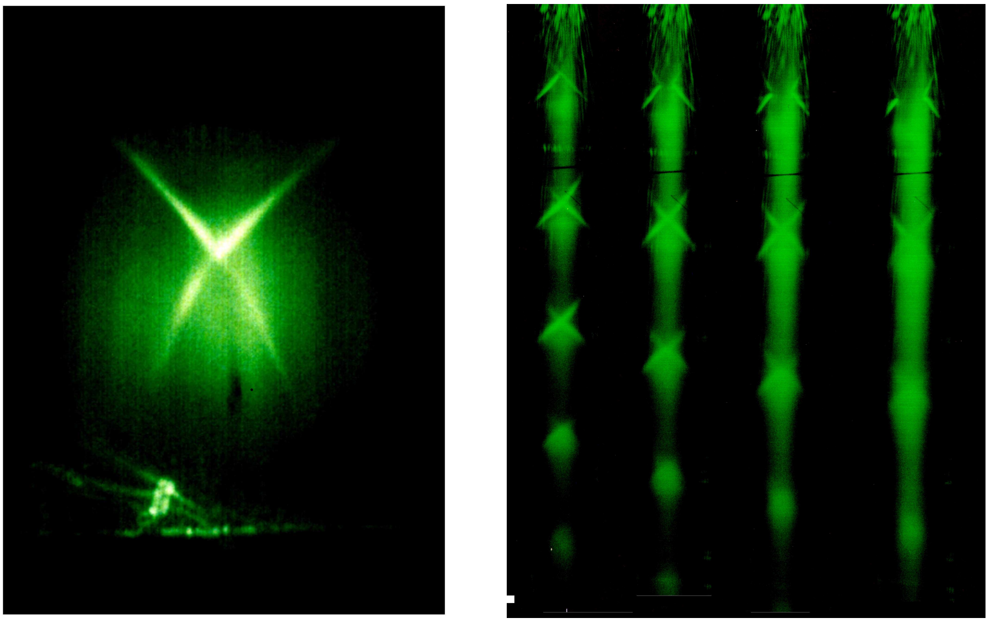

Figure 5. (a) First shock after nozzle; flow is upward. (b) Four jets visualized one at a time with a sheet of light; flow is downward. Several shocks are observed.

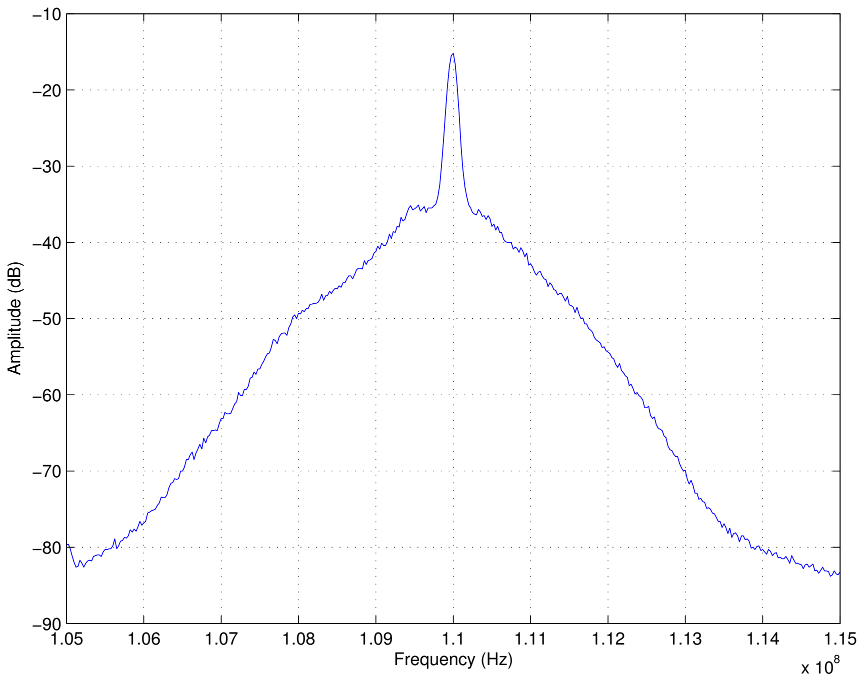

Figure 6. Fluctuations perpendicular to the jet on the axis.

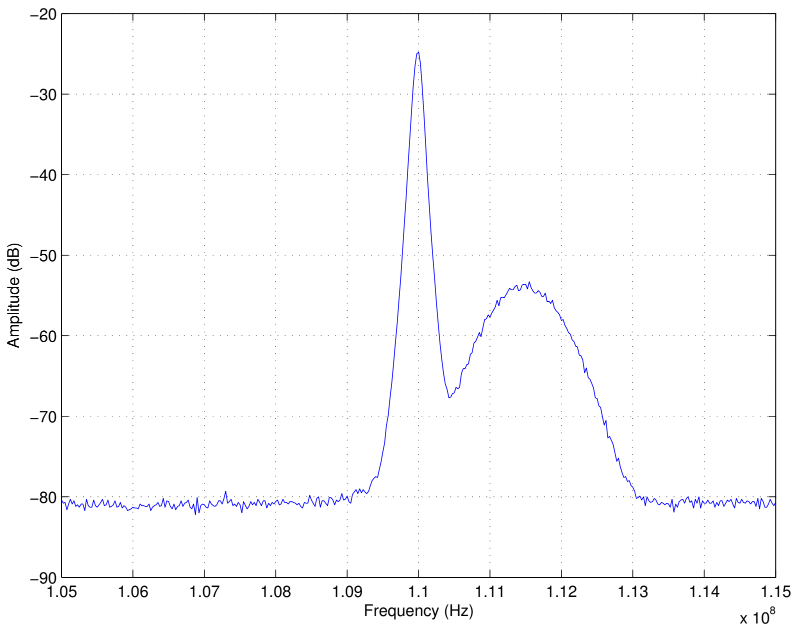

Figure 7. Fluctuations perpendicular to the flow outside the jet.

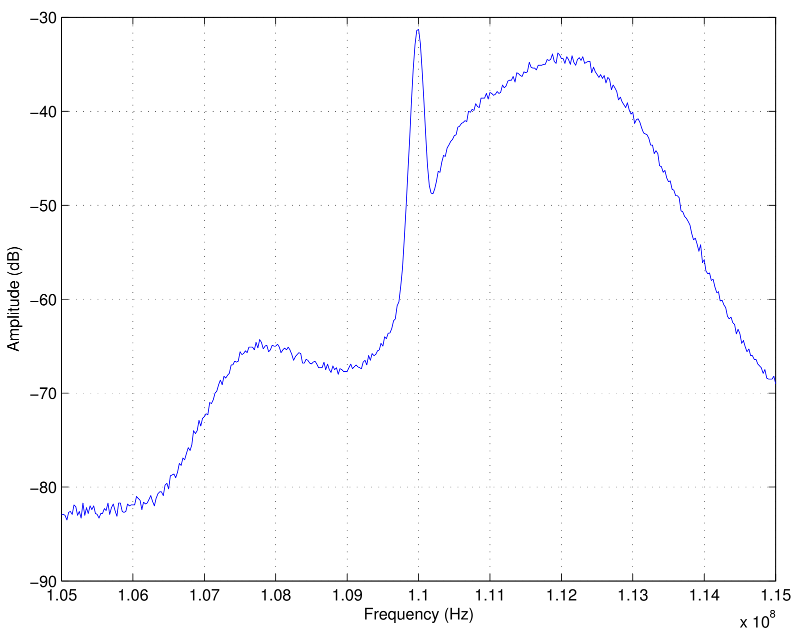

Figure 8. Fluctuations parallel to the flow on the jet axis.

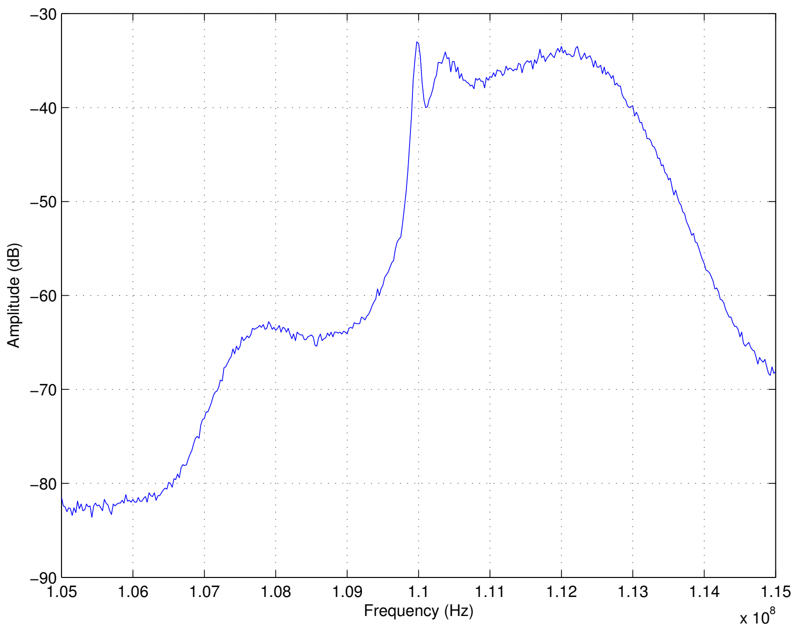

Figure 9. New low-frequency peak appears at some locations along jet center.

The spectral density in Figure 8 corresponds to fluctuations at a point on the centerline of the jet that propagate parallel to the flow. It can be seen that the entropic part of the peak is shifted because the scattering molecules are convected with the flow. Through the change of frequency of the entropic peak, that is, the Doppler shift, we can determine the local mean speed of the flow. An acoustic peak, due to a reflection on the optical setup can be observed on the left hand side of the spectrum. The spectrum in Figure 9 shows that at certain locations along the axis an additional low frequency peak can be observed.

In all the figures shown above, the spectral densities were obtained through a spectrum analyzer. The resolution in frequency is quite poor. To ameliorate these results, we have deheterodyned the signal to shift the reference frequency to zero, acquired it with a computer and treated it with parametric periodgrams of the Burg type [Alvarado 2004].

Figure 10 shows the spectral density at various locations along the centerline for fluctuations perpendicular to the flow. Three peaks are visible. The acoustic peak is always at the same location. The entropic peak changes with the local speed of the flow, and the new peak appears and disappears along the centerline and changes slightly its frequency. It is interesting to note that when the new peak has its highest amplitude, the acoustic peak disappears and vice versa.

By comparing the photographs obtained with the visualization and the graph of the amplitude of the new peak as a function of position as seen in Figure 11, we have been able to determine that the regions of maximum amplitude for the low frequency peak correspond to the crossover between expansion and compression in the shocks. The x coordinate is given in multiples of the nozzle diameter. We are convinced that the peak is related to the interaction of the flow with the shocks. A more detailed study of the origin of this peak is underway.

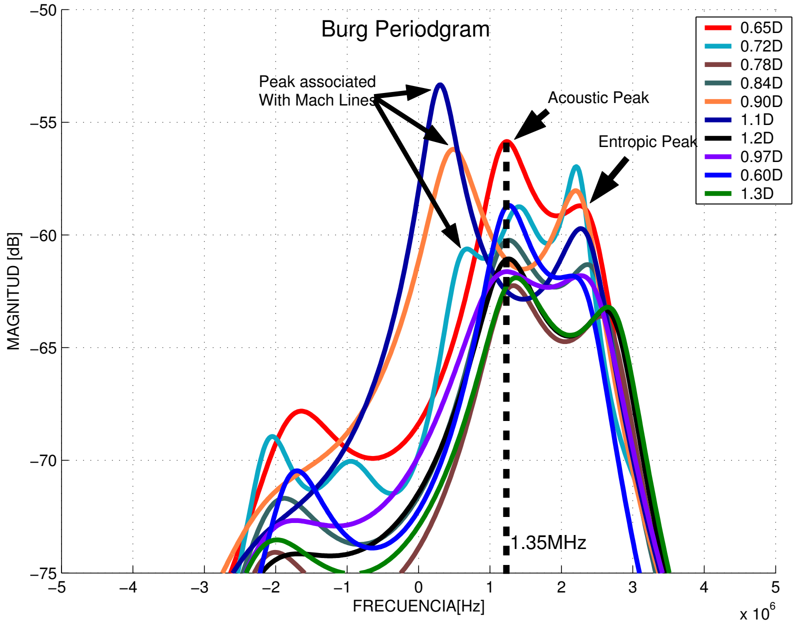

Figure 10. Spectral densities calculated by means of Burgs parametric periodgrams at several locations along jet center.

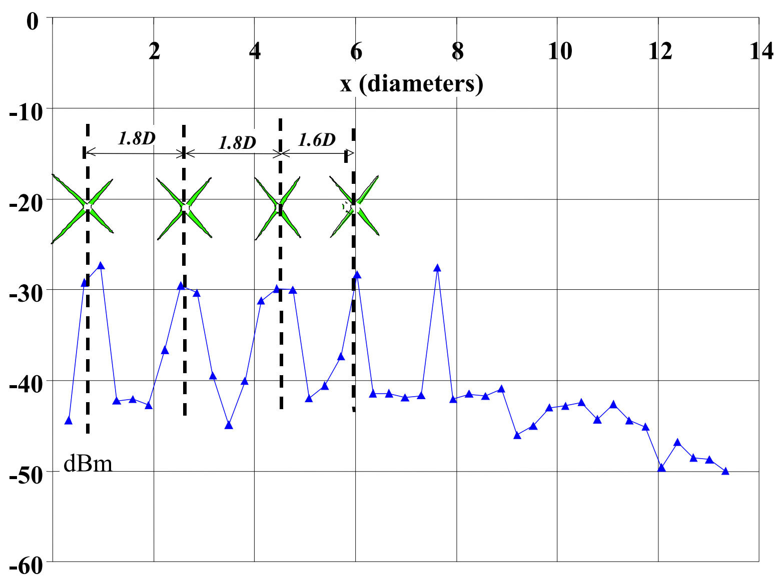

Figure 11. Comparison of new peak amplitude with jet shock structure.

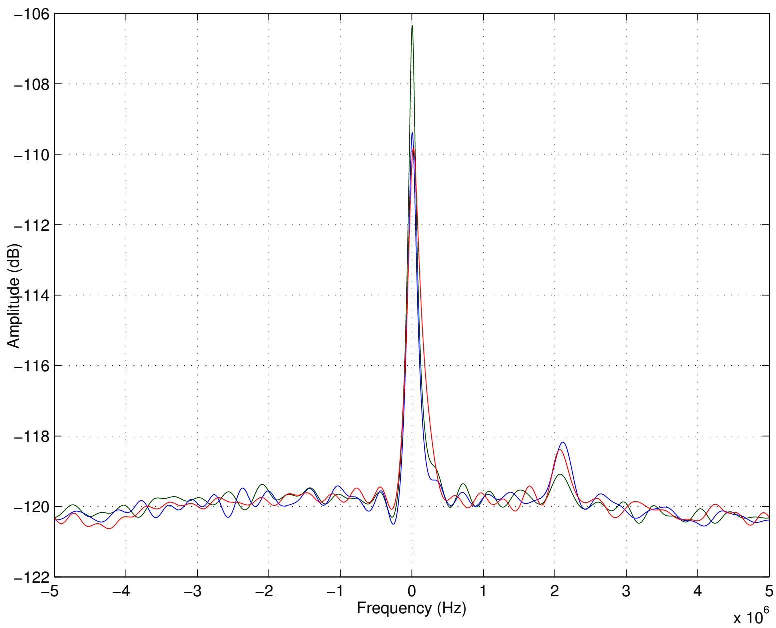

Figure 12. Spectral densities outside the flow at same location for different wavevector directions. Maximum amplitude corresponds to propagation direction of acoustic wave.

The signal of the photodetector depends on the wave vector and is thus sensitive not only to the wavenumber, but also to the direction of propagation of the fluctuations. Figure 12 shows how the amplitude changes when measurements are taken at the same point (outside the flow), for the same wavenumber but different direction of propagation.

It can be observed that the amplitude changes with the direction. If we consider that the maximum amplitude corresponds to the direction of propagation of the acoustic wave, we can determine the acoustic radiation pattern of the jet inside and outside the flow. Figure 13 shows a preliminary radiation pattern obtained by seeking, at each point in the jet, the direction where the acoustic peak has the highest amplitude.

5. Conclusions

We have shown that heterodyne detection can measure density fluctuations in the flow and differentiate among three different phenomena: acoustic waves propagating at the speed of sound, entropy fluctuations that are convected by the flow, and low frequency fluctuations that appear close to the shock structure. The mean local velocity of the jet can also be measured through the Doppler shift of the spectrum. The signal is sensitive to the direction of propagation of the fluctuations, so the technique can also be used to determine the acoustic radiation pattern inside and outside of the jet, and eventually to localize the acoustic sources.

Moreover, Rayleigh scattering can be used to visualize certain aspects of the shock structure. To better understand the origin of the low frequency peak we are planning two kinds of experiments. It is well known that a certain noise known as screech is produced by the interaction of the flow with shocks. The

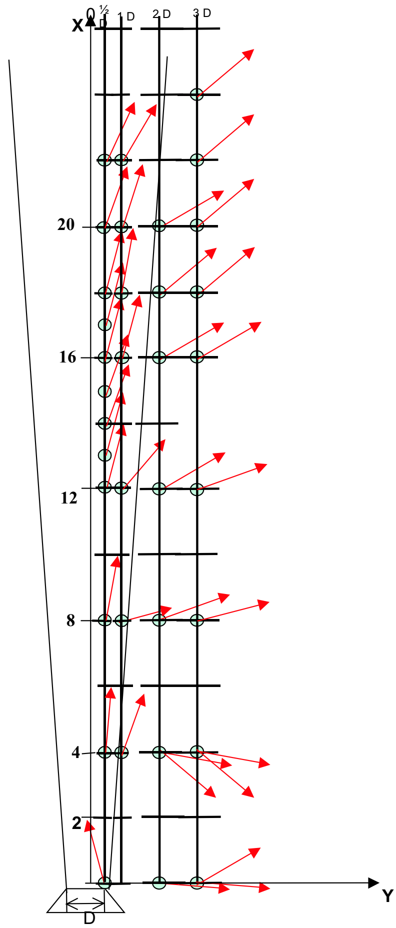

Figure 13. Preliminary acoustic radiation pattern inside and outside the flow.

Strouhal number of the screech is well documented. We aim to detect the screech outside the flow with a transducer and correlate it with our measurements.

Furthermore, we aim to find all the positions in the flow where the peak appears, as well as the direction of maximum amplitude to understand how the flow behaves outside the centerline.

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