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DYNAMIC ANALYSIS OF BANANA FIBER REINFORCED HIGH-DENSITY POLYETHYLENE/POLY (ϵ-CAPROLACTONE) COMPOSITES

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Banana fiber reinforced HDPE/PCL (high-density polyethylene/poly (ϵ-caprolactone)) composites have been prepared in the laboratory for the evaluation of mechanical properties. The tensile strength and Young's modulus of the HDPE/PCL blend attains saturation at approximately 20% by weight of PCL addition. Therefore, the 80:20 blend of HDPE/PCL has been selected as the matrix material. It has also been observed that the ductility of the HDPE/PCL blend drops rapidly by addition of more than 15% by weight of PCL. Reinforcement of the banana fiber in an 80:20 HDPE/PCL blend matrix shows improvement in ductility, tensile strength and Young's modulus. In order to study the dynamic response of this banana fiber reinforced HDPE/PCL composite plate, a multiquadric radial basis function (MQRBF) is developed. MQRBF is applied for spatial discretization and a Newmark implicit scheme is used for temporal discretization. The discretization of the differential equations generates a greater number of algebraic equations than the unknown coefficients. To overcome this ill-conditioning, the multiple linear regression analysis, based on the least square error norm, is employed to obtain the coefficients. Simple supported and clamped boundary conditions are considered. Numerical results are compared with those obtained by other analytical methods.

1. Introduction

Fiber reinforced composites are used extensively in various industries such as automobiles, aerospace, and defence and are replacing various metallic structures because of many advantages, such as high strength to weight ratio, enhanced corrosion resistance, and longer fatigue life. In this context, banana fiber reinforced composites have recently gained importance. Banana fibers are mainly used for light-weight composites and green composites in agricultural industries. These composites can also be used for door panels, room partitions, wall cladding, food packaging, home appliances, automotive parts, building and construction, and electrical housing.

Banana fibers are a waste product of banana cultivation, are easily available, and may be reinforced in thermosets and thermoplastics. Fabrication of banana fiber reinforced low-density polyethylene/poly-caprolactone composites has been reported recently by Sandeep and Misra [2007]. When the quantity of banana fibers is increased, tensile and flexural properties of the resol matrix resin [Joseph et al. 2002] is also increased. Addition of banana fibers to brittle phenol formaldehyde resin makes the matrix ductile. The interfacial shear strength indicates a strong adhesion between the lignocellulosic banana fibers and the phenol formaldehyde resin. Reinforcement of bamboo and sisal fibers in thermoplastic resin has also been reported [Chen et al. 1998; Ali et al. 2003].

Keywords: banana fiber, multiquadric radial basis function, eigenvalue.

However, the theoretical analysis of banana fiber reinforced composite plates by conventional methods poses some problems. Generally, finite elements and finite difference methods are used for analysis of composites. The finite element method is not very suitable as it consumes more time in mesh generation than in execution. The vast applications of composite plates in industries have emphasized the development of new efficient numerical technique for the analysis of composite plates. Therefore, to avoid these problems, new methods known as meshless methods have been developed recently. These methods are the meshless local Petrov–Galerkin (MLPG) method [Atluri and Zhu 1998], partition of unity method [Babuska and Melenk 1997], element free Galerkin method (EFGM) [Belystcho et al. 1994], method of finite spheres [De and Bathe 2000], and wavelet Galerkin method [Qian and Weiss 1993]. Kansa [1990] and Franke and Schaback [1997] developed the concept of solving PDEs using radial basis functions (RBF). Fedoseyev et al. [2002] improved the accuracy of the solution by placing the interior knots. Hardy [1971] used the multiquadric radial basis function method (MQRBF) for the interpolation of geographically scattered data. The discretization process in MQRBF is based on the employment of a set of randomly distributed boundary and internal nodes. Coleman [1996] employed RBFs in the analysis of elliptic boundary value problems. Chen et al. [2004] studied the free vibration analysis of circular and rectangular plates by employing the RBF in the imaginary-part fundamental solution. Misra et al. [2007] have recently presented the static and dynamic analysis of anisotropic plates by using the MQRBF method and the multiple linear regression analysis technique.

In light of the above discussion, it appears that some experimental and theoretical investigations on banana fiber reinforced composites could offer some interesting results. This paper reports the fabrication, chemical treatment, and evaluation of properties of banana fiber reinforced high-density polyethylene/polycaprolactone composites. High-density polyethylene (HDPE) and poly (ϵ-caprolactone) (PCL) in 80:20 ratios were selected to make a matrix material. Since the melting point of PCL is 60° C, it could help in the processing of HDPE. With the properties thus obtained, a collocation method based on the MQRBF method is employed for the dynamic analysis of banana fiber reinforced HDPE/PCL composites due to self-weight.

Notation

a, b	Dimension of plates
h	Thickness of plates
R	Aspect ratio (a/b)
ν_{LT}	Major Poisson's ratio
ρ	Mass density of plates
m	Mass of the plate
C_v^*, C_v	Viscous damping, dimensionless viscous damping
$D_{11}, D_{22}, D_{12}, D_{66}$	Flexural rigidity of plates
E_L, E_T	Young's modulus in x^*, y^* direction respectively
G_{LT}	Shear modulus relative to $x^* - y^*$ direction
q, Q	Transverse load, dimensionless transverse load
t^*, t	Time, dimensionless time
w^*	Displacement in z^* direction

w	Dimensionless displacement in z direction
ω	Natural frequency of vibration
ρ_0	Surface density ($\rho_0 = \rho h$)
λ	Eigen value

2. Materials and methods

2.1. ***Materials.*** The banana fibers were obtained from Regional Research Laboratory (CSIR) Jorhat, India. Sodium hydroxide and HDPE (HD50MA 180) were obtained from Indian Petrochemicals Corporation Limited, Vadodara. PCL was obtained from Union Carbide Company, USA.

The melt flow index of HDPE was 17.3 (at 190° C/2.16 Kg), the density was 0.98 g/cm^3 and the melting temperature was 130° C. PCL is a low melting aliphatic polyester, which is biodegradable and miscible with many polymers. The properties of PCL were as follows: melt flow index 2.2 gm/10 min (at 125°C/2.16 Kg), density 1.145 g/cm^3, melting temperature 60° C.

2.2. ***Chemical treatment of banana fibers.*** Organic solvents were used to separate the fibers. Better separation of fibers was achieved by using toluene followed by washing with petroleum ether. After separation, the fibers were dipped in sodium hydroxide for achieving better mechanical properties. Banana fibers were pale yellowish brown. When treated with NaOH the color became dark brown.

2.3. ***Fabrication of composites.*** A Klockner Windsor single screw extruder having an L/D ratio of 21:1 and screw diameter of 30 mm was used for blending HDPE and PCL. For proper mixing of these polymers, the following optimum parameters were used: screw speed, 35 rpm; barrel zone temperature, 135, 140 and 150° C; die zone temperature, 150° C. After mixing, the mixture was quenched in water. Banana fibers were separated and dried in oven for 3 hours at 80° C. These banana fibers were used for composite fabrication. The untreated and 10% alkali treated banana fibers are designated by UB and TB respectively in the present paper. A prefix indicating the weight % of banana fibers is appended to these letter designations. For example, 9 UB and 13 TB represent blends with 9 weight % untreated banana fibers and 13 weight % alkali treated banana fibers respectively.

2.4. ***Evaluation of properties.*** A 10 KN capacity Zwick tensile testing machine model Z010 was used to determine the mechanical properties of these banana fiber reinforced composites. The tex and denier of the fibers were determined by measuring the weight of fibers of known length. The diameter of the fibers was calculated from the density and denier of the fibers. Properties of the banana fibers have been

Treatment with	Designation	% Wt. Loss	Density (g/cm^3)	Tex (g/1000 m)	Denier (g/9000 m)	Diameter (mm)
Untreated	UB	—	1.4	14.2	127.8	0.113
5% NaOH	AlB5	20.31	1.45	13.1	117.9	0.107
10% NaOH	AlB10	25.4	1.48	13.0	117	0.105

Table 1. Properties of banana/modified banana fibers.

Sample designation	HDPE (g)	PCL (g)	Banana Fiber (g)	Wt % of Banana Fibers
M	160	40	0	0
5UB	160	40	10 (untreated)	4.76
5TB	160	40	10 (treated)	4.76
9UB	160	40	20 (untreated)	9.1
9TB	160	40	20 (treated)	9.1
13UB	160	40	30 (untreated)	13
13TB	160	40	30 (treated)	13

Table 2. Composition of the banana fiber reinforced composites.

shown in Table 1. The compositions of banana fiber reinforced composites are given in Table 2. The resulting tensile strength, tensile modulus, and percentage elongation at tensile fracture are presented in Table 3. The matrix (HDPE:PCL, 80:20) has been designated as M, while ML and MT represent the longitudinal and transverse fibers composites respectively.

3. Theoretical analysis

3.1. ***The multiquadric radial basis function method.*** Consider a general differential equation:

$$Aw = f(x, y) \quad \text{in } \Omega, \tag{1}$$

$$Bw = g(x, y) \quad \text{on } \partial\Omega. \tag{2}$$

The governing equation for a free flexural vibration of a uniform thin plate is written as follows [Chen et al. 2004]:

$$\nabla^4 w = \lambda^4 w(x), \quad x \in \Omega,$$

where w is the lateral displacement, $\lambda^4 = \omega^2 \rho_0 h/D$, λ is the frequency parameter, ω is the circular frequency, ρ_0 is the surface density, D is the flexural rigidity of the isotropic plate, and h is the thickness of the plate.

In the present work, it is assumed that the banana fiber reinforced composite plate behaves as an orthotropic plate because the diameter of each banana fiber is so small that its contribution to the strength is predominantly along its longitudinal direction. In such a case, Equations (1) and (2) for a uniform thin

Sample designation	Tensile strength (GPa)	Tensile modulus (GPa)	Elongation at break (%)
M	21.4	11.41	4.50
ML	80.8	18.0	7.23
MT	32.9	8.0	6.78

Table 3. Mechanical properties of composites.

orthotropic composite plate can be reduced to

$$Aw = \lambda^4 w, \tag{3}$$

$$Bw = 0, \tag{4}$$

where A is a linear differential operator, B is a linear boundary operator imposed on boundary conditions for the orthotropic plate, and λ is an eigenvalue.

Let $\{P_i = (x_i, y_i)\}_{i=1}^{N}$ be N collocation points in the domain Ω of which $\{(x_i, y_i)\}_{i=1}^{N_I}$ are interior points and $\{(x_i, y_i)\}_{i=N_I+1}^{N}$ are boundary points. In the MQRBF method, the approximate solution for the differential Equation (1) with boundary conditions Equation (2) can be expressed as:

$$w(x, y) = \sum_{j=1}^{N} w_j \varphi_j(x, y),$$

with a multiquadric radial basis:

$$\varphi_j = \sqrt{(x - x_j)^2 + (y - y_j)^2 + c^2} = \sqrt{r_j^2 + c^2},$$

where $\{w_j\}_{j=1}^{N}$ are the unknown coefficients to be determined, and $\varphi_j(x_j, y_j)$ is a basis function. Other widely used radial basis functions are:

$$\begin{aligned}
&\varphi(r) = r^3, && \text{cubic,}\\
&\varphi(r) = r^2 \log(r), && \text{thin plate splines,}\\
&\varphi(r) = (1 - r)^m + p(r), && \text{Wendland functions,}\\
&\varphi(r) = e^{-(cr)^2}, && \text{Gaussian,}\\
&\varphi(r) = (c^2 + r^2)^{-1/2}, && \text{inverse multiquadratics.}
\end{aligned}$$

Here $r = \|P - P_j\|$ is the Euclidean norm between points $P = (x, y)$ and $P_j = (x_j, y_j)$. The Euclidian distance r is real and nonnegative, and c is a positive and constant shape parameter. c is smaller in the domain and larger at the boundary [Ling and Kansa 2004].

3.2. *Calculation of eigenvalues.* Let N_B be the total number of points on the boundary $\partial\Omega$, N_I is the total number of points inside Ω, and $N = N_I + N_B$. Applying the RBF in Equation (3) gives

$$\sum_{j=1}^{N} w_j A\varphi\|P_i - P_j\| = \lambda^4 \sum_{j=1}^{N} w_j \varphi\|P_i - P_j\|, \tag{5}$$

where $i = 1, 2, \ldots, N_I$.

Define

$$\boldsymbol{L} = [A\phi(\|P_i - P_j\|)]_{N_I x N},$$

$$\boldsymbol{M} = [\phi\|P_i - P_j\|)]_{N_I x N}.$$

Applying the multiquadric radial basis functions (MQRBF) in Equation (4) gives

$$\sum_{j=1}^{N} w_j B\phi(\|P_i - P_j\|) = 0,$$

where $i = N_{I+1}, N_{I+2}, \ldots, N$, and

$$\begin{aligned} \boldsymbol{K} &= [B\phi(\|P_i - P_j\|)]_{N_B x N}, \\ \boldsymbol{w} &= [w_1 w_2 w_3 \cdots w_N]^T. \end{aligned} \tag{6}$$

Equations (5) and (6) can be written as:

$$\begin{aligned} \boldsymbol{L}\boldsymbol{w} &= \lambda^4 \boldsymbol{M}\boldsymbol{w}, \\ \boldsymbol{K}\boldsymbol{w} &= 0. \end{aligned}$$

The general eigenvalue problem becomes (in matrix form):

$$\begin{bmatrix} \boldsymbol{L} \\ \boldsymbol{K} \end{bmatrix} \boldsymbol{w} = \lambda^4 \begin{bmatrix} \boldsymbol{M} \\ 0 \end{bmatrix} \boldsymbol{w}.$$

The algorithm of Ferreira et al. [2005] for solving this standard eigenvalue problem has been used in the present analysis as follows:

$$\boldsymbol{L}^1 \boldsymbol{w}^{\mathbf{2}} = \lambda^4 \boldsymbol{w}^{\mathbf{2}},$$

where

$$\boldsymbol{L}^1 = \boldsymbol{L}\boldsymbol{D}^{-1} \begin{bmatrix} \boldsymbol{I}_{\boldsymbol{N_I x N_I}} & \boldsymbol{0}_{\boldsymbol{N_B x N_I}} \end{bmatrix}, \qquad \boldsymbol{w}^{\mathbf{2}} = \begin{bmatrix} w_1 w_2 w_3 \cdots w_N \end{bmatrix}^T, \qquad \boldsymbol{D} = \begin{bmatrix} \boldsymbol{M}\boldsymbol{K} \end{bmatrix}.$$

4. Results and discussion

The effect of PCL addition on the mechanical properties of the HDPE matrix is shown in Figures 1 and 2. It is apparent that both the tensile modulus and tensile strength increase with percent of PCL addition, and attain almost saturation at about 20% PCL addition by weight. The modulus increases by 41% and the tensile strength by 29% on introducing 20% by weight of PCL into the HDPE matrix. This may be attributed to the higher degree of crystallinity of PCL than HDPE, and better room temperature mechanical properties. It was for this reason that HDPE/PCL blend in 80:20 ratios was selected for the preparation of the banana fiber reinforced composite.

Figures 3 and 4 show the improvement of the tensile modulus and tensile strength of the NaOH treated banana fiber reinforced HDPE/PCL (80:20) blend composite plates. There is an increase of 11% in the tensile modulus and 15% in the tensile strength at 15% volume fraction of fibers.

Figure 5 offers an interesting result. There is a sharp drop in the percentage elongation between 15 and 20% of PCL addition in the HDPE/PCL blend. This shows that the characteristic of the blend changes drastically from ductile to brittle in this interval. However, when the banana fibers were introduced into this blend, the ductile characteristics were improved appreciably, as shown in Figure 6.

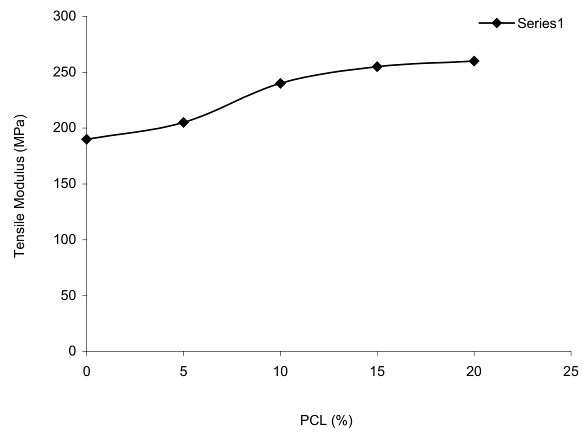

Figure 1. Effect of PCL content on tensile modulus of HDPE/PCL blend.

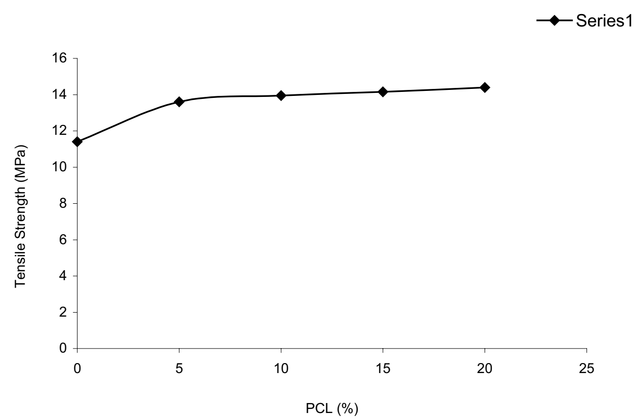

Figure 2. Effect of PCL content on tensile strength of HDPE/PCL blend.

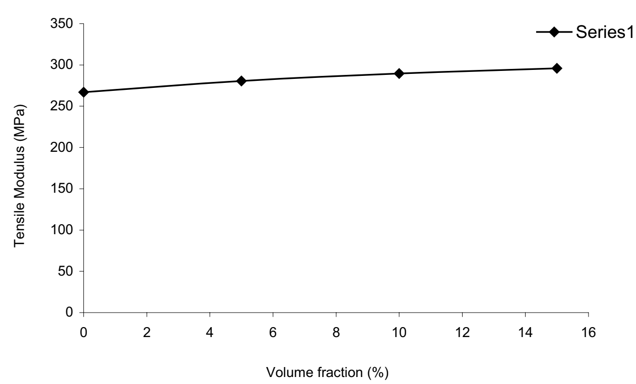

Figure 3. Effect of weight fraction of NaOH treated banana fibers on tensile modulus of HDPE/PCL (80:20) blend.

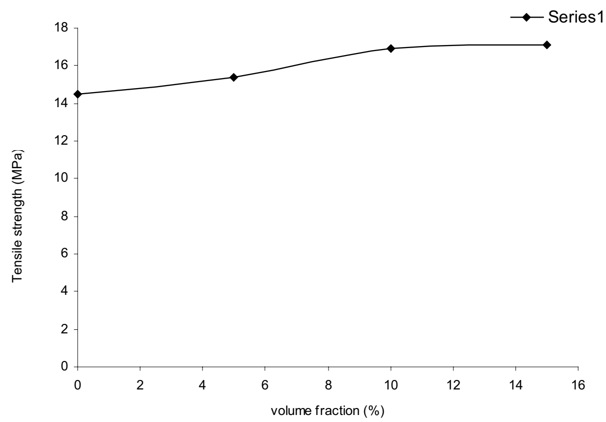

Figure 4. Effect of weight fraction of NaOH treated banana fibers on tensile strength of HDPE/PCL (80:20) blend.

Using the MQRBF method, the dynamic response and natural frequencies of banana fiber reinforced HDPE/PCL composites are obtained. Parameters of the composite plate are as follows:

$$E_L = 18\,\text{GPa}, \qquad E_T = 8\,\text{GPa}, \qquad G_{LT} = 6.6176\,\text{GPa}, \qquad \nu_{LT} = 0.36.$$

The subscripts L and T denote the normal and transverse directions of the fiber.

4.1. ***Case study 1: dynamic response of composites.*** Figure 7 shows the geometry, coordinate system, and loading in banana fiber reinforced composites. Neglecting the transverse shear and rotary inertia, the equation describing the banana fiber reinforced composites is expressed in nondimensional form as:

$$\left(w_{xxxx} + 2R^2\eta w_{xxyy} + R^4\psi w_{yyyy}\right) + w_{tt} + c_v w_t - Q(x, y, t) = 0, \tag{7}$$

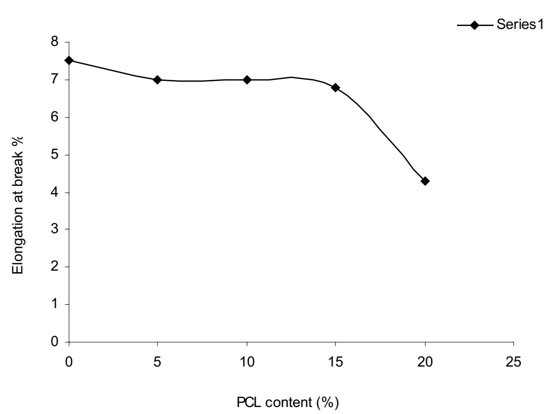

Figure 5. Effect of PCL content on elongation at break of HDPE/PCL blend.

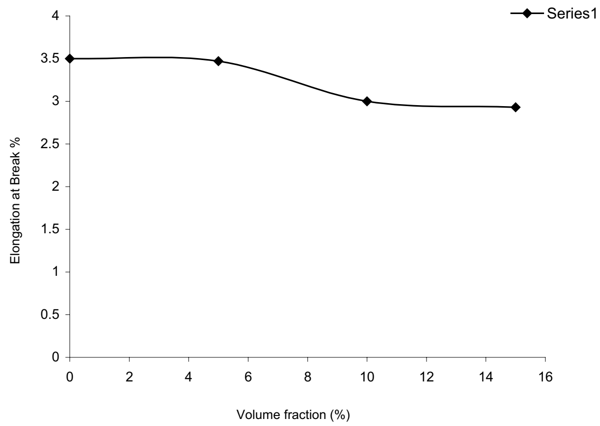

Figure 6. Effect of weight fraction of NaOH treated banana fibers on elongation at break of HDPE/PCL blend.

where the subscript denotes the partial derivative with respect to the variables. The nondimensional quantities are defined by

$$
\begin{aligned}
w &= w^*/h, & Q &= qa^4/(D_{11}h),\\
x &= x^*/a, & C_v &= (C_v^*/M)\sqrt{(\rho a^4 h)/D_{11}},\\
y &= y^*/b, & M &= \rho abh,\\
R &= a/b, & \eta &= (D_{12}+2D_{66})/D_{11},\\
t &= t^*\sqrt{D_{11}/(\rho a^4 h)}, & \psi &= D_{22}/D_{11}.
\end{aligned}
$$

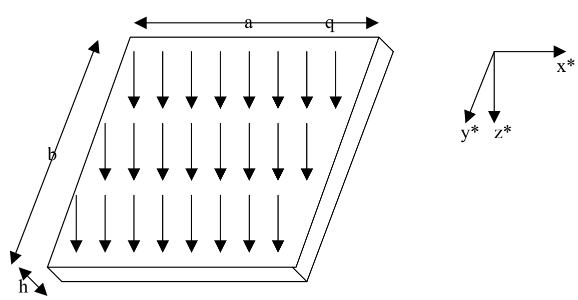

Figure 7. Geometry of the rectangular plate.

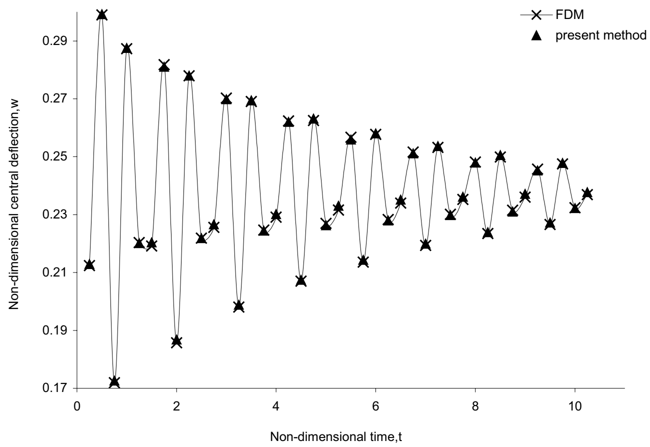

Figure 8. Damped response of a simple supported banana fiber reinforced square orthotropic plate for $C_v = 1.25$.

Boundary conditions for the plate simply supported at all four edges are:

$$\begin{aligned} x &= 0, 1, & w &= 0, \\ x &= 0, 1, & w_{xx} &= 0, \\ y &= 0, 1, & w &= 0, \\ y &= 0, 1, & w_{xx} &= 0. \end{aligned} \tag{8a}$$

Boundary conditions for the clamped edge plate are:

$$\begin{aligned} x &= 0, 1, & w &= 0, \\ x &= 0, 1, & w_x &= 0, \\ y &= 0, 1, & w &= 0, \\ y &= 0, 1, & w_y &= 0. \end{aligned} \tag{9a}$$

The governing Equation (7) is solved using the multiquadric radial basis function and boundary conditions, Equations (8) and (9), for simply supported and clamped edge plates respectively, and has been presented in Appendix A. In this collocation method, the domain is discretized into $n \times n$ grid points. Inside the domain, there are $(n-2) \times (n-2)$ grid points and the remaining grid points are at the boundaries. There are eight boundary conditions: four boundary conditions for x and four for y. The governing equations generate $(n-2)^2$ equations and every boundary condition creates n equations. Therefore, the total number of equations becomes $(n-2)^2 + 8n$. Since the total number of unknown coefficients is n^2, the number of equations is more than the number of unknown coefficients. Hence, this method creates ill-conditioning. To overcome this ill-conditioning, multiple linear regression analysis (Appendix B) based on least-square error norms is employed. A computer program based on the finite difference method (FDM) proposed by Chandrashekhara [2001] is also developed.

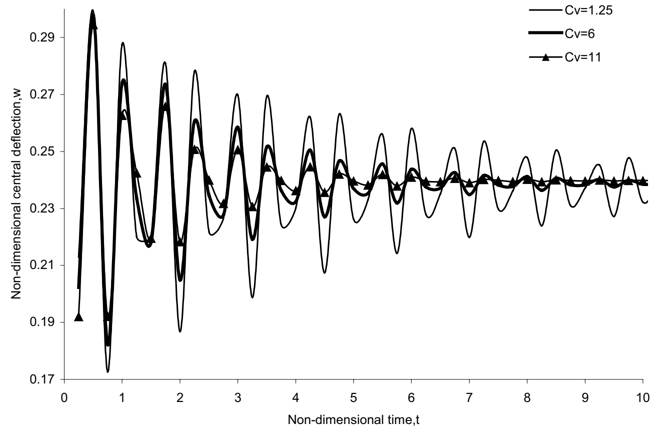

Figure 9. Damped response of a smiple supported banana fiber reinforced orthotropic plate for aspect ratio $R = 1$.

The damped response of the simple supported composite plates obtained by the present method and finite difference method has been compared and shown in Figure 8 for nondimensional load $Q = 100$ and nondimensional viscous damping factor $C_v = 1.25$. There is good agreement within the results. According to the present method, at an aspect ratio of 1.0 the maximum deflections at $C_v = 1.25$, 6.0, and 11.0 are 0.2992, 0.298, and 0.2946, respectively. However, the fluctuations in deflection with $C_v = 6.0$ and 11.0, decay rapidly after the nondimensional time 8.0, as shown in Figure 9. Furthermore, at an aspect ratio of 2.0 the maximum deflections at $C_v = 1.25$, 6.0, and 11.0 are 0.0556, 0.0553, and 0.0549, respectively (Figure 10). As aspect ratio increases from 1.0 to 2.0, nondimensional deflection decreases as shown in Figures 9 and 10. Figure 11 shows the damped response of the simple supported pure HDPE/PCL (80:20) blend orthotropic plate for $R = 1$ and $Q = 100$. The following features are observed after analyzing Figures 9 and 11, and Table 4 for the simple supported boundary condition:

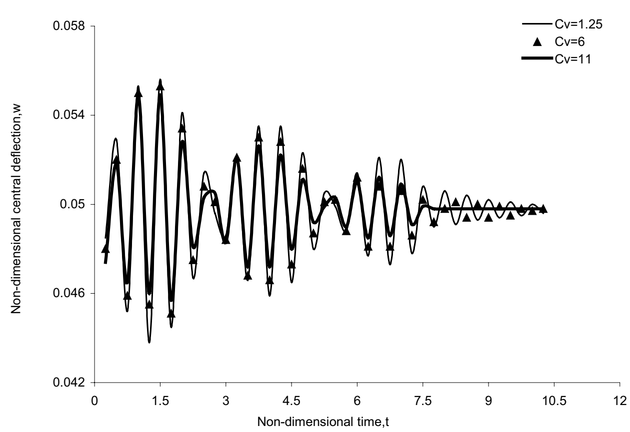

Figure 10. Damped response of a simple supported banana fiber reinforced orthotropic plate for aspect ratio $R = 2$.

Material	$Cv = 1.25$		$Cv = 6.0$		$Cv = 11.0$	
	Maximum Deflection	Stability Time	Maximum Deflection	Stability Time	Maximum Deflection	Stability Time
Pure HDPE/PCL (80:20) blend	0.9665	Did not stabilize after nondimensional time 10	0.9159	Stabilize after 5.75	0.8602	Stabilize after 3.75
Banana fibers-reinforced high density polyethylene polycaprolactone composites	0.2992	Did not stabilize after nondimensional time 10	0.298	Stabilize after 10	0.2946	Stabilize after 7.25

Table 4. Damped response of a simple supported orthotropic plate.

- When the damping coefficient increases from 1.25–11, nondimensional maximum deflection decreases in the banana fiber reinforced composite plate and HDPE/PCL blend orthotropic plate.
- At damping coefficients $Cv = 1.25$, 6 and 11, nondimensional maximum deflections are 0.2992, 0.298, 0.2946, and 0.9665, 0.9159, 0.8602 in the banana fiber reinforced composite plate and HDPE/PCL blend orthotropic plate respectively.
- The HDPE/PCL blend orthotropic plate stabilized early compared to the banana fiber reinforced composite plate.

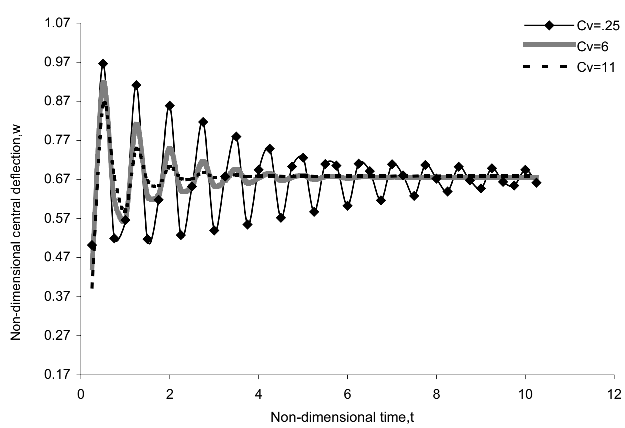

Figure 11. Damped response of a simple supported pure HDPE/PCL (80:20) blend orthotropic plate for $R = 1$ and $Q = 100$.

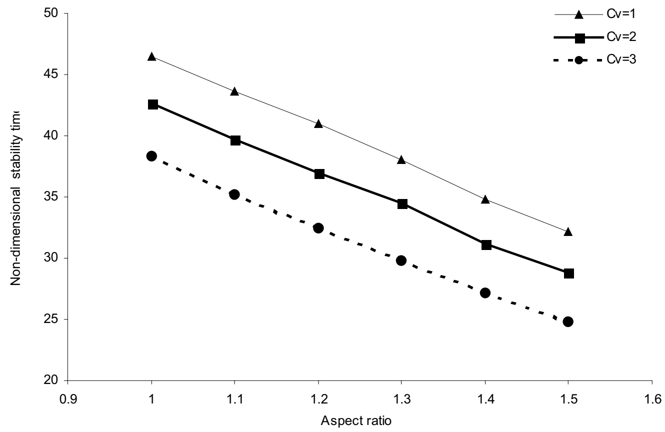

Figure 12. Nondimensional stability time at various aspect ratios for a simple supported banana fiber reinforced orthotropic plate.

It may be noted here that after introducing the banana fibers, the tensile strength of the composite increases. Therefore, the maximum nondimensional deflection decreases in the banana fiber reinforced composite plate compared to HDPE/PCL blend orthotropic plate.

Amash and Zugenmaier [2000] have reported that by incorporation of cellulose fiber, the damping properties of a polypropylene/cellulose composite decreases and stiffness improves. Pothan et al. [2003] reported that in a composite system, damping is affected through the incorporation of fibers. This is due mainly to shear stress concentrations at the fiber ends in association with the additional viscoelastic energy dissipation in the matrix material. Another contributing factor could be the elastic nature of the fiber. When interfacial bonding improves, damping property decreases.

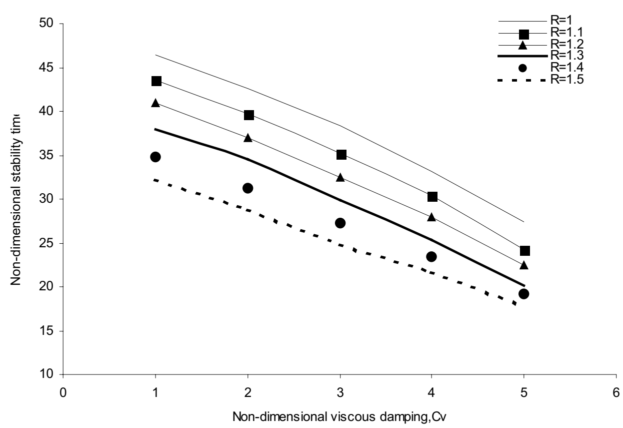

Figure 13. Nondimensional stability time at various nondimensional viscous damping ratios for a simple supported banana fiber reinforced orthotropic plate.

Material	$Cv = 1.25$		$Cv = 6.0$		$Cv = 11.0$	
	Maximum Deflection	Stability Time	Maximum Deflection	Stability Time	Maximum Deflection	Stability Time
Pure HDPE/PCL (80:20) blend	0.2551	Did not stabilize after nondimensional time 10	0.2548	Stabilize after 8	0.254	Stabilize after 6.75
Banana fibers-reinforced high density polyethylene polycaprolactone composites	0.0908	Did not stabilize after nondimensional time 10	0.0904	Stabilize after 10.5	0.0899	Stabilize after 10.25

Table 5. Damped response of a clamped edge orthotropic plate.

Figure 12 shows that as the aspect ratio increases, nondimensional stability time decreases. This may be due to the fact that for a constant load per unit area, the stiffness of the plate increases with an increase in the aspect ratio.

Figure 13 shows the variation of nondimensional stability time with nondimensional viscous damping factor for various aspect ratios. It is observed that as the nondimensional viscous damping factor increases, nondimensional stability time decreases. Furthermore, the effect of C_v on the nondimensional

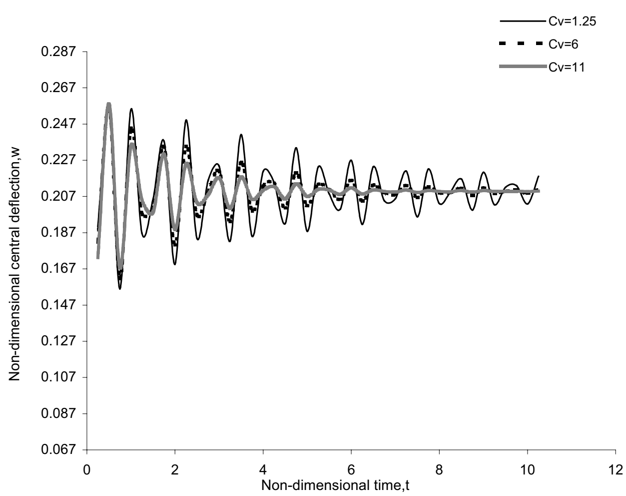

Figure 14. Damped response for a clamped edges pure HDPE/PCL (80:20) blend orthotropic plate for $R = 1$ and $Q = 100$.

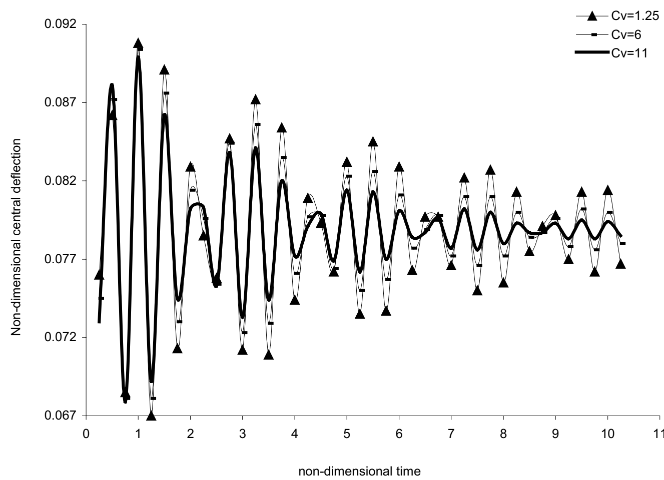

Figure 15. Damped response of a clamped supported banana fiber reinforced orthotropic plate for aspect ratio, $R = 1$.

stability time is more pronounced than the effect of the aspect ratio, as seen in Figures 12 and 13. Similarly, Figures 14, 15 and Table 5 show the damped responses for the clamped edges pure HDPE/PCL (80:20) blend and composite orthotropic plates, at $R = 1$ and $Q = 100$. The pure HDPE/PCL (80:20) blend orthotropic plate behaves identically under the clamped edge boundary conditions and the simple supported boundary conditions, but in the clamped edge boundary conditions the HDPE/PCL blend orthotropic plate stabilizes more slowly than the simple supported boundary conditions. The maximum deflections in the clamped edges composite plate are 0.0908, 0.0904, and 0.0899 at $C_v = 1.25$, 6.0, and 11.0, respectively, as shown in Figure 15. It should be noted that the deflections in clamped edge conditions are much less than those in simple supported edge conditions.

4.2. ***Case study 2: natural frequency of banana fiber reinforced with high-density polyethylene / polycaprolactone composites.*** The equation governing the free vibration of orthotropic plates is:

$$(D_{11}/D_{22})w_{xxxx} + 2(H/D_{22})w_{xxyy} + w_{yyyy} = \lambda^4 w(x, y), \tag{10}$$

where $\lambda^4 = \omega^2 \rho_0 h/D_{22}$ and $H = (D_{12} + 2x D_{66})$. The natural frequency of the simply supported orthotropic plate is obtained by [Ashton and Whitney 1970] as:

$$\omega_{mn} = \frac{\pi^2}{R^2 b^2}\frac{1}{\sqrt{\rho}}\sqrt{D_{11}m^4 + 2(D_{12} + 2D_{66})m^2 n^2 R^2 + D_{22}n^4 R^4},$$

where m and n are integers. Different natural frequencies are obtained at different combinations of m and n. On the other hand, Hearmon [1959] has expressed the natural frequencies of a clamped supported plate as:

$$\omega_{mn} = \frac{1}{a^2\sqrt{\rho}}\sqrt{D_{11}\alpha_1^4 + 2(D_{12} + 2D_{66})\alpha_2 R^2 + \alpha_3{}^4 D_{22} R^4},$$

where α_1, α_2, and α_3 are the coefficients, and their values have been given in Table 6. Substituting the property values of banana fiber reinforced high-density polyethylene/polycaprolactone composites,

α_1	α_3	α_2	m	n
4.730	4.730	151.3	1	1
4.730	$(n+0.5)\pi$	$12.30\alpha_3(\alpha_3-2)$	1	2, 3, 4
$(m+0.5)\pi$	4.730	$12.30\alpha_1(\alpha_1-2)$	2, 3, 4	1
$(m+0.5)\pi$	$(n+0.5)\pi$	$(\alpha_1\alpha_3(\alpha_1-2)(\alpha_3-2))$	2, 3, 4	2, 3, 4

Table 6. Coefficients for natural frequency calculations.

The natural frequencies of free vibration at simple supported and clamped edge boundary conditions have been evaluated by the present method and by the [Ashton and Whitney 1970] and [Hearmon 1959] methods. The results are presented in Tables 7 and 8 respectively, which show a good agreement.

5. Conclusions

Preparation of banana fiber reinforced HDPE/PCL composite plates, evaluation of their mechanical properties, and theoretical dynamic analyses under self-weight of the composite plate have been presented in this work. The tensile strength and Young's modulus of the HDPE/PCl blend attain saturation at approximately 20% by weight of PCL addition. Therefore, the 80:20 blend of HDPE/PCL has been selected for the matrix materials. It has also been observed that the ductility of the HDPE/PCL blend drops rapidly by addition of more than 15% by weight of PCL. Reinforcement of banana fibers into the 80:20 HDPE/PCL blend matrix shows improvement in ductility, tensile strength and Young's modulus.

Furthermore, the MQRBF method is used to analyze the dynamic behavior of the HDPE/PCL blend orthotropic plate and the banana fiber reinforced HDPE/PCL composite plate due to self-weight. This method is more advantageous than the finite element and finite difference methods because it does not

a/b	Mode	m	n	$k^{\dagger}$	k (Present)
1	1st	1	1	2.6624	2.6731
	2nd	1	2	5.7968	5.8102
	3rd	1	3	10.8533	10.8624
	4th	2	1	7.2355	7.2541
2	1st	1	1	1.4492	1.4498
	2nd	1	2	4.4698	4.4735
	3rd	1	3	9.4751	9.4902
	4th	2	1	2.6624	2.6731
3	1st	1	1	1.2059	1.2261
	2nd	1	2	4.2111	4.2109
	3rd	1	3	9.2123	9.2345
	4th	2	1	1.7749	1.7830

Table 7. The normalized fundamental frequency $\omega = \frac{k\pi^2}{b^2}\sqrt{\frac{D_{22}}{\rho}}$ of a simple supported banana fiber reinforced orthotropic plate. †[Ashton and Whitney 1970].

a/b	ω_{mn}	m	n	Hearmon k	Present k
1	ω_{11}	1	1	46.9840	48.1430
	ω_{12}	1	2	84.2114	85.4823
	ω_{13}	1	3	142.7529	142.8692
	ω_{21}	2	1	105.8893	106.9134
	ω_{22}	3	1	194.9038	195.6231
	ω_{14}	1	4	221.5417	223.4689
2	ω_{11}	1	1	107.0414	108.0613
	ω_{12}	1	2	265.6229	266.8829
	ω_{13}	1	3	503.1788	503.6571
	ω_{21}	2	1	158.8229	158.9673
	ω_{22}	3	1	243.9031	245.0341
	ω_{14}	1	4	819.3282	820.5245
3	ω_{11}	1	1	216.5571	217.6572
	ω_{12}	1	2	572.9067	572.8692
	ω_{13}	1	3	1106.70	1107.48
	ω_{21}	2	1	261.9165	261.4561
	ω_{22}	3	1	339.6262	341.3216
	ω_{14}	1	4	1817.40	1819.51

Table 8. The normalized fundamental frequency $\omega = \frac{k}{a^2}\sqrt{\frac{D_{22}}{\rho}}$ of a clamped supported banana fiber reinforced orthotropic plate.

require generation of a mesh. In this collocation method, the number of equations generated is more than the number of unknowns. Therefore, multiple regression analysis is used to overcome this incompatibility. The present results are compared with those obtained by the finite difference method as well as the results available in the literature. The banana fiber reinforced composite plate shows less deflection due to high tensile strength but takes more time to stabilize compared to HDPE/PCL blend orthotropic plate. After the incorporation of fibers, the damping properties of the banana fiber reinforced composite plate decrease and stiffness increases. The natural frequencies are determined for the self-weight of the composite plate. This method is found to be effective in the dynamic response studies of the composite plates.

Appendix A: Multiquadric method for governing differential equation

Substitution of the radial basis functions in Equation (9) gives

$$\left(\sum_{j=1}^{N} w_j \frac{\partial^4}{\partial x^4}\varphi_j + 2\lambda^2\eta \sum_{j=1}^{N} w_j \frac{\partial^4}{\partial x^2 y^2}\varphi_j + \sum_{j=1}^{n} \lambda^4 \psi w_j \frac{\partial^4}{\partial y^4}\varphi_j\right) + \sum_{j=1}^{N} w_j \frac{\partial^2}{\partial t^2}\varphi_j - C_v w_j \frac{\partial}{\partial t}\varphi_j - Q = 0.$$

Substitution of the radial basis functions in Equation (10) gives

$$\left(D_{11}/D_{22}\sum_{j=1}^{N} w_j \frac{\partial^4}{\partial x^4}\varphi_j + 2H/D_{22}\sum_{j=1}^{N} w_j \frac{\partial^4}{\partial x^2 y^2}\varphi_j + \sum_{j=1}^{N} w_j \frac{\partial^4}{\partial y^4}\varphi_j\right) = \lambda^4 \sum_{j=1}^{N} w_j \varphi_j.$$

Boundary conditions for simple supported edges:

$$x = 0, 1 \qquad \sum_{j=1}^{N} w_j \varphi_j = 0, \qquad\qquad y = 0, 1 \qquad \sum_{j=1}^{N} w_j \varphi_j = 0,$$

$$x = 0, 1 \qquad \sum_{j=1}^{N} w_j \frac{\partial^2}{\partial x^2}\varphi_j = 0, \qquad\qquad y = 0, 1 \qquad \sum_{j=1}^{N} w_j \frac{\partial^2}{\partial y^2}\varphi_j = 0.$$

Boundary conditions for clamped edges:

$$x = 0, 1 \qquad \sum_{j=1}^{N} w_j \varphi_j = 0, \qquad\qquad y = 0, 1 \qquad \sum_{j=1}^{N} w_j \varphi_j = 0,$$

$$x = 0, 1 \qquad \sum_{j=1}^{N} w_j \frac{\partial}{\partial x}\varphi_j = 0, \qquad\qquad y = 0, 1 \qquad \sum_{j=1}^{N} w_j \frac{\partial}{\partial y}\varphi_j = 0.$$

Appendix B: Multiple regression analysis

$A \cdot a = p$, where A is an $(l \times k)$ coefficient matrix, a is a $(k \times 1)$ vector, and p is a $(l \times 1)$ load vector. Approximating the solution by introducing the error vector e, one gets $p = Aa + e$, where e is an $(l \times 1)$ vector. To minimize the error norm, function S is defined as $S(a) = e^T e = (p - Aa)^T (p - Aa)$. The least-square norm must satisfy $(\partial S/\partial a)_a = -2A^T p + 2A^T Aa = 0$. This can be expressed as $a = (A^T A)^{-1} A^T P$ or $a = B \cdot p$. The matrix B is evaluated once and stored for subsequent usages.

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