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A numerical approach is proposed in this paper for safety assessment of mechanical structures with age-related degradation. Deterioration occurs in mechanical structures due to various environmental attacks and operation conditions which leads to changes in structural performance and structural resistance capacity. By combining techniques such as degradation evaluation methods, damage mechanics, and the finite element method, the proposed numerical method can be used to analyse the responses and damages of mechanical structures with age-related degradation. Ageing degradation effects can be taken into account by introducing degradation factor into formulations of finite element method and damage mechanics. The numerical results for a notched bar and a plate with a central hole are provided to validate the proposed numerical method.

1. Introduction

The ageing of mechanical structures can be defined as partial or total loss of their capacity to achieve the purpose for which they were constructed via a slow, progressive and irreversible process that occurs over a period of time. Ageing can lead to changes in engineering properties and may affect the static and dynamic responses, structural resistance/capacity, failure mode, and location of failure initiation. The ageing effects may impact the ability of mechanical system to withstand various challenges from operation, environment, and natural events.

Although engineering materials such as steel and concrete are inherently durable, some engineering structures may need to be improved because of deficiencies in their design and manufacture or as a result of environmental attacks. The ageing process can directly affect mechanical structures by changing the characteristics of the materials of which they are made and leading to a loss in their resistance capacity. Corrosion, irradiation, elevated temperature, or fatigue effects are the main problems for mild steel. Common problems to concrete include alkali-aggregate reaction (AAR), freezing and thawing, leaching, sulphate attack, cracking due volume changes led by temperature variation, corrosion of concrete, debonding of steel, etc.

The safety of existing aged mechanical structures is an important research topic owing to ageing processes altering their strength and stiffness as well as revised predictions of the maximum loads associated with severe operation conditions and environmental attacks. Probabilistic degradation evaluation approaches, damage/fracture mechanics methods, nondestructive detection methods, etc., can be used for this purpose.

Time dependent changes in engineering structures are random in nature. Safety evaluation of new and existing structures can be conducted rationally within a probabilistic framework [Shinozuka 1983].

Keywords: mechanical structures, ageing, age-related degradation, finite element method, damage mechanics, numerical analysis.

Probability methods are widely used for condition assessment of existing structures. The mathematical formalism of a probabilistic risk assessment (PRA) provides a means for identifying ageing structural components that may play a significant role in mitigating structural risk. Structural condition assessments supporting a decision regarding continued service can be rendered more sufficient if guided by the logic of a PRA.

Degradation effects can be quantified with fragility curves developed for both undegraded and degraded components. Fragility analysis is a technique for assessing, in probabilistic terms, the capability of an engineered system to withstand a specified event. Fragility modelling requires a focus on the behavior of the system as a whole and specifically on things that can go wrong with the system. The fragility modelling process leads to a median-centered (or likely) estimate of system performance coupled with an estimate of the variability or uncertainty in performance. Braverman et al. [2004] reported on a fragility analyses performed for concrete structural members and other passive components of nuclear power plants (NPPs). Structural performance in the presence of uncertainties is depicted by a fragility curve (or conditional probability of failure). Ellingwood [1998] addressed the issues related to structural ageing in probabilistic risk assessment of NPPs and proposed a probabilistic framework to assess degradation in certain critical structural components or system capacities due to reinforcement corrosion or concrete deterioration from aggressive environmental influences. Naus et al. [1999] summarized the research program addressing the ageing of nuclear power plant concrete structures. A reliability-based methodology was developed that can be used to facilitate quantitative assessments of current and future structural reliability and performance of concrete structures in NPPs. The methodology is able to take into account the nature of past and future loads, and randomness in strength and in degradation results from environmental factors. A methodology was presented by Tekie and Ellingwood [2003] for developing fragilities of concrete gravity dams to assess their performance against seismic hazards. A robust system reliability evaluation method was proposed for ageing redundant structures by Wang et al. [1997]. Using the advanced first-order second-moment reliability method, the element-level reliability indices are calculated for individual piles for several strength limit states. The system-level reliability indices of the pile group are calculated considering the lateral and vertical deflection limit states. An alternative method to evaluate the safety of engineering structures is by using damage/fracture methods. When engineering materials are subjected to unfavourable conditions such as cold and hot working processes, temperature variations, chemical actions, radiation, mechanical loading, or environmental conditions, microscopic defects and cracks may develop inside the materials. Such damage causes reduction in strength and stiffness that may lead to failure and shorten the operating life of the structures. Such deterioration in mechanical properties of a material is known as a damage process [Valliappan et al. 1990].

Because of the significant influence of damage on engineering material properties, a number of studies have been carried out on modelling and numerical methods for crack growth in structures under various loading conditions. Valliappan and his coworkers have been one of the pioneers in developing numerical methods for the analysis of engineering structures using damage mechanics. For example, from the viewpoint of continuum damage mechanics concept, Valliappan and Zhang [1996] addressed the problem of the effect of microscopic defects and cracks within materials in order to study the behavior of structural components under different loading conditions. A formulation for elasto-plastic analysis of damage mechanics problems was developed based on the principles of thermodynamics and the associated finite element method. A theoretical formulation for isotropic and anisotropic elasto-plastic analysis of static

and dynamic damage mechanics problems was developed by Zhang and Valliappan [1998a]. The formulations were based on the combination of concepts between the internal energy dissipation and the damage elasto-plasticity energy potential in terms of a set of internal state variables to explain microstructural changes in damaged materials. And the formulations were introduced into numerical algorithms of the finite element techniques for static and dynamic analysis of structures including the effect of damage [Zhang and Valliappan 1998b].

This paper presents a numerical approach for the safety assessment of mechanical structures with age-related degradation by combining techniques such as degradation evaluation methods, damage mechanics, and the finite element method. A notched bar and a plate with a central hole are solved as the examples in the numerical analysis using the proposed method. The results of the analysis validate that the proposed method is an efficient tool for safety assessment of aged mechanical structures.

2. Numerical method for safety assessment of aged mechanical structures

A dynamic two-dimensional finite element method coupled with damage mechanics is developed to evaluate damage initiation and propagation for aged mechanical structures. To take into account the effect of aged-related degradation on the safety of aged mechanical structures, ageing degradation is quantified according to various mechanical and environmental attacks under various operation conditions. The quantified age-related degradation factor is then included in the damage model and the finite element formulation.

2.1. Quantification of ageing degradation of engineering materials. Age-related degradation of engineering materials such as steel and concrete is a complicated process. Degradation of aged steel can be classified as either material or physical damage. Material damage occurs when the microstructure of the metal is modified causing changes in its mechanical properties. For example, degradation mechanisms that can potentially cause material damage to containment steels include: (i) low-temperature exposure, (ii) high-temperature exposure, (iii) intergranular corrosion, (iv) dealloying corrosion, (v) hydrogen embrittlement, and (vi) neutron irradiation. Primary degradation mechanisms that potentially can cause physical damage to containment pressure boundary components include: (i) general corrosion (atmospheric, aqueous, galvanic, stray-electrical current, and general biological); (ii) localized corrosion (filiform, crevice, pitting, and localized biological); (iii) mechanically assisted degradation (erosion, fretting, cavitation, corrosion fatigue, surface flaws, arc strikes, and overload conditions); (iv) environmentally induced cracking (stress-corrosion and hydrogen-induced); and (v) fatigue. Degradation of mild steel reinforcing concrete can occur as a result of corrosion, irradiation, elevated temperature, or fatigue effects. Prestressing concrete is susceptible to the same degradation mechanism as mild steel concrete, primarily due to tendon relaxation and concrete creep and shrinkage.

Ageing is a time-dependent process. Ageing degradation of engineering materials, which accumulates over time by various processes depending on the operating environment and service conditions, will reduce the strength of structures or their components. Generally, the ageing degradation of the strength of a structure or a structural component can be expressed by

$$R(t) = R_0G(t) \quad (1)$$

in which R_0 is the component capacity in the undegraded (original) state and $G(t)$ is a time dependent degradation function defining the fraction of initial strength remaining at time t . The degradation mechanisms are uncertain, experimental data are lacking, and thus the function $G(t)$ should be treated as stochastic. However, as it has been found that the variability in $G(t)$ is of minor importance when compared to mean degradation and local process characteristics, it is assumed that $G(t)$ is deterministic and equal to mean $E[g(t)] = G(t)$ [Mori and Ellingwood 1993].

Corrosion is one of the main reasons for degradation of aged steel. In this research time-dependent ageing degradation function for steel is assumed to be given by Broomfield [1997]

$$G(t) = 1 - \frac{1}{1 + Ae^{-Bt}} \quad (2)$$

in which A and B are constant parameters and t is the age of the structure.

It is worth pointing out that Equation (1) is applicable to all structural materials whereas (2) is specifically for steel. However, similar to (2) expressions are available for other materials such as concrete. In fact, the authors have adopted one such function for the stability evaluation of aged concrete gravity dams.

2.2. Dynamic finite element analysis of aged mechanical structures using damage mechanics. Since the safety margin reserved in the critical mechanical structures is large, they are considered to be safe even against hostile loadings larger than expected in their design. However, past experience shows that engineering structures are susceptible to ageing degradation under various attacks which may affect the mechanical characteristics of the structures. Whether the aged mechanical structures can still withstand the challenges from hostile environments and natural events is of great concern to engineers. Therefore, time-dependent degradation effects should be included when structural safety and reliability are evaluated for mechanical structures with age-related degradation.

In the past many investigators have studied the effect of cracking on the dynamic response of engineering structures using the concept of fracture mechanics. The concept of fracture mechanics requires complete details of initiation and propagation of cracks within the structure and the location. Besides, the numerical modelling of such individual crack propagation requires special techniques such as quarter-point element, remeshing, etc. [Murti and Valliappan 1986]. Therefore the application of fracture mechanics is limited and for cases in which extensive microcracking may develop, it may not be suitable, especially in dynamic analysis.

The concept of continuum damage mechanics can be used to study the effect of microcracking on the dynamic responses of engineering structures [Valliappan and Zhang 1996]. Damage mechanics provides an average measure of material degradation due to microcracking, interfacial debonding, nucleation, and coalescence of voids. In the microcracking of brittle materials under tensile stress, damage is regarded as elastic degradation. This material degradation is reflected in the nonlinear behaviour of the structures.

2.2.1. Finite element equations of motion for aged mechanical structures. Modelling of mechanical structures can be done using the two-dimensional finite element method. The equation of motion for the dynamic analysis including damage can be written as

$$\mathbf{M}\ddot{\vec{U}} + \mathbf{C}^*\dot{\vec{U}} + \vec{P}^*(\vec{U}) = \vec{F}_s + \vec{F}_d, \quad (3)$$

where \mathbf{M} is the mass matrix of the system and consists of element mass matrices \mathbf{m}^e

$$\mathbf{m}^e = \int_{\Omega_e} \rho^{(e)} \mathbf{N}^T \mathbf{N} d\Omega,$$

where $\rho^{(e)}$ and \mathbf{N} represent mass density and shape function matrix for an element, respectively, \mathbf{C}^* is the damping matrix of the system, $\vec{P}^*(\vec{U})$ represents the vector of restoring forces, and \vec{F}_s and \vec{F}_d are static and dynamic loading vectors, respectively. $\vec{P}^*(\vec{U})$ is a nonlinear function of displacement and stress-strain history depending on the constitutive law. It can be given by

$$\vec{P}^*(\vec{U}) = \mathbf{K}^* \vec{U},$$

where \mathbf{K}^* is the system stiffness and is obtained from the assemblage of an element stiffness matrices \mathbf{k}^e

$$\mathbf{k}^e = \int_{\Omega_e} \mathbf{B}^T \mathbf{T}_\sigma \tilde{\mathbf{E}}^{*(e)} \mathbf{T}_\sigma^T \mathbf{B} d\Omega,$$

where \mathbf{T}_σ is the coordinate transformation matrix, $\tilde{\mathbf{E}}^*$ is the damaged constitutive matrix taking into account the age-related degradation in the orthotropic damage space, and \mathbf{B} is the strain-displacement matrix.

2.2.2. Damage model and damage evolution. The formulation of a damage model first requires the definition of threshold of damage, which is the condition that initiates the damage. Secondly, the evolution of damage with loading must be also defined, and it is a function of a measure of strains, stresses, or energy. This damage evolution can be any of the following forms

$$\dot{D} = \dot{D}(\sigma_{ij}(\varepsilon_{ij}), D, \dots) \quad \text{or} \quad D = D(\sigma_{ij}(\varepsilon_{ij}), \dots), \quad (4)$$

where σ_{ij} is the state of stress at a particular point and D is the damage tensor at that point. Also \dot{D} represents the rate of damage. The most common damage kinetic equation that is used widely is based on a power function of tensile normal stress and was introduced first by [Kachanov \[1980\]](#):

$$\dot{D} = \begin{cases} A_L \left(\frac{\sigma}{1-D} \right)^n, & \text{for } \sigma > \sigma_d, \\ 0, & \text{for } \sigma \leq \sigma_d, \end{cases}$$

where $A_L > 0$ and $n > 1$ are material constants depending on the rate of loading, σ is the uniaxial tensile stress, and σ_d is the stress at damage threshold. For this model one needs experimental results to obtain parameters A_L and n , but these results are not available for all kinds of loading. As proposed by [Bazant and Lin \[1988\]](#) and applied by [Ghrib and Tinawi \[1995\]](#), a second model based on [Equation \(4\)](#) (right) can be used for dynamic analysis.

In the materials, which eventually exhibit strain softening that leads to a complete loss of strength, the secant modulus decreases with increasing strain [\[Lubliner et al. 1989\]](#). A widely used assumption is a triangular stress-strain diagram for uniaxial loading. This gives a linear strain softening relationship. But various experimental evidences indicate that it is more realistic to assume a strain-softening curve with a steep initial decline followed by an extended tail [\[Lubliner et al. 1989\]](#). Then an exponential

strain-softening model can be given by

$$\sigma(\varepsilon) = \begin{cases} E\varepsilon, & \text{for } \varepsilon \leq \varepsilon_0, \\ f'_t(2e^{-a(\varepsilon-\varepsilon_0)} - e^{-2a(\varepsilon-\varepsilon_0)}), & \text{for } \varepsilon_0 < \varepsilon < \varepsilon_{cr}, \\ 0, & \text{for } \varepsilon \geq \varepsilon_{cr} \end{cases} \quad (5)$$

in which f'_t is the tensile strength and ε_0 is the corresponding strain threshold, E is the modulus of elasticity, a is a dimensionless constant, and ε_{cr} is the maximum strain. For aged mechanical structures, E and f'_t in Equation (5) are given by Equation (1) due to ageing degradation. In this study the value ε_{cr} is calculated when its corresponding stress is equal to $0.02f'_t$. Then

$$\varepsilon_{cr} = \varepsilon_0 + \frac{\ln\left(\frac{2+\sqrt{4-4\lambda}}{2\lambda}\right)}{a}.$$

When $\lambda = 0.02$, the maximum strain ε_{cr} is calculated by

$$\varepsilon_{cr} = \varepsilon_0 + \frac{4.6}{a}.$$

The fracture energy per unit area, G_f , is defined as $G_f = l_{ch}g_t$, where g_t is total area under stress-strain curve

$$g_t = \int_0^\infty \sigma(\varepsilon) d\varepsilon = \frac{3f'_t}{2a} + \frac{f'_t}{2E},$$

and l_{ch} is the characteristic length. From the above two equations the constant a can be given by

$$a = \frac{3}{\varepsilon_0 \left(\frac{2EG_f}{l_{ch}f'^2_t} - 1 \right)} \geq 0.$$

Based on the hypothesis of strain energy equivalence, the anisotropic damage parameters can be defined in terms of Young's modulus [Valliappan et al. 1990]

$$D_i = 1 - \sqrt{\frac{E_i^*}{E_i}},$$

and hence from Equation (5), the proper definition of damage for the uniaxial case is

$$D_i = 1 - \sqrt{\left(\frac{\varepsilon_0}{\varepsilon}\right)(2e^{-a(\varepsilon-\varepsilon_0)} - e^{-2a(\varepsilon-\varepsilon_0)})}.$$

In the two equations above i represents the i -th principal direction.

3. Numerical analysis

Two examples – a notched bar and a plate with a central hole – are used for the numerical analysis to validate the proposed method for safety assessment of aged mechanical structures.

Figure 1 shows the model of the notched bar and its finite element mesh discretization. The dimension of the bar is 10 cm × 30 cm. Distributed step loading of 1560 kN/cm⁻¹, which is shown in Figure 2, is applied at one end of the bar in longitudinal direction. The notched bar is modeled as a two-dimensional

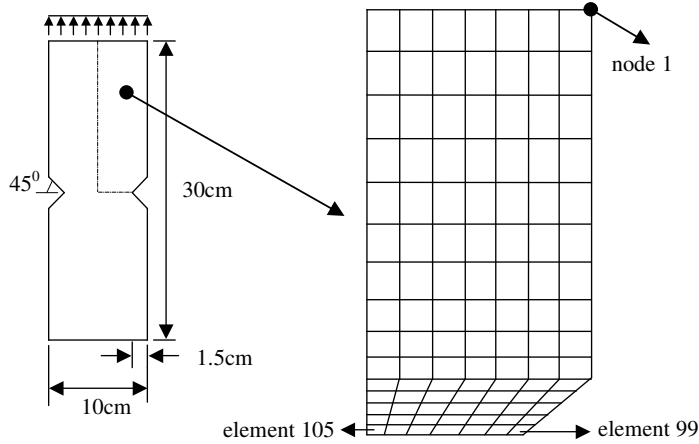


Figure 1. Notched bar and its finite element mesh discretization.

plane stress case and is discretized with four noded isoparametric elements. Because of the symmetries of geometry and loading, the notched bar is discretized with 105 elements and 128 nodes. The material properties of the bar are chosen as follows: modulus of elasticity $E = 2.0 \times 10^{11}$ Pa; Poisson’s ratio $\nu = 0.28$; density $\rho = 7850.0 \text{ kg/m}^{-3}$; tensile strength $f_t' = 4.0 \times 10^8$ Pa, and fracture energy for unit area $G_f = 2.6 \times 10^6 \text{ N/m}^{-1}$.

To include ageing degradation effect in the numerical analysis, a degradation function given by Equation (2) is used to modify the modulus of elasticity and tensile strength of the material. Parameters A and B in Equation (2) are the two constants that define the ageing degradation effect. In order to determine their effect on the system responses and damages, in the present study parameter A is considered to be a fuzzy variable, while parameter B is considered to be a deterministic parameter. It was found that B is more sensitive to the ageing degradation. Therefore the values of 0.125 and 0.25 were assigned to parameter B in the numerical analysis to simulate the degradation processes under different environmental and working conditions. The value of A is set to 100. Figure 3 shows the deterioration curves of different B values used in the numerical analysis.

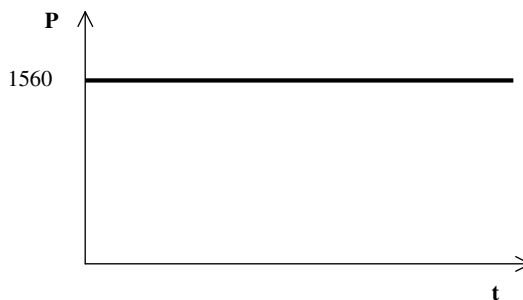


Figure 2. Step loading: $P = 1560 \text{ kN/cm}$.

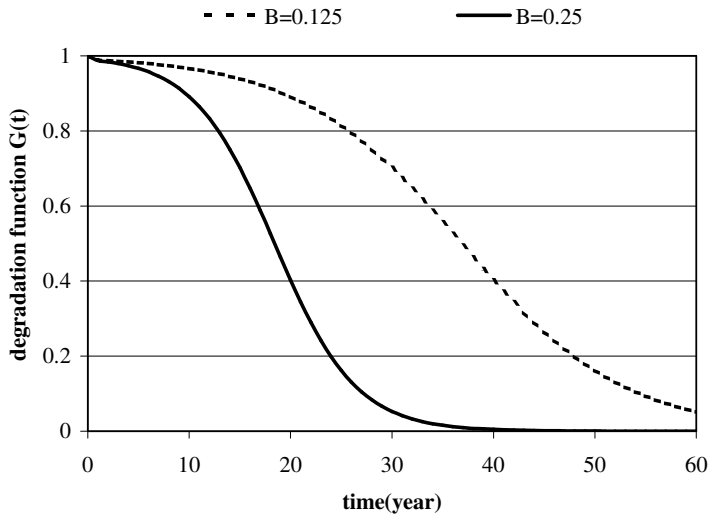


Figure 3. Degradation function of steel.

In this study the equation of motion (Equation (3)) in the time domain is solved using Newmark’s integration method, which is unconditionally convergent. In selecting the time step the natural frequency of the structure and the type of loading must be considered. However, in the present case, because of the step loading, it is necessary to consider only the loading, and hence the time step is set as 0.01 s to check the system responses and damages. Structural responses and damage parameters were obtained for both cases of $B = 0.125$ (up to 40 years) and $B = 0.25$ (up to 20 years) to check the effect of ageing degradation on the behaviour of the notched bar.

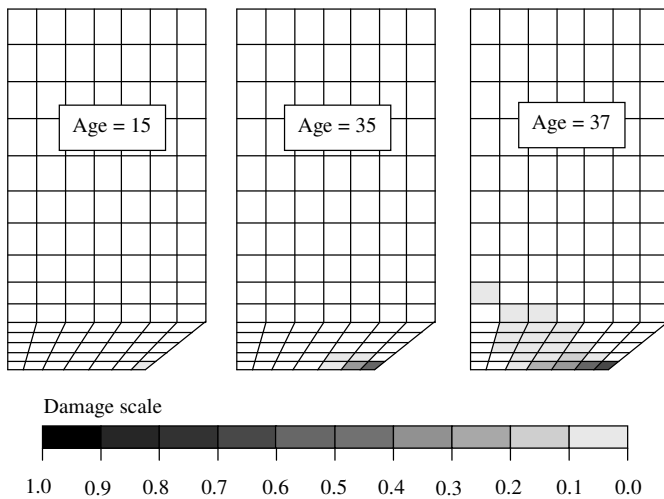


Figure 4. Damage patterns of the notched bar ($B = 0.125$).

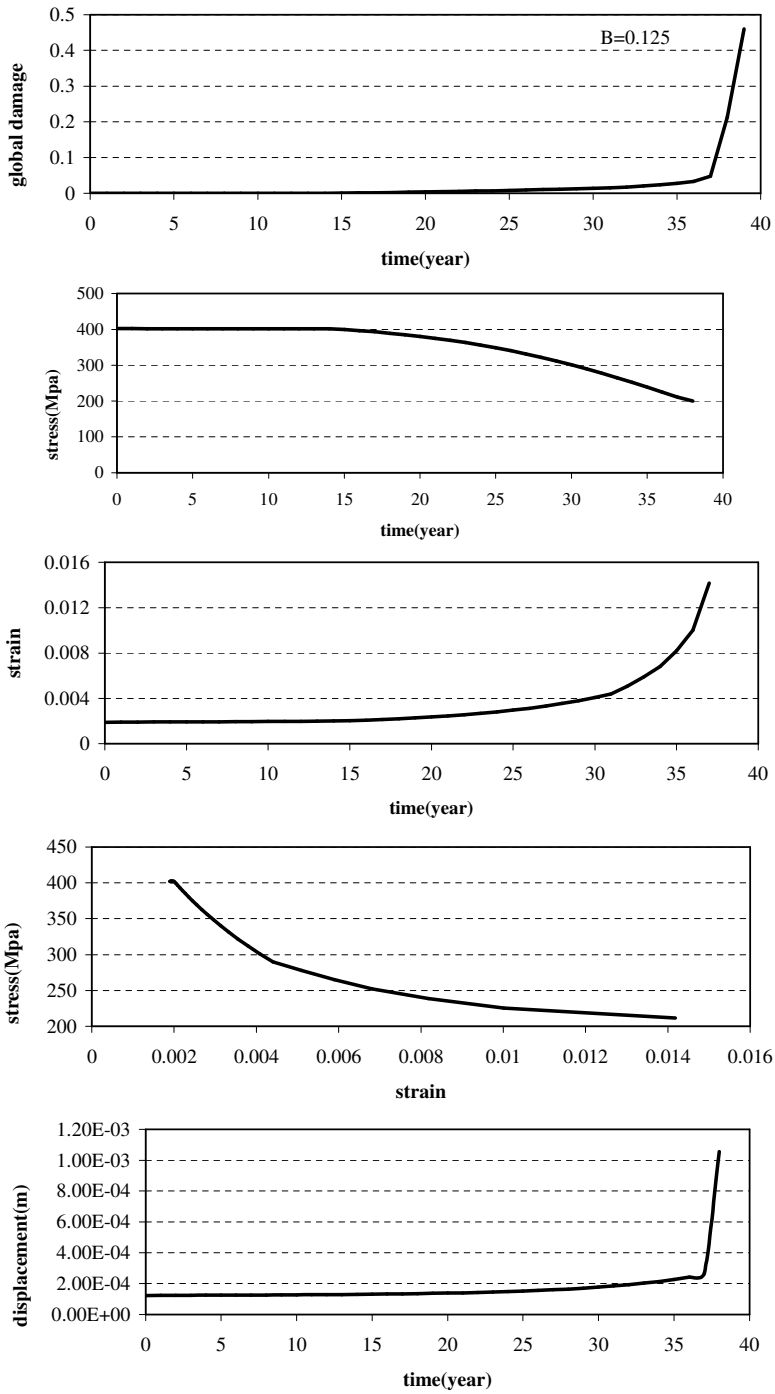


Figure 5. Global damage, displacement of node 1, and major principal stress, major principal strain, and stress-strain relationship of element 99, respectively, of the notched bar ($B = 0.125$).

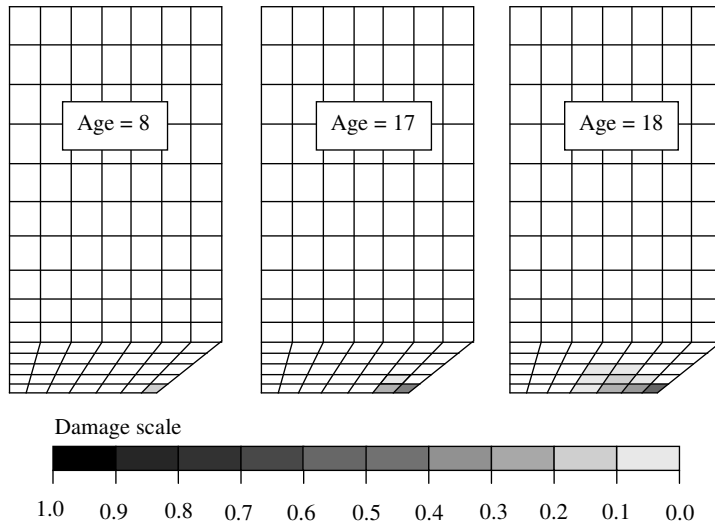


Figure 6. Damage patterns of the notched bar ($B = 0.25$).

For the case of $B = 0.125$ Figure 4 shows the damage pattern of the notched bar at times of 15, 35, and 37 years, and Figure 5 shows the global damage index. The global damage index is defined by the following equation

$$D_{\text{global}} = \sqrt{\sum_{e=1}^{\text{nel}} \int_{\Omega_e} (D^{(e)})^2 d\Omega_e} / \sqrt{\sum_{e=1}^{\text{nel}} \int_{\Omega_e} d\Omega_e} \quad (6)$$

in which $D^{(e)}$ is the damage of the e -th element and ‘nel’ represents the total number of the damaged elements. In determining the damage to the entire structure, it is desirable to take into account the cumulative effect of the damage occurring in parts of the structure, which are individual elements in the finite element method. It is well known that there is a size effect when discretization techniques are used. In the present study this cumulative effect is represented by the expression given in Equation (6) for the purpose of illustrating the damage to the structure due to ageing degradation. If one desires, any other alternative expression can be developed for this purpose.

These two figures show that the damage occurs when the bar is 15 years of age and then extends gradually to the direction of 45° as the age of the bar increases. After 37 years, the damage increases significantly which leads to the failure of the notched bar. Figure 5 shows the displacement at node 1. As the age of the notched bar increases, the strength of the material decreases which results in an increase of displacement and damage. Also shown in Figure 5 are the major principal stress, strain, and their relationship of element 99, respectively.

Figure 6 shows the damage pattern of the notched bar at 8, 17, and 18 years for the case of $B = 0.25$, and Figure 7 shows the global damage index. Because the notched bar has suffered more ageing degradation than the case of $B = 0.125$, the damage initiates at the age of 8 years and increases dramatically to total failure after 18 years. Also shown in Figure 7 are the displacement of node 1, the major principal stress, strain, and their relationship of element 99, respectively.

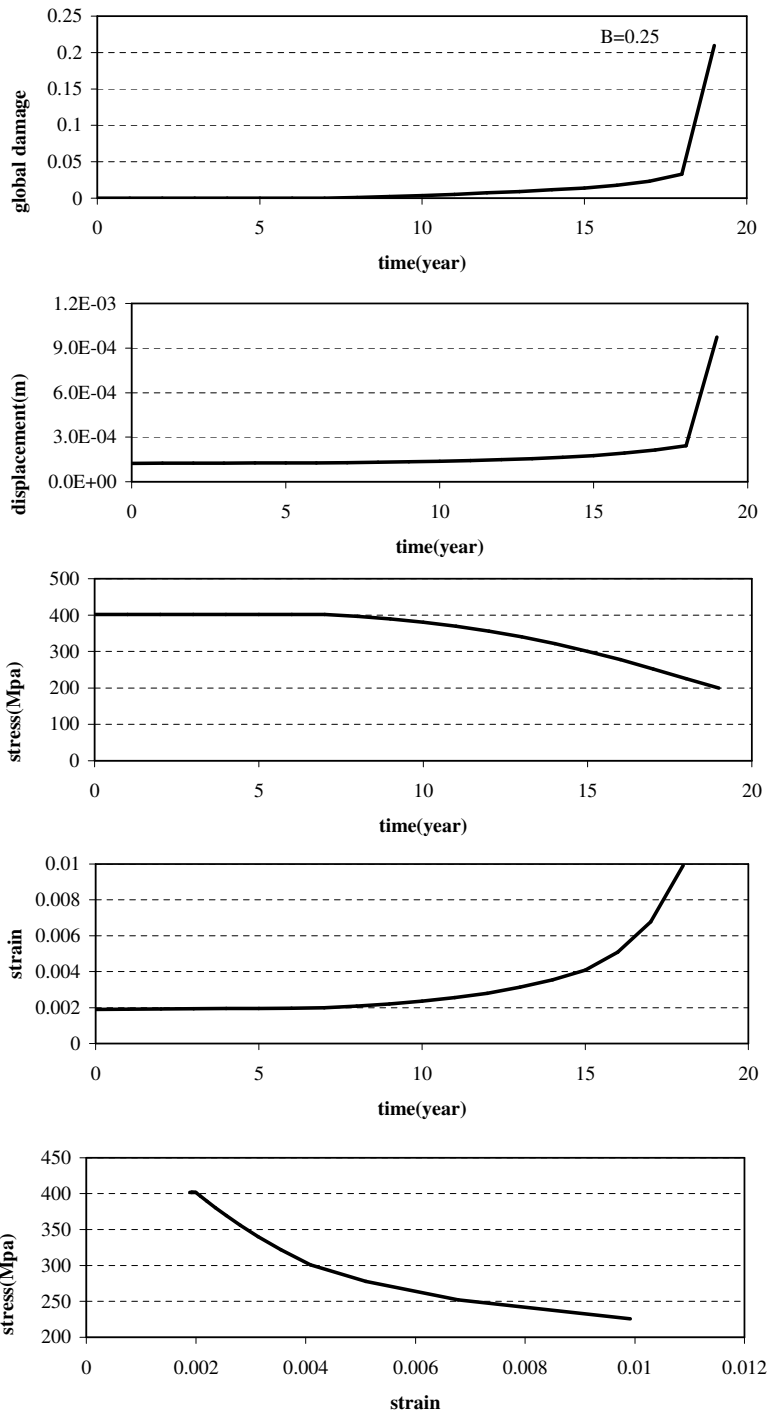


Figure 7. Global damage, displacement of node 1, and major principal stress, major principal strain, and stress-strain relationship of element 99, respectively, of the notched bar ($B = 0.25$).

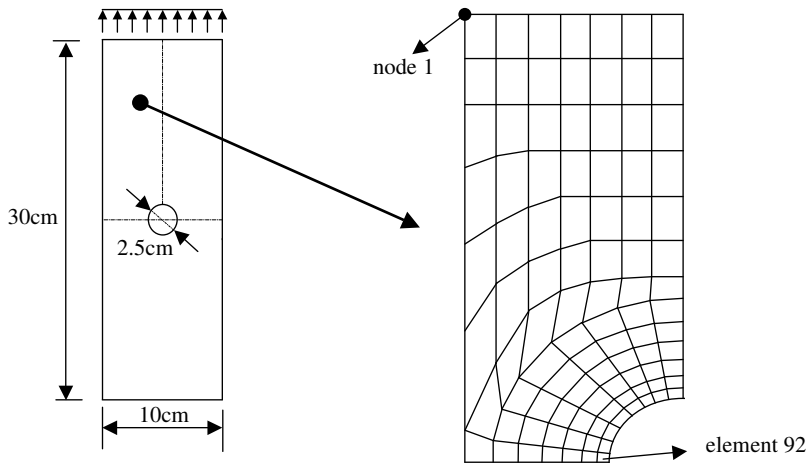


Figure 8. A plate with a central hole and its finite element mesh discretization.

Figure 8 shows the plate model and its finite element mesh. The dimension of the plate is 10 cm \times 30 cm, and the diameter of the central hole is 2.5 cm. Distributed step loading is also used in the analysis of which the amplitude is 1380 kN/cm^{-1} . Four noded isoparametric elements are used in finite element discretization, and the plate is discretized into 92 elements and 120 nodes due to the symmetries of geometry and loading. The material properties of the plate are assumed to be the same as those of the notched bar. The analysis is carried out for the two different deterioration rate $B = 0.125$ and $B = 0.25$, respectively.

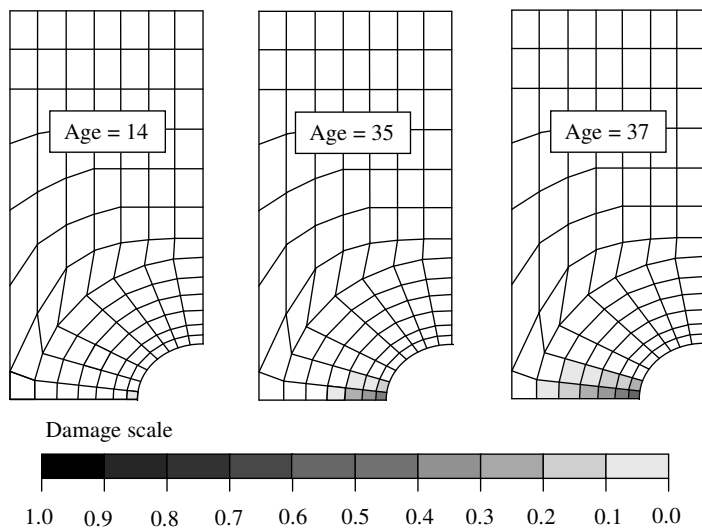


Figure 9. Damage patterns of the plate ($B = 0.125$).

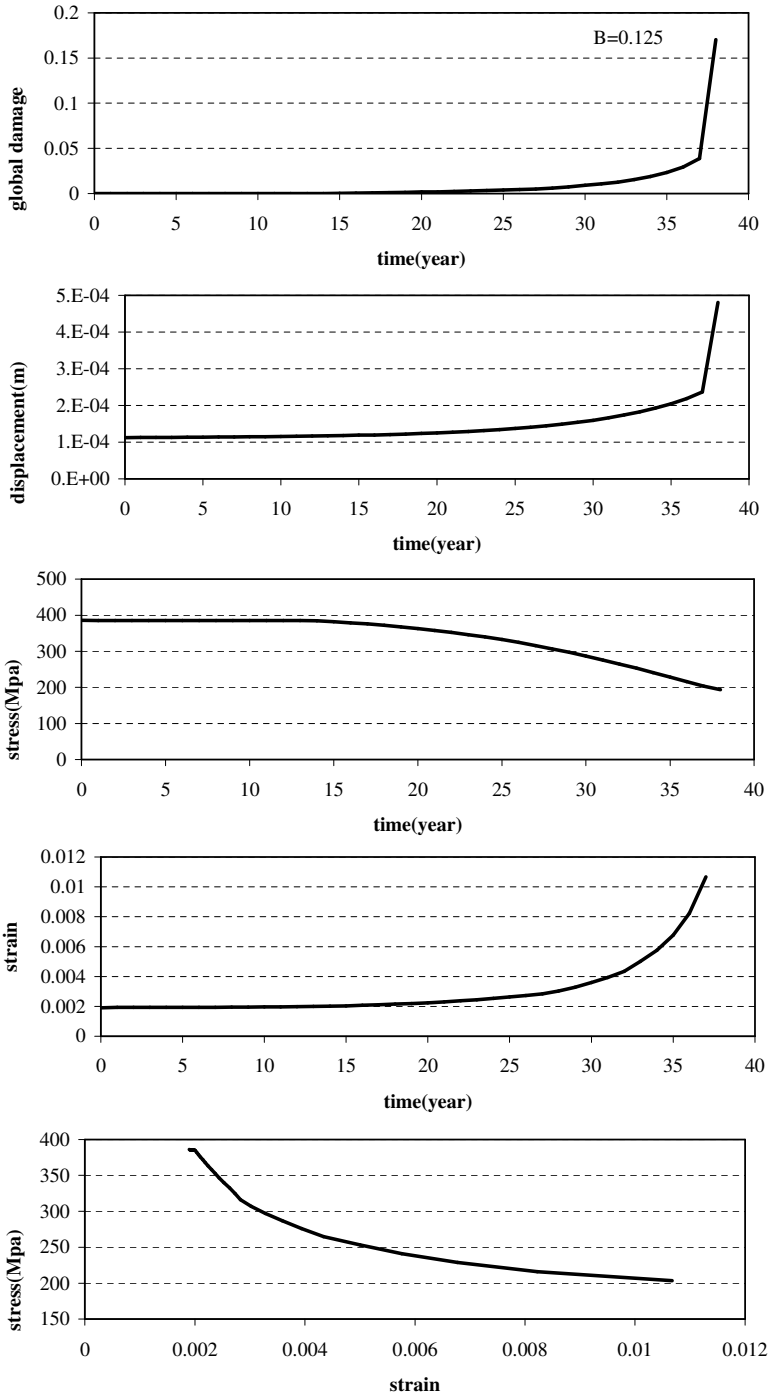


Figure 10. Global damage, displacement of node 1, and major principal stress, major principal strain, and stress-strain relationship of element 92, respectively, of the notched bar ($B = 0.125$).

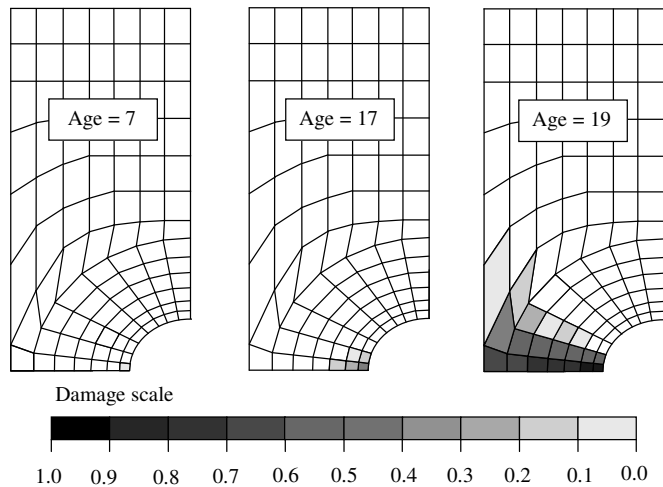


Figure 11. Damage patterns of the plate ($B = 0.25$).

Figure 9 shows the damage pattern of the plate model at 14, 35, and 37 years and Figure 10 shows the global damage index for the case of $B = 0.125$. Damage occurs firstly in element 92 at 14 years and then extends in the direction orthogonal to the loading as the plate suffers more ageing deterioration. Damage increases significantly after 37 years and leads to total failure of the plate. Figure 10 shows the displacement of node 1 at different times and the major principal stress, strain, and their relationship of element 92, respectively.

For the case of $B = 0.25$ Figures 11 and 12 show the damage pattern of the plate and the global damage index at 7, 17 and 19 years. Because of more severe ageing degradation for this case, the plate is damaged initially in element 92 at 7 years and the damage increases gradually up to 18 years. The damage increases sharply at 19 years which then leads to total failure. The displacement at node 1 is shown in Figure 12. It shows the same trend as the global damage index. Figure 12 also shows the major principal stress, strain, and their relationship of element 92, respectively.

4. Conclusions

Ageing is a natural phenomenon via a slow process for mechanical structures due to environmental attacks and operation conditions. Age-related degradation can reduce the strength, change physical properties of the mechanical structures, which may result in the decrease of the their capacity to withstand operation conditions and hostile natural events.

The method presented in this paper is for numerical analysis of mechanical structures with ageing degradation effect. Using techniques such as the finite element method, damage mechanics, and ageing degradation evaluation methods, this method can be used for safety assessment of aged mechanical structures by obtaining initiation and propagation of structural damage over a long period of time. The proposed method was validated by numerical analyses of a notched bar and a plate with a central hole. Under step loading condition, structural responses, damage initiation and propagation were studied in detail for different levels of degradation over a period of several decades. Numerical results show ageing

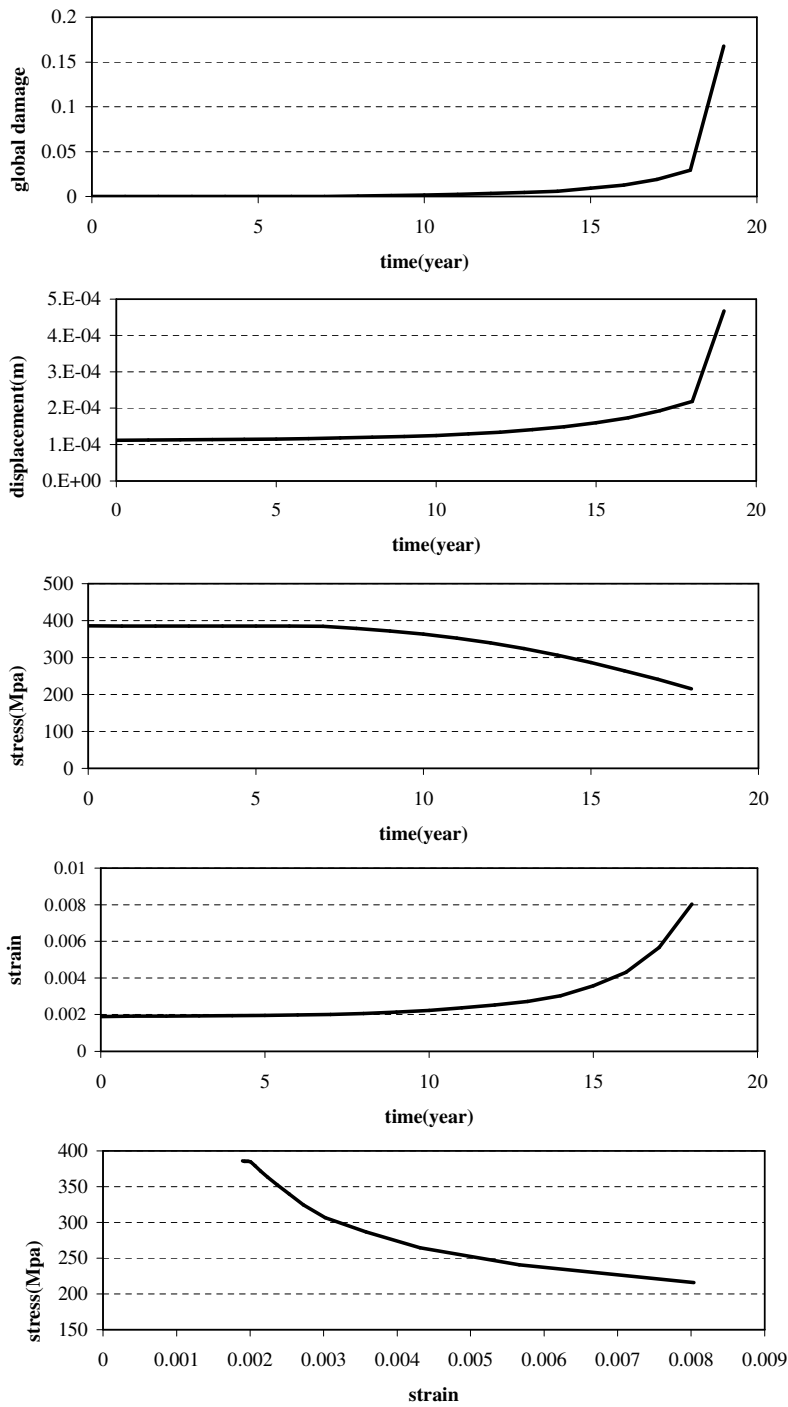


Figure 12. Global damage, displacement of node 1, and major principal stress, major principal strain, and stress-strain relationship of element 92, respectively, of the notched bar ($B = 0.25$).

degradation can significantly affect structural performance and more severe damages occur for mechanical structures when ageing degradation is included.

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