Journal of
Mechanics of
Materials and Structures

INTERACTION BETWEEN A SCREW DISLOCATION AND A PIEZOELECTRIC CIRCULAR INCLUSION WITH VISCOUS INTERFACE

Xu Wang, Ernian Pan and A. K Roy

Volume 3, № 4

April 2008

mathematical sciences publishers

JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES
Vol. 3, No. 4, 2008

INTERACTION BETWEEN A SCREW DISLOCATION AND A PIEZOELECTRIC CIRCULAR INCLUSION WITH VISCOUS INTERFACE

XU WANG, ERNIAN PAN AND A. K ROY

Exact closed-form solutions in terms of elementary functions are derived for the problem of a screw dislocation embedded in an unbounded piezoelectric matrix interacting with a piezoelectric circular inclusion with a linear viscous interface. By means of the complex variable method, the original boundary value problem is reduced to an inhomogeneous first-order partial differential equation whose solution can be expressed in terms of elementary functions. The time dependent electroelastic fields such as stresses, strains, electric fields, and electric displacements are then obtained. In particular the image force acting on the piezoelectric screw dislocation, due to its interaction with the circular viscous interface, is presented. Some special cases of practical importance are discussed to verify and to illustrate the obtained solution. Finally we present a specific example of a screw dislocation located in a piezoelectric PZT-5 matrix interacting with a piezoelectric $BaTiO_3$ fiber to graphically demonstrate the influence of the viscosity of the interface on the mobility of the screw dislocation.

1. Introduction

Due to their intrinsic electromechanical coupling behaviors, piezoelectric ceramics have been widely used in applications such as sensors, filters, ultrasonic generators, and actuators. More recently, the use of piezoelectric materials has gone beyond the traditional application domain of small electric devices due to the emergence of piezoelectric composites. Nowadays, piezoelectric materials have been employed as integrated active structural elements. These adaptive structures are capable of monitoring and adapting to their environments, providing a *smart* response to external conditions. Investigations on piezoelectric materials in the presence of defects such as dislocations, cracks, and inclusions are many [Pak 1990a; 1990b; 1992; Meguid and Deng 1998; Liu et al. 1999; Ru and Mao 1999; Lee et al. 2000; Chen et al. 2002; Wang et al. 2003; Wang and Pan 2007; Xiao et al. 2007] due to the fact that these defects can adversely influence the performance of the piezoelectric devices. Recently He and Lim [2003] analyzed the electromechanical response of a piezoelectric fibrous composite with a viscous interface described by the linear law of rheology [Ray and Ashby 1971; Suo 1997]. Their results demonstrated that the interfacial sliding could alter the local mechanical and electrical fields in the piezoelectric composite, and could further lead to significant change in overall electromechanical response of the composites with time.

The present paper investigates the interaction between a screw dislocation and a piezoelectric circular inclusion with a viscous interface described by the linear law of rheology [Ray and Ashby 1971; Suo 1997; He and Lim 2003]. The viscosity of the interface investigated in this research originates from the

Keywords: piezoelectricity, circular inclusion, screw dislocation, viscous interface, image force.
This work is supported in part by Air Force Research Laboratory (06-S531-060-C1).

microscopically diffusion controlled sliding mechanism [Ray and Ashby 1971; Suo 1997], or from an artificially introduced thin viscous layer for damping purpose. This study is confined to the quasistatic assumption, ignoring the inertial force in the piezoelectric inclusion and matrix. By means of the complex variable method, the original boundary value problem is reduced to an inhomogeneous first-order partial differential equation for an analytic function defined within the circular inclusion. A closed-form solution in terms of elementary functions to the partial differential equation is obtained after a transformation is introduced. It is stressed that the usage of the complex variable combined with the real time variable is very novel in the literature.

2. Basic formulations

Consider a circular piezoelectric inclusion (or fiber) of radius R embedded in an unbounded piezoelectric matrix, as shown in Figure 1. Both the inclusion and matrix are of 6 mm material with symmetry about the fiber axis. The inclusion/matrix interface is a viscous one characterized with a law of linear rheology [Ray and Ashby 1971; Suo 1997; He and Lim 2003] (or equivalently modeled by linear dashpot [Fan and Wang 2003; Wang and Schiavone 2007]). At time $t = 0$, a piezoelectric screw dislocation is introduced into the piezoelectric matrix and fixed at point (x_0, y_0). The screw dislocation is assumed to be straight and infinitely long in the x_3 direction (the fiber axis), experiencing a displacement jump b and an electric potential jump $\Delta\phi$ across the slip plane. The dislocation also has a line force p and line charge q along its core. In this configuration the governing equations and constitutive equations can be simplified considerably as follows.

- Governing field equations:

$$\sigma_{zx,x} + \sigma_{zy,y} = 0, \qquad D_{x,x} + D_{y,y} = 0, \tag{1}$$

- Electric field/electric potential relations:

$$E_x = -\phi_{,x}, \qquad E_y = -\phi_{,y}, \tag{2}$$

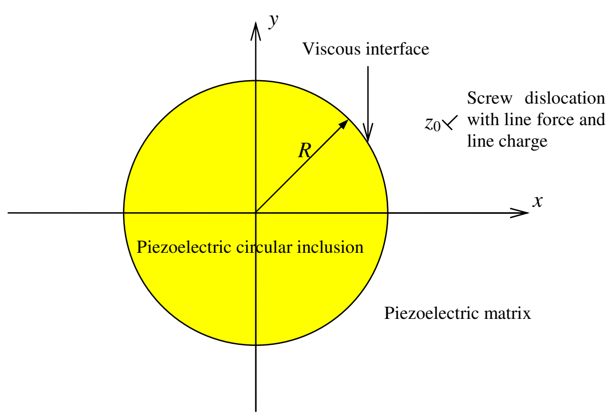

Figure 1. A screw dislocation with a line force and a line charge near a piezoelectric circular inclusion with a viscous interface.

- Linear piezoelectric constitutive equations:

$$\begin{bmatrix}\sigma_{zy}\\ D_y\end{bmatrix}=\begin{bmatrix}c_{44} & -e_{15}\\ e_{15} & \epsilon_{11}\end{bmatrix}\begin{bmatrix}w_{,y}\\ E_y\end{bmatrix},\qquad \begin{bmatrix}\sigma_{zx}\\ D_x\end{bmatrix}=\begin{bmatrix}c_{44} & -e_{15}\\ e_{15} & \epsilon_{11}\end{bmatrix}\begin{bmatrix}w_{,x}\\ E_x\end{bmatrix}, \tag{3}$$

where a comma followed by x or y denotes partial derivatives with respect to x or y, respectively. σ_{zx} and σ_{zy} are the shear stress components, D_x and D_y the electric displacement components, E_x and E_y the electric fields, w the out of plane displacement, ϕ the electric potential, and c_{44}, e_{15}, and ϵ_{11} are, respectively, the elastic modulus, piezoelectric constant, and dielectric permittivity. In the following analysis the piezoelectrically stiffened elastic constant $\tilde{c}_{44}=c_{44}+e_{15}^2/\epsilon_{11}$ will be also used. In Equation (1) we have neglected the inertial effect of the piezoelectric material due to the fact that the viscous response comes from the interface only.

The displacement and electric potential can be expressed in terms of an analytic function vector $\boldsymbol{f}(z,t)=\left[f_1(z,t),\ f_2(z,t)\right]^T$, $z=x+iy$, as

$$\begin{bmatrix}w\\ \phi\end{bmatrix}=\mathrm{im}\{\boldsymbol{f}(z,t)\},$$

where im stands for the imaginary part. Since the viscous interface exhibits the time effect, the analytic function vector $\boldsymbol{f}(z,t)$ depends not only on the complex variable z but also on the time t. In terms of the analytic function vector $\boldsymbol{f}(z,t)$, the strains, electric fields, stresses, and electric displacements in the Cartesian coordinate system can be expressed as

$$\begin{bmatrix}\gamma_{zy}+i\gamma_{zx}\\ -E_y-iE_x\end{bmatrix}=\frac{\partial\boldsymbol{f}(z,t)}{\partial z},\qquad \begin{bmatrix}\sigma_{zy}+i\sigma_{zx}\\ D_y+iD_x\end{bmatrix}=\boldsymbol{C}\,\frac{\partial\boldsymbol{f}(z,t)}{\partial z}, \tag{4}$$

where the strains γ_{zx} and γ_{zy} are related to the out of plane displacement w through $\gamma_{zx}=w_{,x}$, and $\gamma_{zy}=w_{,y}$, and $\boldsymbol{C}$ is the extended stiffness matrix given by

$$\boldsymbol{C}=\begin{bmatrix}c_{44} & e_{15}\\ e_{15} & -\epsilon_{11}\end{bmatrix}.$$

The strains, electric fields, stresses, and electric displacements in the polar coordinate system can also be expressed in terms of the analytic function vector $\boldsymbol{f}(z,t)$ as

$$\begin{bmatrix}\gamma_{z\theta}+i\gamma_{zr}\\ -E_\theta-iE_r\end{bmatrix}=\frac{z}{|z|}\frac{\partial\boldsymbol{f}(z,t)}{\partial z},\qquad \begin{bmatrix}\sigma_{z\theta}+i\sigma_{zr}\\ D_\theta+iD_r\end{bmatrix}=\boldsymbol{C}\,\frac{z}{|z|}\frac{\partial\boldsymbol{f}(z,t)}{\partial z}. \tag{5}$$

In this paper, the superscripts (1) and (2) (or the subscripts 1 and 2) will be used to denote, respectively, the physical quantities in the inclusion and matrix. The analytic function vector defined in the inclusion is denoted by $\boldsymbol{g}(z,t)=\left[g_1(z,t),\ g_2(z,t)\right]^T$, whilst that in the unbounded matrix is denoted by $\boldsymbol{h}(z,t)=\left[h_1(z,t),h_2(z,t)\right]^T$.

3. Complex potentials and field components

The boundary conditions on the viscous interface $r = R$ can be expressed as [He and Lim 2003]

$$\begin{aligned} &\sigma_{zr}^{(1)} = \sigma_{zr}^{(2)},\ D_r^{(1)} = D_r^{(2)}, \\ &\phi^{(1)} = \phi^{(2)}, \qquad\qquad r = R \quad \text{and} \quad t > 0, \\ &\sigma_{zr}^{(1)} = \eta\big(\dot{w}^{(2)} - \dot{w}^{(1)}\big), \end{aligned} \tag{6}$$

where a dot over the quantity denotes differentiation with respect to time t, and η is the interface slip coefficient, which can be measured through properly designed experiment.

Equation $(6)_1$ for the continuity condition of traction and normal electric displacement across the interface can be equivalently expressed as

$$\boldsymbol{C}_1\boldsymbol{g}^+(z,t) + \boldsymbol{C}_1\bar{\boldsymbol{g}}^-\big(R^2/z,t\big) = \boldsymbol{C}_2\boldsymbol{h}^-(z,t) + \boldsymbol{C}_2\bar{\boldsymbol{h}}^+\big(R^2/z,\big), \qquad |z| = R,$$

It follows from the above expression that

$$\begin{aligned} \boldsymbol{h}(z,t) &= \boldsymbol{C}_2^{-1}\boldsymbol{C}_1\bar{\boldsymbol{g}}\big(R^2/z,t\big) + \boldsymbol{h}_0(z) - \bar{\boldsymbol{h}}_0\big(R^2/z\big), \\ \bar{\boldsymbol{h}}\big(R^2/z,t\big) &= \boldsymbol{C}_2^{-1}\boldsymbol{C}_1\boldsymbol{g}(z,t) - \boldsymbol{h}_0(z) + \bar{\boldsymbol{h}}_0\big(R^2/z\big), \end{aligned} \tag{7}$$

where

$$\begin{aligned} &\boldsymbol{h}_0(z) = \big[h_{10}(z), h_{20}(z)\big]^T = \frac{\hat{\boldsymbol{b}} - i\boldsymbol{C}_2^{-1}\hat{\boldsymbol{f}}}{2\pi}\ln(z - z_0), \\ &z_0 = x_0 + iy_0, \qquad \hat{\boldsymbol{b}} = \big[b\ \ \Delta\phi\big]^T, \qquad \hat{\boldsymbol{f}} = \big[p\ \ -q\big]^T, \end{aligned}$$

which is time independent, is the complex potential for a piezoelectric screw dislocation in a homogeneous piezoelectric material [Pak 1990b].

Equation $(6)_2$ for the continuity condition of the electric potential across the interface can be equivalently expressed as

$$g_2^+(z,t) - \bar{g}_2^-\big(R^2/z,t\big) = h_2^-(z,t) - \bar{h}_2^+\big(R^2/z,t\big), \qquad |z| = R.$$

It follows from the above expression that

$$\begin{aligned} h_2(z,t) &= -\bar{g}_2\big(R^2/z,t\big) + h_{20}(z) + \bar{h}_{20}\big(R^2/z\big), \\ \bar{h}_2\big(R^2/z,t\big) &= -g_2(z,t) + h_{20}(z) + \bar{h}_{20}\big(R^2/z\big), \end{aligned} \qquad |z| = R. \tag{8}$$

In view of (7) and (8), the three analytic functions $h_1(z,t)$, $h_2(z,t)$, and $\bar{g}_2(R^2/z,t)$ defined in the matrix can be expressed in terms of a single analytic function $\bar{g}_1(R^2/z,t)$ defined in the matrix as

$$
\begin{aligned}
h_1(z,t) &= \frac{c_{44}^{(1)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(1)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\bar{g}_1\big(R^2/z,t\big) \\
&\qquad +h_{10}(z)-\bar{h}_{10}\big(R^2/z\big)+\frac{2\big(\epsilon_{11}^{(2)}e_{15}^{(1)}-\epsilon_{11}^{(1)}e_{15}^{(2)}\big)}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\bar{h}_{20}\big(R^2/z\big), \\
h_2(z,t) &= \frac{c_{44}^{(1)}e_{15}^{(2)}-c_{44}^{(2)}e_{15}^{(1)}}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\bar{g}_1\big(R^2/z,t\big) \\
&\qquad +h_{20}(z)+\frac{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}-\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}-e_{15}^{(2)}\big)}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\bar{h}_{20}\big(R^2/z\big), \\
\bar{g}_2\big(R^2/z,t\big) &= \frac{c_{44}^{(2)}e_{15}^{(1)}-c_{44}^{(1)}e_{15}^{(2)}}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\bar{g}_1\big(R^2/z,t\big) \\
&\qquad +\frac{2\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\bar{h}_{20}\big(R^2/z\big).
\end{aligned}
\tag{9}
$$

Similarly the three analytic functions $\bar{h}_1(R^2/z,t)$, $\bar{h}_2(R^2/z,t)$, and $g_2(z,t)$ defined in the inclusion can be expressed in terms of a single analytic function $g_1(z,t)$ defined in the inclusion as

$$
\begin{aligned}
\bar{h}_1\big(R^2/z,t\big) &= \frac{c_{44}^{(1)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(1)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}g_1(z,t) \\
&\qquad +\bar{h}_{10}\big(R^2/z\big)-h_{10}(z)+\frac{2\big(\epsilon_{11}^{(2)}e_{15}^{(1)}-\epsilon_{11}^{(1)}e_{15}^{(2)}\big)}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}h_{20}(z), \\
\bar{h}_2\big(R^2/z,t\big) &= \frac{c_{44}^{(1)}e_{15}^{(2)}-c_{44}^{(2)}e_{15}^{(1)}}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}g_1(z,t) \\
&\qquad +\bar{h}_{20}\big(R^2/z\big)+\frac{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}-\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}-e_{15}^{(2)}\big)}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}h_{20}(z), \\
g_2(z,t) &= \frac{c_{44}^{(2)}e_{15}^{(1)}-c_{44}^{(1)}e_{15}^{(2)}}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}g_1(z,t)+\frac{2\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}h_{20}(z).
\end{aligned}
\tag{10}
$$

Equation $(6)_3$ for the law of linear rheology can be equivalently expressed as

$$
\begin{aligned}
&c_{44}^{(1)}zg_1^{\prime +}(z,t)+e_{15}^{(1)}zg_2^{\prime +}(z,t)-c_{44}^{(1)}R^2z^{-1}\bar{g}_1^{\prime -}\big(R^2/z,t\big) \\
&\qquad -e_{15}^{(1)}R^2z^{-1}\bar{g}_2^{\prime -}\big(R^2/z,t\big)=R\eta\Big[\frac{\partial h_1^-(z,t)}{\partial t}-\frac{\partial\bar{h}_1^+\big(R^2/z,t\big)}{\partial t}-\frac{\partial g_1^+(z,t)}{\partial t}+\frac{\partial\bar{g}_1^-\big(R^2/z,t\big)}{\partial t}\Big],
\end{aligned}
\tag{11}
$$

where $|z|=R$, and where the superscript comma means the derivative with respect to the complex variable.

Inserting (9) and (10) into (11), and with $|z| = R$, we finally obtain

$$\begin{aligned}&\frac{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\epsilon_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}zg_1^{\prime +}(z,t)+R\eta\frac{\big(c_{44}^{(1)}+c_{44}^{(2)}\big)\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+\big(e_{15}^{(1)}+e_{15}^{(2)}\big)^2}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\frac{\partial g_1^{+}(z,t)}{\partial t}\\&\qquad+\frac{2\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}e_{15}^{(1)}}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}zh_{20}'(z)\\&=\frac{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\epsilon_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}R^2z^{-1}\bar{g}_1^{\prime -}\big(R^2/z,t\big)\\&\qquad+R\eta\frac{\big(c_{44}^{(1)}+c_{44}^{(2)}\big)\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+\big(e_{15}^{(1)}+e_{15}^{(2)}\big)^2}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\frac{\partial \bar{g}_1^{-}\big(R^2/z,t\big)}{\partial t}\\&\qquad\qquad+\frac{2\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}e_{15}^{(1)}}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}R^2z^{-1}\bar{h}_{20}'\big(R^2/z\big).\end{aligned}\tag{12}$$

Apparently the left hand side of Equation (12) is analytic within the circle $|z| = R$, while the right hand side of (12) is analytic outside the circle including the point at infinity. By employing Liouville's theorem, the left and right hand sides should be identically zero. Consequently we obtain the inhomogeneous first-order partial differential equation for $g_1(z, t)$

$$z\frac{\partial g_1(z,t)}{\partial z}+t_0\frac{\partial g_1(z,t)}{\partial t}=-\frac{\beta z}{z-z_0},\qquad |z|\le R,\tag{13}$$

where t_0 is the characteristic time and β a constant, defined, respectively, by

$$t_0=R\eta\frac{\big(c_{44}^{(1)}+c_{44}^{(2)}\big)\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+\big(e_{15}^{(1)}+e_{15}^{(2)}\big)^2}{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\epsilon_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}}>0,\qquad \beta=\frac{e_{15}^{(1)}\big[\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}\Delta\phi-i\big(e_{15}^{(2)}p+c_{44}^{(2)}q\big)\big]}{\pi\big(\tilde{c}_{44}^{(1)}c_{44}^{(2)}\epsilon_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}\big)}.$$

It is of interest to notice that the resulting first-order partial differential equation (13) for a circular viscous interface is different from that for a straight viscous interface [Wang et al. 2007; Wang and Pan 2008]. Once we introduce the transformation $\zeta = \ln z$, (13) changes into

$$\frac{\partial g_1(\zeta,t)}{\partial \zeta}+t_0\frac{\partial g_1(\zeta,t)}{\partial t}=-\frac{\beta e^{\zeta}}{e^{\zeta}-z_0},\tag{14}$$

whose structure is in a sense similar to that of the resulting differential equation for a straight viscous interface [Wang et al. 2007; Wang and Pan 2008].

Equation (14) demonstrates that it has a homogeneous solution in the form $g_1(\zeta - t/t_0)$. In view of the form of the homogeneous solution to (14), the solution to the original partial differential equation (13) can be conveniently given by

$$g_1(z,t)=\alpha\ln(z-e^{t/t_0}z_0)-\beta\ln(z-z_0),\qquad |z|\le R,\tag{15}$$

where α is an unknown constant to be determined, and the term $-\alpha t/t_0$ in $g_1(z, t)$ representing the rigid body displacement and equipotential has been ignored. It is mentioned that the first term in $g_1(z, t)$,

$$\alpha\ln(z-e^{t/t_0}z_0)-at/t_0=\alpha\ln(e^{-t/t_0}z-z_0),$$

is a homogeneous solution to (13), while the second term in $g_1(z, t)$, $-\beta \ln(z-z_0)$, is a particular solution to (13). At the initial moment $t = 0$, when the piezoelectric screw dislocation is just introduced into the matrix, the displacement across the interface has no time to experience a jump due to the dashpot [Fan and Wang 2003; Wang and Schiavone 2007]. Therefore at $t = 0$ the interface is a perfect one. As a result we have

$$g_1(z,0) = \frac{\boldsymbol{J}_1(\boldsymbol{C}_1+\boldsymbol{C}_2)^{-1}(\boldsymbol{C}_2\hat{\boldsymbol{b}} - i\hat{\boldsymbol{f}})}{\pi}\ln(z-z_0), \qquad \boldsymbol{J}_0 = \begin{bmatrix}1 & 0\end{bmatrix}, \quad \boldsymbol{J}_2 = \begin{bmatrix}0 & 1\end{bmatrix}. \tag{16}$$

In view of Equations (15) and (16), the constant α can be uniquely determined to be

$$\alpha = \beta + \frac{\boldsymbol{J}_1(\boldsymbol{C}_1+\boldsymbol{C}_2)^{-1}(\boldsymbol{C}_2\hat{\boldsymbol{b}} - i\hat{\boldsymbol{f}})}{\pi}. \tag{17}$$

Once $g_1(z, t)$ has been determined, after some tedious but straightforward mathematical operations we finally arrive at $g_2(z, t)$ defined in the inclusion and $h_1(z, t)$ and $h_2(z, t)$ defined in the matrix as

$$\begin{aligned}
g_2(z,t) &= \frac{\alpha(c_{44}^{(2)}e_{15}^{(1)} - c_{44}^{(1)}e_{15}^{(2)})}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\ln\big(z-e^{t/t_0}z_0\big) + \frac{\beta c_{44}^{(1)}}{e_{15}^{(1)}}\ln\big(z-z_0\big),\\
h_1(z,t) &= \frac{\bar{\alpha}\big[c_{44}^{(1)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(1)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)\big]}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\ln\Big(\frac{z-e^{-t/t_0}R^2/\bar{z}_0}{z}\Big)\\
&\quad -\Big[\frac{\tilde{c}_{44}^{(1)}\epsilon_{11}^{(1)}e_{15}^{(2)}\boldsymbol{J}_2}{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\epsilon_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}}+\frac{\boldsymbol{J}_1}{2}\Big]\frac{\hat{\boldsymbol{b}}+i\boldsymbol{C}_2^{-1}\hat{\boldsymbol{f}}}{\pi}\ln\Big(\frac{z-R^2/\bar{z}_0}{z}\Big)\\
&\qquad +\frac{\boldsymbol{J}_1\big(\hat{\boldsymbol{b}}-i\boldsymbol{C}_2^{-1}\hat{\boldsymbol{f}}\big)}{2\pi}\ln\big(z-z_0\big),\\
h_2(z,t) &= \frac{\bar{\alpha}\big(c_{44}^{(1)}e_{15}^{(2)}-c_{44}^{(2)}e_{15}^{(1)}\big)}{c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)}\ln\Big(\frac{z-e^{-t/t_0}R^2/\bar{z}_0}{z}\Big)\\
&\quad +\frac{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\epsilon_{11}^{(1)}-c_{44}^{(1)}\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}}{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\epsilon_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}}\frac{\boldsymbol{J}_2\big(\hat{\boldsymbol{b}}+i\boldsymbol{C}_2^{-1}\hat{\boldsymbol{f}}\big)}{2\pi}\ln\Big(\frac{z-R^2/\bar{z}_0}{z}\Big)\\
&\qquad +\frac{\boldsymbol{J}_2\big(\hat{\boldsymbol{b}}-i\boldsymbol{C}_2^{-1}\hat{\boldsymbol{f}}\big)}{2\pi}\ln\big(z-z_0\big).
\end{aligned} \tag{18}$$

Notice that the last term in the expressions of $h_1(z, t)$ and $h_2(z, t)$ is the singular part due to the screw dislocation. If both the inclusion and the matrix are purely elastic materials, that is, $e_{15}^{(1)} = e_{15}^{(2)} = 0$, then

$g_1(z,t)$, $g_2(z,t)$ defined in the inclusion and $h_1(z,t)$, $h_2(z,t)$ defined in the matrix are reduced to

$$g_1(z,t)=\frac{c_{44}^{(2)}b-ip}{\pi\big(c_{44}^{(1)}+c_{44}^{(2)}\big)}\ln\big(z-e^{t/t_0}z_0\big),\qquad g_2(z,t)=\frac{\epsilon_{11}^{(2)}\Delta\phi-iq}{\pi\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)}\ln\big(z-z_0\big),$$

$$h_1(z,t)=\frac{c_{44}^{(1)}\big(c_{44}^{(2)}b+ip\big)}{\pi c_{44}^{(2)}\big(c_{44}^{(1)}+c_{44}^{(2)}\big)}\ln\Big(\frac{z-e^{-t/t_0}R^2/\bar{z}_0}{z}\Big)-\frac{c_{44}^{(2)}b+ip}{2\pi c_{44}^{(2)}}\ln\Big(\frac{z-R^2/\bar{z}_0}{z}\Big)+\frac{c_{44}^{(2)}b-ip}{2\pi c_{44}^{(2)}}\ln\big(z-z_0\big),$$

$$h_2(z,t)=\frac{\big(\epsilon_{11}^{(1)}-\epsilon_{11}^{(2)}\big)\big(\epsilon_{11}^{(2)}\Delta\phi+iq\big)}{2\pi\epsilon_{11}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)}\ln\Big(\frac{z-R^2/\bar{z}_0}{z}\Big)+\frac{\epsilon_{11}^{(2)}\Delta\phi-iq}{2\pi\epsilon_{11}^{(2)}}\ln\big(z-z_0\big),$$

where

$$t_0=\frac{R\eta\big(c_{44}^{(1)}+c_{44}^{(2)}\big)}{c_{44}^{(1)}c_{44}^{(2)}}. \tag{19}$$

In this special case $g_2(z,t)$ and $h_2(z,t)$ are in fact independent of the time t due to the fact that there is no piezoelectric effect.

Substituting Equations (15) and (18) into (4) or (5), we can arrive at the explicit expressions of strains, stresses, electric fields, and electric displacements induced by the piezoelectric screw dislocation, which are listed in the online supplement. For example the strains, electric fields, stresses, and electric displacements within the piezoelectric circular inclusion are given by

$$\begin{aligned}
\gamma_{zy}^{(1)}+i\gamma_{zx}^{(1)}&=\frac{\alpha}{z-e^{t/t_0}z_0}-\frac{\beta}{z-z_0},\\
E_y^{(1)}+iE_x^{(1)}&=\frac{\alpha\big(c_{44}^{(1)}e_{15}^{(2)}-c_{44}^{(2)}e_{15}^{(1)}\big)}{\big[c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)\big]\big(z-e^{t/t_0}z_0\big)}-\frac{\beta c_{44}^{(1)}}{e_{15}^{(1)}\big(z-z_0\big)},\\
\sigma_{zy}^{(1)}+i\sigma_{zx}^{(1)}&=\frac{\alpha\big(\tilde{c}_{44}^{(1)}c_{44}^{(2)}\epsilon_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}\big)}{\big[c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)\big]\big(z-e^{t/t_0}z_0\big)},\\
D_y^{(1)}+iD_x^{(1)}&=\frac{\alpha\big(\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}e_{15}^{(1)}+\tilde{c}_{44}^{(1)}\epsilon_{11}^{(1)}e_{15}^{(2)}\big)}{\big[c_{44}^{(2)}\big(\epsilon_{11}^{(1)}+\epsilon_{11}^{(2)}\big)+e_{15}^{(2)}\big(e_{15}^{(1)}+e_{15}^{(2)}\big)\big]\big(z-e^{t/t_0}z_0\big)}-\frac{\beta\tilde{c}_{44}^{(1)}\epsilon_{11}^{(1)}}{e_{15}^{(1)}\big(z-z_0\big)}.
\end{aligned} \tag{20}$$

It is clearly observed from the above expression that in general the strains, electric fields, stresses, and electric displacements inside the piezoelectric circular inclusion are time dependent due to appearance of the term e^{t/t_0}. In addition it is found from $(20)_2$ that the electric fields within the piezoelectric circular inclusion will be time independent when the condition $c_{44}^{(1)}e_{15}^{(2)}=c_{44}^{(2)}e_{15}^{(1)}$ is satisfied. As time elapses, the strains, electric fields, stresses, and electric displacements within the piezoelectric circular inclusion will finally arrive at the steady state

$$\begin{aligned}
&\gamma_{zy}^{(1)}+i\gamma_{zx}^{(1)}=-\frac{\beta}{z-z_0}, && E_y^{(1)}+iE_x^{(1)}=-\frac{c_{44}^{(1)}}{e_{15}^{(1)}}\frac{\beta}{z-z_0},\\
&\sigma_{zy}^{(1)}+i\sigma_{zx}^{(1)}=0, && D_y^{(1)}+iD_x^{(1)}=-\frac{\beta\tilde{c}_{44}^{(1)}\epsilon_{11}^{(1)}}{e_{15}^{(1)}(z-z_0)}, \qquad |z|\le R,\ t\to\infty.
\end{aligned}$$

It is observed from the above expression that the internal stresses will eventually vanish due to the dashpot. The relationship $E_y^{(1)} + iE_x^{(1)} = (\gamma_{zy}^{(1)} + i\gamma_{zx}^{(1)})c_{44}^{(1)}/e_{15}^{(1)}$ observed in the above expression is just in agreement with the vanishing internal stress condition when $t \to \infty$.

4. Image force on the screw dislocation

Furthermore, by employing the Peach–Koehler formulation [Pak 1990b; Lee et al. 2000] and the previously derived field components in the piezoelectric matrix (see Equation A2 of the the online supplement), it is also convenient to arrive at the image force acting on the screw dislocation due to its interaction with the circular viscous interface. For example if we assume that the piezoelectric screw dislocation with $b \neq 0$, $p = q = \Delta\phi = 0$ lies on the positive real x axis (that is, $x_0 > R$, $y_0 = 0$), then a rather concise closed-form expression of the time dependent image force on the screw dislocation can be finally derived as

$$F_x(t) = \frac{b^2R^2}{2\pi x_0}\left[\frac{2\big(\tilde{c}_{44}^{(1)}c_{44}^{(2)}\epsilon_{11}^{(1)} + c_{44}^{(1)}\tilde{c}_{44}^{(2)}\epsilon_{11}^{(2)}\big)}{\big[\big(c_{44}^{(1)} + c_{44}^{(2)}\big)\big(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}\big) + \big(e_{15}^{(1)} + e_{15}^{(2)}\big)^2\big]\big(e^{t/t_0}x_0^2 - R^2\big)} - \frac{c_{44}^{(2)}}{x_0^2 - R^2}\right], \tag{21}$$

where F_x is the x component of the image force (the y component of the image force is zero).

It is of interest to discuss several special cases to verify and to illustrate the obtained solution.

Case 1. If the inclusion and the matrix have the same material properties and poling direction, meaning $c_{44}^{(1)} = c_{44}^{(2)} = c_{44}$, $e_{15}^{(1)} = e_{15}^{(2)} = e_{15}$, and $\epsilon_{11}^{(1)} = \epsilon_{11}^{(2)} = \epsilon_{11}$, then it follows from Equation (21) that the image force on the screw dislocation is

$$F_x(t) = \frac{c_{44}b^2R^2x_0\big(1 - e^{t/t_0}\big)}{2\pi\big(x_0^2 - R^2\big)\big(e^{t/t_0}x_0^2 - R^2\big)} \leq 0,$$

with $t_0 = 2R\eta/c_{44}$.

The above expression indicates that: there is no image force on the screw dislocation at the initial moment $t = 0$; the screw dislocation will always be attracted to the piezoelectric inclusion when $t > 0$; and the image force is independent of the piezoelectric and dielectric properties e_{15} and ϵ_{11}.

Case 2. If the inclusion and matrix have the same material property but are poled in opposite directions, implying $c_{44}^{(1)} = c_{44}^{(2)} = c_{44}$, $e_{15}^{(1)} = -e_{15}^{(2)} = e_{15}$, and $\epsilon_{11}^{(1)} = \epsilon_{11}^{(2)} = \epsilon_{11}$, then it follows from (21) that the image force on the screw dislocation is

$$F_x(t) = \frac{b^2R^2}{2\pi x_0}\left[\frac{\tilde{c}_{44}}{e^{t/t_0}x_0^2 - R^2} - \frac{c_{44}}{x_0^2 - R^2}\right],$$

with $t_0 = 2R\eta/\tilde{c}_{44}$.

It is observed from the above expression that when

$$0 \leq t < t_1, \qquad t_1 = t_0 \ln\left(\frac{\tilde{c}_{44}x_0^2 - \big(\tilde{c}_{44} - c_{44}\big)R^2}{c_{44}x_0^2}\right),$$

the screw dislocation will be repelled from the inclusion, meaning $F_x(t) > 0$, $(0 \leq t < t_1)$. At the moment $t = t_1$, there is no image force on the dislocation, that is, $F_x(t_1) = 0$. When $t > t_1$, the screw dislocation will always be attracted to the inclusion, meaning $F_x(t) < 0$, $(t > t_1)$.

Case 3. If both the inclusion and matrix are purely elastic, that is, $e_{15}^{(1)} = e_{15}^{(2)} = 0$, then it follows from (21) that the image force on the screw dislocation is

$$F_x(t) = \frac{c_{44}^{(2)} b^2 R^2}{2\pi x_0} \Big(\frac{2c_{44}^{(1)}}{c_{44}^{(1)} + c_{44}^{(2)}} \frac{1}{e^{t/t_0} x_0^2 - R^2} - \frac{1}{x_0^2 - R^2} \Big), \tag{22}$$

with t_0 given by Equation (19). We have carefully checked that our closed-form expression of the image force, (22), is consistent with the numerical results from Fan and Wang [2003, Figure 7, Equation (4.12)].

At the initial time $t = 0$, (22) for the image force is

$$F_x(0) = \frac{c_{44}^{(2)} b^2}{2\pi} \frac{c_{44}^{(1)} - c_{44}^{(2)}}{c_{44}^{(1)} + c_{44}^{(2)}} \frac{R^2}{x_0 \big(x_0^2 - R^2\big)}, \tag{23}$$

which is just the result for a screw dislocation interacting with a circular inclusion with a perfect interface [Dundurs 1967; Fan and Wang 2003]. At the time $t = \infty$, (22) for the image force is

$$F_x(\infty) = -\frac{c_{44}^{(2)} R^2 b^2}{2\pi x_0 \big(x_0^2 - R^2\big)} < 0, \tag{24}$$

which is the result for a dislocation interacting with a traction free circular hole. It is observed from (22)–(24) that if the inclusion is stiffer than the matrix (that is, $c_{44}^{(1)} > c_{44}^{(2)}$), there always exists a time $t = t_2$ ($t_2 > 0$), at which $F_x(t_2) = 0$ due to the fact that $F_x(0) > 0$ and $F_x(\infty) < 0$. In addition t_2 can be determined from (22) as

$$t_2 = t_0 \ln\Big(\frac{2c_{44}^{(1)} x_0^2 - R^2 \big(c_{44}^{(1)} - c_{44}^{(2)}\big)}{x_0^2 \big(c_{44}^{(1)} + c_{44}^{(2)}\big)} \Big), \qquad c_{44}^{(1)} > c_{44}^{(2)}.$$

Furthermore, when $0 \le t < t_2$ the screw dislocation will be repelled from the inclusion, while when $t > t_2$ the screw dislocation will be attracted to the inclusion.

Case 4. Lastly we consider a straight interface. The straight interface can be considered as a limit of the circular interface if we let $\delta = x_0 - R$ and assume that $R \to \infty$. Under this condition and after some derivations, (21) is finally reduced to

$$F_x(t) = \frac{b^2}{4\pi\delta} \Big[\frac{2\big(\tilde{c}_{44}^{(1)} c_{44}^{(2)} \epsilon_{11}^{(1)} + c_{44}^{(1)} \tilde{c}_{44}^{(2)} \epsilon_{11}^{(2)}\big)}{\big(c_{44}^{(1)} + c_{44}^{(2)}\big)\big(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}\big) + \big(e_{15}^{(1)} + e_{15}^{(2)}\big)^2} \Big(1 + \frac{t}{2\tilde{t}_0}\Big)^{-1} - c_{44}^{(2)} \Big], \tag{25}$$

where

$$\tilde{t}_0 = \delta\eta \frac{\big(c_{44}^{(1)} + c_{44}^{(2)}\big)\big(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}\big) + \big(e_{15}^{(1)} + e_{15}^{(2)}\big)^2}{\tilde{c}_{44}^{(1)} c_{44}^{(2)} \epsilon_{11}^{(1)} + c_{44}^{(1)} \tilde{c}_{44}^{(2)} \epsilon_{11}^{(2)}}.$$

(25) is just the result derived in [Wang and Pan 2008, Equation (40)] for a straight interface.

Finally we consider a piezoelectric composite with the piezoelectric $BaTiO_3$ being the fiber and the piezoelectric PZT-5 being the matrix. The material properties of $BaTiO_3$ and PZT-5 are listed in Table 1.

Figure 2 illustrates the normalized image force $\tilde{F} = (RF_x)/(b^2 c_{44}^{(2)})$ on the screw dislocation at the four different times $t/t_0 = 0, 0.05, 0.5$, and ∞. It is observed that at the initial time $t = 0$ the screw dislocation is always repelled from the inclusion ($F_x > 0$), while at the times $t = 0.5t_0$ and $t = \infty$ the

Compound	c_{44} (10^{10} N/m^2)	e_{15} (C/m^2)	ϵ_{11} (10^{-9} F/m)	$\tilde{c}_{44}$ (10^{10} N/m^2)
PZT-5	2.11	12.3	8.1103	3.9754
$BaTiO_3$	4.4	11.4	9.8722	5.7164

Table 1. The material properties of PZT-5 and $BaTiO_3$ [Wang et al. 2003].

screw dislocation is always attracted to the inclusion ($F_x < 0$). At the time $t = 0.05t_0$ we observe that there exists a transient equilibrium position ($F_x = 0$) at the point $x_0 = 1.066R$ very close to the circular interface. In addition, the equilibrium position is unstable due to the fact that $F_x < 0$ for $x_0 < 1.066R$ and $F_x > 0$ for $x_0 > 1.066R$. In fact, the relationship between time and the transient unstable equilibrium position ($F_x = 0$) can be easily determined from Equation (21) as

$$\frac{x_0}{R} = \sqrt{\frac{c-1}{c-e^{t/t_0}}}, \qquad c > 1,\, 0^+ \le t \le t_0 \ln(c),$$

where

$$c = \frac{2\big(\tilde{c}_{44}^{(1)} c_{44}^{(2)} \epsilon_{11}^{(1)} + c_{44}^{(1)} \tilde{c}_{44}^{(2)} \epsilon_{11}^{(2)}\big)}{c_{44}^{(2)}\big[\big(c_{44}^{(1)} + c_{44}^{(2)}\big)\big(\epsilon_{11}^{(1)} + \epsilon_{11}^{(2)}\big) + \big(e_{15}^{(1)} + e_{15}^{(2)}\big)^2\big]}.$$

In this example $c = 1.4277$. Then it is found that when $0^+ \le t \le 0.3561t_0$ there always exists a transient unstable equilibrium position for the screw dislocation. We demonstrate in Figure 3 the transient unstable equilibrium position as a function of time. One can observe from Figure 3 that as the time evolves from $t = 0^+$ to $t = 0.3561t_0$ the transient unstable equilibrium position moves along the positive x direction from $x_0 = R$ to infinity.

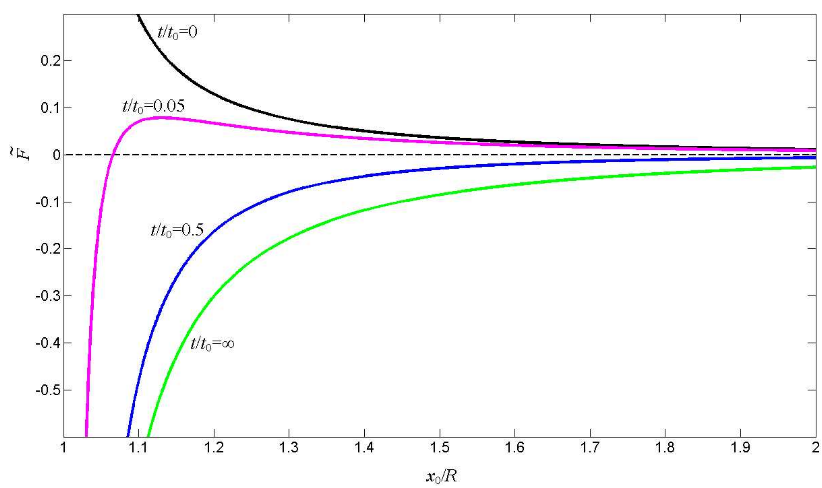

Figure 2. The normalized image force $\tilde{F} = (RF_x)/(b^2 c_{44}^{(2)})$ on the screw dislocation at the four times $t/t_0 = 0, 0.05, 0.5$, and ∞. The piezoelectric composite is composed of the piezoelectric $BaTiO_3$ fiber and the piezoelectric PZT-5 matrix.

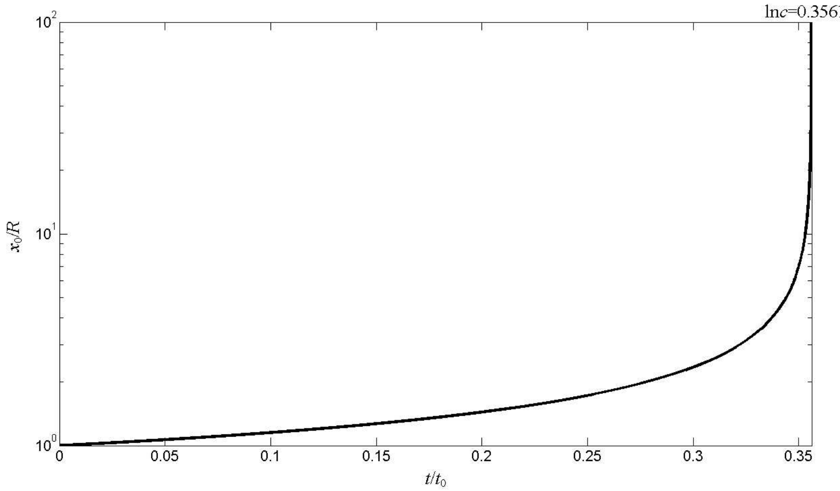

Figure 3. The transient unstable equilibrium position for the screw dislocation as a function of time. The piezoelectric composite is composed of the piezoelectric $BaTiO_3$ fiber and the piezoelectric PZT-5 matrix.

5. Conclusions

A theoretical analysis was performed for a screw dislocation with a line force and a line charge interacting with a piezoelectric circular inclusion with a viscous interface described by a linear dashpot. The exact closed-form solutions were obtained by the complex variable and analytical continuation technique. In this investigation the screw dislocation was assumed to be within the matrix, whilst the solution to the situation in which the dislocation is located within the circular inclusion can be identically derived. Starting from the derived closed-form solution, we can further investigate the interaction of a matrix crack with the circular viscous interface. Finally we mention that if the viscoelastic effect modeled by both the linear spring and dashpot is introduced into the circular interface, a closed-form solution cannot be obtained for the interaction problem due to the additional introduction of the linear spring. In this case, however, infinite series form solutions to the interaction problem can be derived (see solutions in [Fan and Wang 2003] for the Kelvin and Maxwell type viscoelastic interfaces). In fact, it is in principle impossible to obtain closed-form solutions for the dislocation/inclusion interaction problem even when only the linear spring model is introduced into the circular interface [Ru and Schiavone 1997].

Acknowledgements

The authors are greatly indebted to the reviewers for their very helpful comments and suggestions.

References

[Chen et al. 2002] B. J. Chen, Z. M. Xiao, and K. M. Liew, "A screw dislocation in a piezoelectric bi-material wedge", *Int. J. Eng. Sci.* **40**:15 (2002), 1665–1685.

[Dundurs 1967] J. Dundurs, "On the interaction of a screw dislocation with inhomogeneities", *Recent Adv. Eng. Sci.* **2** (1967), 223–233.

[Fan and Wang 2003] H. Fan and G. F. Wang, "Interaction between a screw dislocation and viscoelastic interfaces", *Int. J. Solids Struct.* **40**:4 (2003), 763–776.

[He and Lim 2003] L. H. He and C. W. Lim, "Electromechanical responses of piezoelectric fiber composites with sliding interface under anti-plane deformations", *Compos. Part B Eng.* **34**:4 (2003), 373–381.

[Lee et al. 2000] K. Y. Lee, W. G. Lee, and Y. E. Pak, "Interaction between a semi-infinite crack and a screw dislocation in a piezoelectric material", *J. Appl. Mech.* (*Trans. ASME*) **67**:1 (2000), 165–170.

[Liu et al. 1999] J. Liu, S. Du, and B. Wang, "A screw dislocation interacting with a piezoelectric bimaterial interface", *Mech. Res. Commun.* **26**:4 (1999), 415–420.

[Meguid and Deng 1998] S. A. Meguid and W. Deng, "Electro-elastic interaction between a screw dislocation and an elliptical inhomogeneity in piezoelectric materials", *Int. J. Solids Struct.* **35**:13 (1998), 1467–1482.

[Pak 1990a] Y. E. Pak, "Crack extension force in a piezoelectric material", *J. Appl. Mech.* (*Trans. ASME*) **57** (1990), 647–653.

[Pak 1990b] Y. E. Pak, "Force on a piezoelectric screw dislocation", *J. Appl. Mech.* (*Trans. ASME*) **57** (1990), 863–869.

[Pak 1992] Y. E. Pak, "Circular inclusion problem in antiplane piezoelectricity", *Int. J. Solids Struct.* **29**:19 (1992), 2403–2419.

[Ray and Ashby 1971] R. Ray and M. F. Ashby, "On grain boundary sliding and diffusional creep", *Metall. Mater. Trans. B* **2**:4 (1971), 1113–1127.

[Ru and Mao 1999] C. Q. Ru and X. Mao, "Conducting cracks in a piezoelectric ceramic of limited electrical polarization", *J. Mech. Phys. Solids* **47**:10 (1999), 2125–2146.

[Ru and Schiavone 1997] C. Q. Ru and P. Schiavone, "A circular inclusion with circumferentially inhomogeneous interface in antiplane shear", *P. Roy. Soc. Lond. A Mat.* **453**:1967 (1997), 2551–2572.

[Suo 1997] Z. Suo, "Motion of microscopic surface in materials", *Adv. Appl. Mech.* **33** (1997), 193–294.

[Wang and Pan 2007] X. Wang and E. Pan, "A moving screw dislocation interacting with an imperfect piezoelectric bimaterial interface", *Phys. Status Solidi B* **244**:6 (2007), 1940–1956.

[Wang and Pan 2008] X. Wang and E. Pan, "Interaction between a screw dislocation and a viscoelastic piezoelectric bimaterial interface", *Int. J. Solids Struct.* **45**:1 (2008), 245–257.

[Wang and Schiavone 2007] G. F. Wang and P. Schiavone, "Influence of thin film on a screw dislocation in a substrate with viscoelastic interface", *Math. Mech. Solids* **12**:1 (2007), 119–128.

[Wang et al. 2003] X. Wang, Z. Zhong, and F. L. Wu, "A moving conducting crack at the interface of two dissimilar piezoelectric materials", *Int. J. Solids Struct.* **40**:10 (2003), 2381–2399.

[Wang et al. 2007] X. Wang, E. Pan, and W. J. Feng, "Time-dependent Green's functions for an anisotropic bimaterial with viscous interface", *Eur. J. Mech. A Solids* **26**:5 (2007), 901–908.

[Xiao et al. 2007] Z. M. Xiao, H. X. Zhang, and B. J. Chen, "A piezoelectric screw dislocation interacts with interfacial collinear rigid lines in piezoelectric bimaterials", *Int. J. Solids Struct.* **44**:1 (2007), 255–271.

Received 27 Sep 2007. Accepted 3 Feb 2008.

XU WANG: xuwang@uakron.edu
Deparment of Civil Engineering University of Akron, Akron, OH 44325-3905, United States
and
Department of Applied Mathematics, University of Akron, Akron, OH 44325-3905, United States

ERNIAN PAN: pan2@uakron.edu
Department of Civil Engineering University of Akron, Akron, OH 44325-3905, United States
and
Department of Applied Mathematics, University of Akron, Akron, OH 44325-3905, United States

A. K ROY: Ajit.roy@wpafb.af.mil
Air Force Research Laboratory, AFRL/MLBCM, Bldg 654, 2941 Hobson Way, Wright-Patterson AFB, OH 45433-7750, United States