SUPPLEMENT TO

INTERACTION BETWEEN A SCREW DISLOCATION AND A PIEZOELECTRIC CIRCULAR INCLUSION WITH VISCOUS INTERFACE

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The time-dependent electroelastic fields inside and outside the piezoelectric inclusion:

The time-dependent strains, electric fields, stresses and electric displacements inside the piezoelectric circular inclusion are given below

$$
\begin{aligned}
&\gamma_{zy}^{(1)}+\mathrm{i}\gamma_{zx}^{(1)}=\frac{\alpha}{z-\mathrm{e}^{t/t_0}z_0}-\frac{\beta}{z-z_0},\\
&E_y^{(1)}+\mathrm{i}E_x^{(1)}=\frac{\alpha(c_{44}^{(1)}e_{15}^{(2)}-c_{44}^{(2)}e_{15}^{(1)})}{\left[c_{44}^{(2)}(\in_{11}^{(1)}+\in_{11}^{(2)})+e_{15}^{(2)}(e_{15}^{(1)}+e_{15}^{(2)})\right](z-\mathrm{e}^{t/t_0}z_0)}-\frac{\beta c_{44}^{(1)}}{e_{15}^{(1)}(z-z_0)},\\
&\sigma_{zy}^{(1)}+\mathrm{i}\sigma_{zx}^{(1)}=\frac{\alpha(\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)})}{\left[c_{44}^{(2)}(\in_{11}^{(1)}+\in_{11}^{(2)})+e_{15}^{(2)}(e_{15}^{(1)}+e_{15}^{(2)})\right](z-\mathrm{e}^{t/t_0}z_0)},\\
&D_y^{(1)}+\mathrm{i}D_x^{(1)}=\frac{\alpha(\tilde{c}_{44}^{(2)}\in_{11}^{(2)}e_{15}^{(1)}+\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)})}{\left[c_{44}^{(2)}(\in_{11}^{(1)}+\in_{11}^{(2)})+e_{15}^{(2)}(e_{15}^{(1)}+e_{15}^{(2)})\right](z-\mathrm{e}^{t/t_0}z_0)}-\frac{\beta\tilde{c}_{44}^{(1)}\in_{11}^{(1)}}{e_{15}^{(1)}(z-z_0)}.
\end{aligned}
\tag{A1}
$$

The time-dependent strains, electric fields, stresses and electric displacements in the piezoelectric matrix are given below

$$\gamma_{zy}^{(2)}+\mathrm{i}\gamma_{zx}^{(2)}=\frac{\bar{\alpha}R^2\left[c_{44}^{(1)}(\in_{11}^{(1)}+\in_{11}^{(2)})+e_{15}^{(1)}(e_{15}^{(1)}+e_{15}^{(2)})\right]}{\left[c_{44}^{(2)}(\in_{11}^{(1)}+\in_{11}^{(2)})+e_{15}^{(2)}(e_{15}^{(1)}+e_{15}^{(2)})\right]z(\mathrm{e}^{t/t_0}\bar{z}_0z-R^2)}$$

$$-\frac{R^2\left[\dfrac{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)}\mathbf{J}_2}{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)}}+\dfrac{\mathbf{J}_1}{2}\right](\hat{\mathbf{b}}+\mathrm{i}\mathbf{C}_2^{-1}\hat{\mathbf{f}})}{\pi z(\bar{z}_0z-R^2)}+\frac{\mathbf{J}_1(\hat{\mathbf{b}}-\mathrm{i}\mathbf{C}_2^{-1}\hat{\mathbf{f}})}{2\pi(z-z_0)},$$

$$E_y^{(2)}+\mathrm{i}E_x^{(2)}=\frac{\bar{\alpha}R^2(c_{44}^{(2)}e_{15}^{(1)}-c_{44}^{(1)}e_{15}^{(2)})}{\left[c_{44}^{(2)}(\in_{11}^{(1)}+\in_{11}^{(2)})+e_{15}^{(2)}(e_{15}^{(1)}+e_{15}^{(2)})\right]z(\mathrm{e}^{t/t_0}\bar{z}_0z-R^2)}$$

$$+\frac{R^2(c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)}-\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)})\mathbf{J}_2(\hat{\mathbf{b}}+\mathrm{i}\mathbf{C}_2^{-1}\hat{\mathbf{f}})}{2\pi(\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)})z(\bar{z}_0z-R^2)}-\frac{\mathbf{J}_2(\hat{\mathbf{b}}-\mathrm{i}\mathbf{C}_2^{-1}\hat{\mathbf{f}})}{2\pi(z-z_0)},$$

$$\sigma_{zy}^{(2)}+\mathrm{i}\sigma_{zx}^{(2)}=\frac{\bar{\alpha}R^2(\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)})}{\left[c_{44}^{(2)}(\in_{11}^{(1)}+\in_{11}^{(2)})+e_{15}^{(2)}(e_{15}^{(1)}+e_{15}^{(2)})\right]z(\mathrm{e}^{t/t_0}\bar{z}_0z-R^2)}-\frac{R^2\mathbf{J}_1(\mathbf{C}_2\hat{\mathbf{b}}+\mathrm{i}\hat{\mathbf{f}})}{2\pi z(\bar{z}_0z-R^2)}+\frac{\mathbf{J}_1(\mathbf{C}_2\hat{\mathbf{b}}-\mathrm{i}\hat{\mathbf{f}})}{2\pi(z-z_0)},$$

$$D_y^{(2)}+\mathrm{i}D_x^{(2)}=\frac{\bar{\alpha}R^2(\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)}+\tilde{c}_{44}^{(2)}\in_{11}^{(2)}e_{15}^{(1)})}{\left[c_{44}^{(2)}(\in_{11}^{(1)}+\in_{11}^{(2)})+e_{15}^{(2)}(e_{15}^{(1)}+e_{15}^{(2)})\right]z(\mathrm{e}^{t/t_0}\bar{z}_0z-R^2)}$$

$$-\frac{R^2\left[\dfrac{\tilde{c}_{44}^{(1)}\in_{11}^{(1)}e_{15}^{(2)2}-c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)2}+\tilde{c}_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(1)}\in_{11}^{(2)}}{\tilde{c}_{44}^{(1)}c_{44}^{(2)}\in_{11}^{(1)}+c_{44}^{(1)}\tilde{c}_{44}^{(2)}\in_{11}^{(2)}}\mathbf{J}_2+e_{15}^{(2)}\mathbf{J}_1\right](\hat{\mathbf{b}}+\mathrm{i}\mathbf{C}_2^{-1}\hat{\mathbf{f}})}{2\pi z(\bar{z}_0z-R^2)}+\frac{\mathbf{J}_2(\mathbf{C}_2\hat{\mathbf{b}}-\mathrm{i}\hat{\mathbf{f}})}{2\pi(z-z_0)}.$$

(A2)

Particularly when the inclusion and the matrix have the same material property and same poling direction, i.e., $c_{44}^{(1)}=c_{44}^{(2)}=c_{44}$, $e_{15}^{(1)}=e_{15}^{(2)}=e_{15}$, $\in_{11}^{(1)}=\in_{11}^{(2)}=\in_{11}$, Eqs. (A1) and (A2) will reduce to the following more explicit expressions

$$\begin{aligned}
\gamma_{zy}^{(1)}+\mathrm{i}\gamma_{zx}^{(1)}&=\frac{(c_{44}b+e_{15}\Delta\phi-\mathrm{i}p)(\mathrm{e}^{t/t_0}-1)z_0}{2\pi c_{44}(z-z_0)(z-\mathrm{e}^{t/t_0}z_0)}+\frac{\tilde{c}_{44}\in_{11}b-\mathrm{i}(\in_{11}p-e_{15}q)}{2\pi\tilde{c}_{44}\in_{11}(z-z_0)},\\
E_y^{(1)}+\mathrm{i}E_x^{(1)}&=\frac{-\tilde{c}_{44}\in_{11}\Delta\phi+\mathrm{i}(e_{15}p+c_{44}q)}{2\pi\tilde{c}_{44}\in_{11}(z-z_0)},\\
\sigma_{zy}^{(1)}+\mathrm{i}\sigma_{zx}^{(1)}&=\frac{c_{44}b+e_{15}\Delta\phi-\mathrm{i}p}{2\pi(z-\mathrm{e}^{t/t_0}z_0)},\\
D_y^{(1)}+\mathrm{i}D_x^{(1)}&=\frac{e_{15}(c_{44}b+e_{15}\Delta\phi-\mathrm{i}p)(\mathrm{e}^{t/t_0}-1)z_0}{2\pi c_{44}(z-z_0)(z-\mathrm{e}^{t/t_0}z_0)}+\frac{e_{15}b-\in_{11}\Delta\phi+\mathrm{i}q}{2\pi(z-z_0)},
\end{aligned}\tag{A3}$$

within the piezoelectric circular inclusion, and

$$\gamma_{zy}^{(2)}+\mathrm{i}\gamma_{zx}^{(2)}=\frac{R^2(c_{44}b+e_{15}\Delta\phi+\mathrm{i}p)(1-\mathrm{e}^{t/t_0})\bar{z}_0}{2\pi c_{44}(\bar{z}_0 z-R^2)(\mathrm{e}^{t/t_0}\bar{z}_0 z-R^2)}+\frac{\tilde{c}_{44}\in_{11}b-\mathrm{i}(\in_{11}p-e_{15}q)}{2\pi\tilde{c}_{44}\in_{11}(z-z_0)},$$

$$E_y^{(2)}+\mathrm{i}E_x^{(2)}=\frac{-\tilde{c}_{44}\in_{11}\Delta\phi+\mathrm{i}(e_{15}p+c_{44}q)}{2\pi\tilde{c}_{44}\in_{11}(z-z_0)},\tag{A4}$$

$$\sigma_{zy}^{(2)}+\mathrm{i}\sigma_{zx}^{(2)}=\frac{R^2(c_{44}b+e_{15}\Delta\phi+\mathrm{i}p)(1-\mathrm{e}^{t/t_0})\bar{z}_0}{2\pi(\bar{z}_0 z-R^2)(\mathrm{e}^{t/t_0}\bar{z}_0 z-R^2)}+\frac{c_{44}b+e_{15}\Delta\phi-\mathrm{i}p}{2\pi(z-z_0)},$$

$$D_y^{(2)}+\mathrm{i}D_x^{(2)}=\frac{R^2e_{15}(c_{44}b+e_{15}\Delta\phi+\mathrm{i}p)(1-\mathrm{e}^{t/t_0})\bar{z}_0}{2\pi c_{44}(\bar{z}_0 z-R^2)(\mathrm{e}^{t/t_0}\bar{z}_0 z-R^2)}+\frac{e_{15}b-\in_{11}\Delta\phi+\mathrm{i}q}{2\pi(z-z_0)},$$

within the piezoelectric matrix. In Eqs. (A3) and (A4) $t_0=2R\eta/c_{44}$.

It is observed from Eqs. (A3) and (A4) that when the inclusion and the matrix have the same material property and same poling direction, 1) the electric fields inside and outside the inclusion are in fact time-independent; 2) at time t=0, the expressions of strains, electric fields, stresses and electric displacements are just the result in Pak [1990] for a piezoelectric screw dislocation in a homogeneous piezoelectric material.