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PREFRACTURE ZONE MODELING FOR AN ELECTRICALLY IMPERMEABLE INTERFACE CRACK IN A PIEZOELECTRIC BIMATERIAL COMPOUND

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This work is concerned with the analytical characterization of the electromechanical nonlinear effects in the fields surrounding the tip of an interface crack located between two piezoelectric materials. It is assumed that a prefracture zone arises along a line segment in front of the crack tip. The prefracture zone is modeled by electrical displacement reaching a saturation limit and constant stress distribution. This reduces the problem to a linear interface crack analysis leading to a Hilbert problem, which is solved exactly. The prefracture zone length and the stress magnitude in this zone are found from algebraic and transcendental equations. The latter are derived from the requirement of stresses and electrical displacement to be finite at the end of prefracture zone towards the undamaged ligament. Numerical results for certain material combinations and remote loadings are presented and analyzed. In addition, energy release rate and crack opening displacements are introduced, which offers the possibility of formulating a fracture criterion based on the crack opening displacements.

1. Introduction

Piezoelectric materials have found wide technological applications as transducers, sensors and actuators due to their inherent electromechanically coupled behavior. However, piezoelectric materials are brittle and susceptible to fracture. Various defects, such as grain boundaries, flaws and pores, impurities and inclusions, etc, exist in piezoelectric materials. The defects cause geometric, electric and mechanical discontinuities and thus induce strong stress and electric field concentrations, which may induce crack initiation and crack growth, eventually causing fracture and failure. Structural reliability concerns of electromechanical devices call for a better understanding of the mechanisms of piezoelectric fracture.

Important results about fracture in piezoelectric solids based on linear electroelasticity have been derived by Parton [1976], Pak [1992], Sosa [1992], Suo et al. [1992], Dunn [1994], and many others. However, analysis based on linear electroelasticity cannot explain some discrepancies between theory and experiment [Park and Sun 1995]. Hence, various nonlinear models have been suggested. Narita and Shindo [2001] considered a mechanical yield strip model for a piezoelectric crack under a low stress level. In order to derive a fracture criterion suitable for piezoelectrics, Gao et al. [1997] generalized the essential ideas of Dugdale [1960] and proposed a strip saturation model of electrical yielding by assuming that the electrical polarization is saturated in a line segment in front of the crack tip. Based on general linear constitutive equations, the analysis of the strip saturation model was conducted and extended by Ru [1999], Wang [2000] and Li [2003]. McMeeking [2001] gave comprehensive and suggestive comments on the strip saturation model. Beom and Atluri [2003] proposed a nonlinear domain switching model for a ferroelectric material which has a circular zone of perfect saturation near the crack tip. A strip

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dielectric breakdown model was introduced by Zhang and Gao [2004] for an electrically impermeable crack of semi-infinite length. Zhang [2004] further discussed this model for a crack of finite length.

All of the works mentioned deal with nonlinear crack models for cracks in homogeneous piezoelectric materials. For interface cracks between piezoelectric bimaterials, the situation is more complicated since the field equations are complex and fracture behavior of piezoelectric compounds is far from obvious. Most theoretical studies regarding interface cracks in piezoelectric compounds were performed within the framework of the classical interface crack model (Williams, 1959). This model usually leads to an oscillating singularity at the crack tip and to physically unrealistic overlapping of the crack faces. To eliminate this phenomenon, a contact zone model for a crack between two isotropic materials was suggested by Comninou [1977]. It was developed further for interface cracks in piezoelectric bimaterials by Qin and Mai [1999], Herrmann and Loboda [2000] and Govorukha et al. [2006]. Another way of removing crack tip singularities and modeling fracture processes for interface cracks was introduced by Needleman [1990], Tvergaard and Hutchinson [1996] and Tvergaard [2001] by means of a cohesive zone model. An analysis of the plastic zone for an interface crack was performed by Huang [1992], Wang and Shen [1993] and Pickthall et al. [2002]. Plastic strips or prefracture zones in front of interface crack tips were analyzed by Kaminsky et al. [1999] and Bakirov and Gol'dshtein [2004] for the case of isotropic bimaterials, while Loboda et al. [2007] studied a permeable interface crack between two piezoelectric materials.

In this paper,we want to model the situation where two piezoelectric materials are bonded by a thin ductile interlayer. Since neither infinite strains nor infinite potential gradients can be sustained at the atomic level, both mechanical and electrical nonlinearity of the interlayer are taken into account. Shen et al. [2000] considered simultaneous mechanical and electrical yielding for a mode III interface crack. However, to the authors knowledge no modeling of electrical and mechanical yielding at the same time for an in-plane interface crack in a piezoelectric bimaterial has been done until now. In this paper, such modeling is proposed and an interface crack with mechanical and electrical yield zones is examined.

2. General solution of the basic equations

The constitutive and equilibrium equations for a linear piezoelectric material in the absence of body forces and free charges can be represented in the form [Pak 1992]

$$\Pi_{iJ} = E_{iJKl} V_{K,l}, \qquad \Pi_{iJ,i} = 0, \tag{1}$$

where

$$V_K = \begin{cases} u_k, & K = 1, 2, 3, \\ \varphi, & K = 4, \end{cases} \qquad \Pi_{iJ} = \begin{cases} \sigma_{ij}, & i, J = 1, 2, 3, \\ D_i, & i = 1, 2, 3, \ J = 4, \end{cases} \qquad E_{iJKl} = \begin{cases} c_{ijkl}, & J, K = 1, 2, 3, \\ e_{lij}, & J = 1, 2, 3, \ K = 4, \\ e_{ikl}, & K = 1, 2, 3, \ J = 4, \\ -\varepsilon_{il}, & J = K = 4. \end{cases} \tag{2}$$

Here, u_k, φ, σ_{ij} and D_i are the elastic displacements, electric potential, stresses and electric displacements, respectively, while c_{ijkl}, e_{ijk} and ε_{ij} are the elastic, piezoelectric and dielectric constants. Lowercase subscripts in (1)–(2) and afterwards range from 1 to 3, capital subscripts range from 1 to 4 and Einstein's summation convention is used in (1).

For two-dimensional deformations, where the vector $\boldsymbol{V} = [u_1, u_2, u_3, \varphi]^T$ depends on x_1 and x_3 only (the superscript T denoting the transpose), a general solution of (1) can be obtained, following [Pak 1992], by writing

$$\boldsymbol{V} = \boldsymbol{a} f(z), \tag{3}$$

where f is an arbitrary function of $z = x_1 + px_3$, and $\boldsymbol{a} = [a_1, a_2, a_3, a_4]^T$ is an eigenvector and p an eigenvalue. They can be determined by inserting (3) into $(1)_1$, and then into $(1)_2$, which yields

$$\big[\boldsymbol{Q} + p(\boldsymbol{R} + \boldsymbol{R}^T) + p^2\boldsymbol{T}\big]\boldsymbol{a} = 0, \tag{4}$$

where $\boldsymbol{Q}$, $\boldsymbol{R}$ and $\boldsymbol{T}$ are 4×4 real matrices whose components are defined by $Q_{JK} = E_{1JK1}$, $R_{JK} = E_{1JK3}$, $T_{JK} = E_{3JK3}$.

Since Equation (4) has no real eigenvalues [Suo et al. 1992], we write an eigenvalue of (4) with positive imaginary parts as p_α and the associated eigenvectors of (4) as $\boldsymbol{a}_\alpha$, the subscript α here and afterwards ranging from 1 to 4. The general solution of (1) can then be represented as

$$\boldsymbol{V} = \boldsymbol{A}\boldsymbol{f}(z) + \bar{\boldsymbol{A}}\bar{\boldsymbol{f}}(\bar{z}), \tag{5}$$

where $\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \boldsymbol{a}_4]^T$ is a matrix of eigenvectors, $\boldsymbol{f}(z) = [f_1(z_1), f_2(z_2), f_3(z_3), f_4(z_4)]^T$ with $z_\alpha = x_1 + p_\alpha x_3$ is an arbitrary vector function, and the bar stands for complex conjugation.

By using $(1)_1$, the vector $\boldsymbol{t} = [\sigma_{13}, \sigma_{23}, \sigma_{33}, D_3]^T$ can be represented in the form

$$\boldsymbol{t} = \boldsymbol{B}\boldsymbol{f}'(z) + \bar{\boldsymbol{B}}\bar{\boldsymbol{f}}'(\bar{z}), \tag{6}$$

where the 4×4 matrix $\boldsymbol{B}$ is defined by $B_{J\alpha} = (E_{3JK1} + p_\alpha E_{3JK3})A_{k\alpha}$ (not summed over α) and $\boldsymbol{f}'(z) = [f_1'(z_1), f_2'(z_2), f_3'(z_3), f_4'(z_4)]$.

Consider a bimaterial composed of two different piezoelectric semi-infinite spaces $x_3 > 0$ and $x_3 < 0$, as sketched in Figure 1. The material properties are defined by matrices $E^{(1)}_{iJKl}$ and $E^{(2)}_{iJKl}$. We assume that the vector $\boldsymbol{t}$ is continuous across the whole bimaterial interface $x_3 = 0$. Furthermore, in the undamaged ligament L of the interface, both parts of the bimaterial are mechanically and electrically fixed to each other, in ideal contact. In summary, the boundary conditions at the interface are

$$\begin{aligned} \boldsymbol{t}^{(1)}(x_1, 0) &= \boldsymbol{t}^{(2)}(x_1, 0) \quad \text{for } x_1 \in (-\infty, \infty), \\ \boldsymbol{V}^{(1)}(x_1, 0) &= \boldsymbol{V}^{(2)}(x_1, 0) \quad \text{for } x_1 \in L. \end{aligned}$$

In this case according to (5)–(6) the solution of equations (1) can be written for each domain in the form

$$\begin{aligned} \boldsymbol{V}^{(m)}(x_1, x_3) &= \boldsymbol{A}^{(m)}\boldsymbol{f}^{(m)}(z) + \bar{\boldsymbol{A}}^{(m)}\bar{\boldsymbol{f}}^{(m)}(\bar{z}), \\ \boldsymbol{t}^{(m)}(x_1, x_3) &= \boldsymbol{B}^{(m)}\boldsymbol{f}'^{(m)}(z) + \bar{\boldsymbol{B}}^{(m)}\bar{\boldsymbol{f}}'^{(m)}(\bar{z}), \end{aligned}$$

Here $m = 1$ stands for $x_3 > 0$ and $m = 2$ for $x_3 < 0$; the vector functions $\boldsymbol{f}^{(1)}(z)$ and $\boldsymbol{f}^{(2)}(z)$ are analytic in the upper ($x_3 > 0$) and lower ($x_3 < 0$) half-planes, respectively.

An analysis similar to that of Herrmann and Loboda [2000] leads to the expressions

$$\begin{aligned} [\![\boldsymbol{V}'(x_1, 0)]\!] &= \boldsymbol{D}\boldsymbol{f}'^{(1)}(x_1) + \bar{\boldsymbol{D}}\bar{\boldsymbol{f}}'^{(1)}(x_1), \\ \boldsymbol{t}(x_1, 0) &= \boldsymbol{B}^{(1)}\boldsymbol{f}'^{(1)}(x_1) + \bar{\boldsymbol{B}}^{(1)}\bar{\boldsymbol{f}}'^{(1)}(x_1), \end{aligned}$$

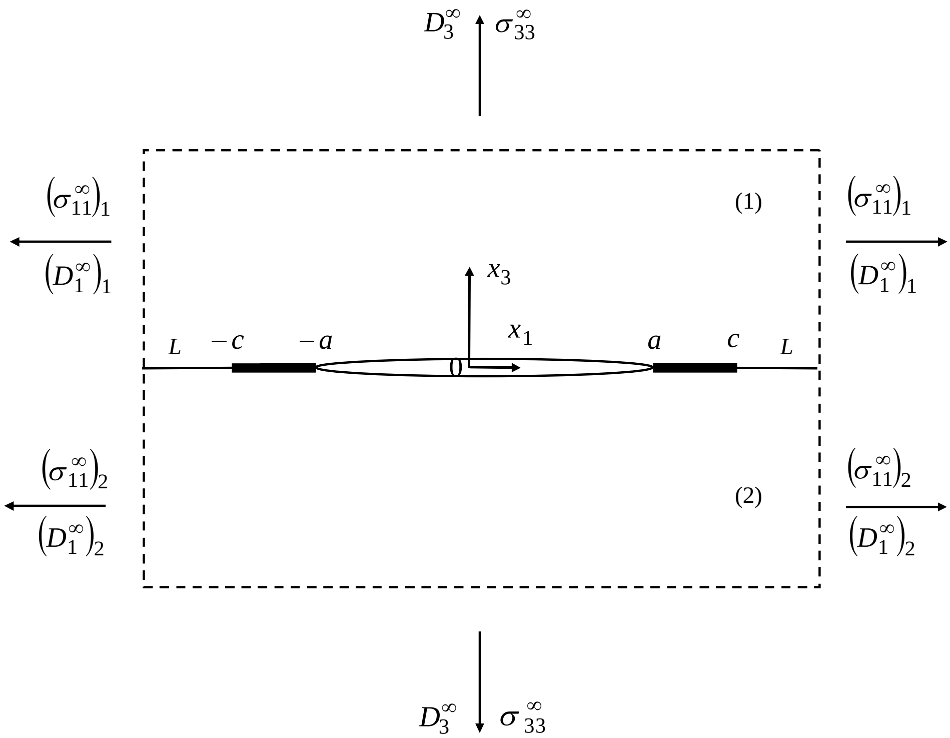

Figure 1. Prefracture zones ahead of the crack tips in a piezoelectric bimaterial compound

where $\boldsymbol{D} = \boldsymbol{A}^{(1)} - \bar{\boldsymbol{A}}^{(2)}(\bar{\boldsymbol{B}}^2)^{-1}\boldsymbol{B}^{(1)}$. Here and afterwards, the double square brackets $[\![\cdot]\!]$ denote the jump of the corresponding function over the material interface, i.e., $[\![\boldsymbol{V}'(x_1, 0)]\!] = \boldsymbol{V}'^{(1)}(x_1, 0) - \boldsymbol{V}'^{(2)}(x_1, 0)$.

Introducing the vector function

$$\boldsymbol{W}(z) = \begin{cases} \boldsymbol{D}\boldsymbol{f}'^{(1)}(z) & \text{for } x_3 > 0, \\ -\bar{\boldsymbol{D}}\bar{\boldsymbol{f}}'^{(1)}(z) & \text{for } x_3 < 0, \end{cases}$$

one obtains

$$[\![\boldsymbol{V}'(x_1, 0)]\!] = \boldsymbol{W}^+(x_1) - \boldsymbol{W}^-(x_1), \tag{7}$$

$$\boldsymbol{t}(x_1, 0) = \boldsymbol{G}\boldsymbol{W}^+(x_1) - \bar{\boldsymbol{G}}\boldsymbol{W}^-(x_1), \tag{8}$$

where $\boldsymbol{G} = \boldsymbol{B}^{(1)}\boldsymbol{D}^{-1}$, $\boldsymbol{W}^+(x_1) = \boldsymbol{W}(x_1 + i0)$, $\boldsymbol{W}^-(x_1) = \boldsymbol{W}(x_1 - i0)$. It follows from (7) that the vector function $\boldsymbol{W}(z)$ is analytical in the whole (x_1, x_3)-plane, including the bonded parts of the material interface.

In the following, our attention is focused on piezoelectric materials of the symmetry class 6mm poled in direction x_3, which have an essential practical significance. In this case for loads which are independent of the coordinate x_2 we can look for fields in the (x_1, x_3)-plane, where the displacement u_2 contained as second component in vector function $\boldsymbol{V}$ decouples from the components (u_1, u_3, φ). Due to this, u_2 can simply be determined after having solved the remaining problem for (u_1, u_3, φ), and therefore our attention will be focused on the plane problem for components (u_1, u_3, φ). For this case, the bimaterial matrix $\boldsymbol{G}$ without its second row and column has the form [Herrmann and Loboda 2000]

$$\boldsymbol{G} = \begin{bmatrix} G_{11} & G_{13} & G_{14} \\ G_{31} & G_{33} & G_{34} \\ G_{41} & G_{43} & G_{44} \end{bmatrix} = \begin{bmatrix} ig_{11} & g_{13} & g_{14} \\ g_{31} & ig_{33} & ig_{34} \\ g_{41} & ig_{43} & ig_{44} \end{bmatrix}, \tag{9}$$

where all the g_{ij} are real.

Consider in addition an arbitrary row matrix $\boldsymbol{S} = [S_1, S_3, S_4]$ and a product $\boldsymbol{St}(x_1, 0)$ which by using (8) with $\boldsymbol{G}$ defined by (9) can be written as

$$\boldsymbol{St}(x_1, 0) = \boldsymbol{SGW}^+(x_1) - \boldsymbol{S\bar{G}W}^-(x_1). \tag{10}$$

Introducing the function

$$F(z) = \boldsymbol{HW}(z)$$

with $\boldsymbol{H} = \boldsymbol{SG}$ and assuming

$$S\bar{\boldsymbol{G}} = -\gamma\, \boldsymbol{SG},$$

where γ is a constant, equation (10) can be written as

$$\boldsymbol{St}(x_1, 0) = F^+(x_1) + \gamma\, F^-(x_1).$$

Here, γ and $\boldsymbol{S}^T$ are an eigenvalue and an eigenvector of the system

$$(\gamma\, \boldsymbol{G}^T + \bar{\boldsymbol{G}}^T)\boldsymbol{S}^T = 0. \tag{11}$$

By the use of (9), the roots of the equation $\det(\gamma\, \boldsymbol{G}^T + \bar{\boldsymbol{G}}^T) = 0$ can be represented in the form

$$\gamma_1 = \frac{1+\delta}{1-\delta}, \quad \gamma_3 = \frac{1}{\gamma_1}, \quad \gamma_4 = 1, \quad \text{where } \delta^2 = \frac{g_{14}^2 g_{33} + g_{13}^2 g_{44} - 2g_{13}g_{14}g_{34}}{g_{11}(g_{33}g_{44} - g_{34}^2)}.$$

Numerical analysis shows that for a group of compound piezoelectric materials the inequality

$$\delta^2 > 0 \tag{12}$$

holds, while for another group this inequality is not valid. This has been mentioned in [Suo et al. 1992]. In the following, attention is paid to piezoelectric materials satisfying inequality (12). In this case, the eigenvector $\boldsymbol{S}_j = \left[S_{j1}, S_{j3}, S_{j4}\right]$ associated with an eigenvalue γ_j $(j = 1, 3, 4)$ can be found from system (11). If one assumes S_{j3} to be real, then S_{j1} is imaginary and S_{j4} is real. Then, the components of the corresponding vectors $\boldsymbol{H}_j = \left[H_{j1}, H_{j3}, H_{j4}\right]$ have the following properties: H_{j1} is real while H_{j3} and H_{j4} are imaginary.

Thus, according to the conclusions above concerning the properties of $\boldsymbol{S}_j$ and $\boldsymbol{H}_j$, and choosing $S_{j3} = 1$, one can write

$$\sigma_{33}(x_1, 0) + m_{j4}D_3(x_1, 0) + im_{j1}\sigma_{13}(x_1, 0) = F_j^+(x_1) + \gamma_j F_j^-(x_1), \tag{13}$$

where

$$F_j(z) = n_{j1}W_1(z) + in_{j3}W_3 + in_{j4}W_4, \tag{14}$$

and $m_{j4} = S_{j4}$, $m_{j1} = -iS_{j1}$, $n_{j1} = H_{j1}$, $n_{j3} = -iH_{j3}$, $n_{j4} = -iH_{j4}$. Here m_{jl}, n_{jl} $(j, l = 1, 3, 4)$ are real values. It is clear from (14) that the functions $F_j(z)$ are analytic in the whole (x_1, x_3)-plane, including the bonded parts of the material interface.

For piezoelectric ceramics of the symmetry class 6mm with poling direction x_3, the relations $m_{41} = 0$, $n_{41} = 0$, $m_{14} = m_{34}$, $m_{11} = -m_{31}$ and $\gamma_3 = 1/\gamma_1$ hold [Herrmann and Loboda 2000]. Using them, we can obtain the solution of the problems (13)–(14) for $j = 3$ from the solution of this problem for $j = 1$. Therefore, only the equations

$$\begin{aligned} \sigma_{33}(x_1, 0) + m_{14} D_3(x_1, 0) + i m_{11} \sigma_{13}(x_1, 0) &= F_1^+(x_1) + \gamma_1 F_1^-(x_1), \\ \sigma_{33}(x_1, 0) + m_{44} D_3(x_1, 0) &= F_4^+(x_1) + F_4^-(x_1), \end{aligned} \tag{15}$$

will be considered below.

Equations (7) and (14) lead to the expressions

$$n_{11}[\![u_1'(x_1, 0)]\!] + i n_{13}[\![u_3'(x_1, 0)]\!] + i n_{14}[\![\varphi'(x_1, 0)]\!] = F_1^+(x_1) - F_1^-(x_1), \tag{16}$$

$$i n_{43}[\![u_3'(x_1, 0)]\!] + i n_{44}[\![\varphi'(x_1, 0)]\!] = F_4^+(x_1) - F_4^-(x_1) \tag{17}$$

for the derivatives of the displacement and electrical potential jumps.

3. Formulation of the problem

Consider two piezoelectric half-spaces $x_3 > 0$ and $x_3 < 0$ having both the symmetry class 6mm with poling direction x_3. It is assumed that the half-spaces are adhered to each other by means of an interlayer of very small thickness. The loading at infinity is given by $\sigma_{33}^{(m)} = \sigma_{33}^{\infty}$, $\sigma_{11}^{(m)} = (\sigma_{11}^{\infty})_m$, $D_3^{(m)} = D_3^{\infty}$ and $D_1^{(m)} = (D_1^{\infty})_m$ ($m = 1$ stands for the upper domain, and $m = 2$ for the lower one). Moreover, stresses $(\sigma_{11}^{\infty})_m$ and electrical displacement $(D_1^{\infty})_m$ are chosen to satisfy continuity conditions at the interface. Assuming the load to be independent of coordinate x_2, the plane strain problem in the (x_1, x_3)-plane can be considered and the relations from the previous section can be used. It is assumed that a crack $(-a, a)$ is situated in the interlayer (Figure 1). The interlayer thickness is assumed to be small compared to the crack length. Therefore, the exact location of the crack in thickness direction, may it be either between the interlayer and one of the piezoelectric materials or inside the interlayer is not resolved in detail in this work. Rather, the interlayer thickness will not be completely taken into consideration. However, the material properties of the interlayer and its influence upon the fracture process will be accounted for.

As mentioned, a linear piezoelectric constitutive model leads to an oscillating singularity in stresses and in electrical displacement at interface crack tips. To avoid these singularities, electromechanical prefracture zones $[-c, -a]$ and $[a, c]$ are introduced in front of the crack tips. We believe that consideration of such zones of electrical saturation zone and mechanical yielding zone might offer a perspective to understand the currently observed discrepancies between theory and experiments. In general, the electrical saturation zone and the mechanical yielding zone would be of different length. However, a complete nonlinear analysis including such electromechanical zones of different length for the discussion of fracture in piezoelectric bimaterial compounds will encounter considerable mathematical difficulties. In stead, as a first step towards understanding the effects of electromechanical nonlinearity, we propose here to consider a strip saturation model where the zones of electrical and mechanical yielding are of the same length. In this zone, electric displacement is limited by some given saturation value, i.e., $D_3 = D_s$, while the stresses in this zones are constant and unknown: $\sigma_{33} = \sigma_0$, $\sigma_{13} = \tau_0$.

In view of this, the interface conditions can be represented in the form

$$[\![\boldsymbol{V}(x_1, 0)]\!] = 0, \;\; [\![t(x_1,0)]\!] = 0 \quad \text{for } x_1 \notin (-c, c), \tag{18}$$

$$\sigma_{33}(x_1, 0) \equiv q_1(x_1) = \begin{cases} \sigma_0, & -c \le x_1 \le -a, \\ 0, & -a < x_1 < a, \\ \sigma_0, & \le x_1 \le c, \end{cases} \tag{19}$$

$$\sigma_{13}(x_1, 0) \equiv q_2(x_1) = \begin{cases} -\tau_0, & -c \le x_1 \le -a \\ 0, & -a < x_1 < a \\ \tau_0, & \le x_1 \le c, \end{cases} \tag{20}$$

$$D_3(x_1, 0) \equiv q_3(x_1) = \begin{cases} D_s, & -c \le x_1 \le -a \\ 0, & -a < x_1 < a, \\ D_s, & \le x_1 \le c. \end{cases} \tag{21}$$

Here, due to continuity, the saturation limit D_s is equal the smaller one of the two materials. The length c of this prefracture zone has to be determined from the above mentioned conditions.

In this way, we have formulated a problem of linear fracture mechanics for a crack $(-c, c)$ between two half-spaces with unknown stress components σ_0, τ_0 and unknown position of the point c.

4. Solution of the problem

Taking into account that $F_j^+(x_1) = F_j^-(x_1)$ $(j = 1, 4)$ for $|x_1| > c$, one can write by means of (15) and the prescribed remote electromechanical loads at infinity the conditions

$$F_1(z)|_{z\to\infty} = \frac{1}{1+\gamma_1}(\sigma_{33}^\infty + m_{14} D_3^\infty), \qquad F_4(z)|_{z\to\infty} = \tfrac{1}{2}(\sigma_{33}^\infty + m_{44} D_3^\infty) \tag{22}$$

for the functions $F_j(z)$.

Using equations (15) and imposing the interface conditions (19)–(21), we obtain for $|x_1| < c$

$$F_1^+(x_1) + \gamma_1 F_1^-(x_1) = p_1(x_1), \qquad F_4^+(x_1) + F_4^-(x_1) = p_4(x_1), \tag{23}$$

where $p_1(x_1) = q_1(x_1) + m_{14} q_3(x_1) + i m_{11} q_2(x_1)$, $p_4(x_1) = q_1(x_1) + m_{44} q_3(x_1)$.

By satisfying conditions at infinity (22), the solution of the problems (23) can be written in the form [Muskhelishvili 1953]

$$\begin{aligned} F_1(z) &= \frac{1}{2\pi i(1+\gamma_1)Y_1(z)} \left\{ 2\pi i(\sigma_{33}^\infty + m_{14} D_3^\infty)(z - 2ci\varepsilon_1) + (1+\gamma_1)\int_{-c}^{c} \frac{Y_1^+(t)p_1(t)\,dt}{t-z} \right\}, \\ F_4(z) &= \frac{1}{2\pi i Y_4(z)} \left\{ \pi z i(\sigma_{33}^\infty + m_{44} D_3^\infty) + \int_{-c}^{c} \frac{Y_4^+(t)p_4(t)\,dt}{t-z} \right\}, \end{aligned} \tag{24}$$

where $Y_1(z) = (z+c)^{0.5-i\varepsilon_1}(z-c)^{0.5+i\varepsilon_1}$, $Y_4(z) = \sqrt{z^2 - c^2}$, $\varepsilon_1 = (\ln \gamma_1)/(2\pi)$.

By introducing the new functions

$$\Phi_{11}(z)=\frac{1}{2\pi i}\int_{-c}^{-a}\frac{Y_1^+(t)\,dt}{t-z},\quad \Phi_{12}(z)=\frac{1}{2\pi i}\int_{a}^{c}\frac{Y_1^+(t)\,dt}{t-z},$$
$$\Phi_{41}(z)=\frac{1}{2\pi i}\int_{-c}^{-a}\frac{Y_4^+(t)\,dt}{t-z},\quad \Phi_{42}(z)=\frac{1}{2\pi i}\int_{a}^{c}\frac{Y_4^+(t)\,dt}{t-z},$$

we reduce the equations (24) to the form

$$F_1(z)=\frac{1}{(1+\gamma_1)Y_1(z)}\big\{(\sigma_{33}^\infty+m_{14}D_3^\infty)(z-2ci\varepsilon_1)+(1+\gamma_1)(\sigma_0+m_{14}D_s-im_{11}\tau_0)\Phi_{11}(z)$$
$$+(1+\gamma_1)(\sigma_0+m_{14}D_s+im_{11}\tau_0)\Phi_{12}(z)\big\} \quad (25)$$

and

$$F_4(z)=\frac{1}{2Y_4(z)}\big\{z(\sigma_{33}^\infty+m_{44}D_3^\infty)+2(\sigma_0+m_{44}D_s)(\Phi_{41}(z)+\Phi_{42}(z))\big\}. \quad (26)$$

Taking into account that $\Phi_{jk}^+(x_1)=\Phi_{jk}^-(x_1)=\Phi_{jk}(x_1)$ $(j=1,4,\ k=1,2)$ for $|x_1|>c$ and using equations (15), we obtain the relations

$$\sigma_{33}(x_1,0)+m_{14}D_3(x_1,0)+im_{11}\sigma_{13}(x_1,0)$$
$$=\frac{1}{Y_1(x_1)}\big\{(\sigma_{33}^\infty+m_{14}D_3^\infty)(x_1-2ci\varepsilon_1)+(1+\gamma_1)(\sigma_0+m_{14}D_s-im_{11}\tau_0)\Phi_{11}(x_1)$$
$$+(1+\gamma_1)(\sigma_0+m_{14}D_s+im_{11}\tau_0)\Phi_{12}(x_1)\big\} \quad (27)$$

and

$$\sigma_{33}(x_1,0)+m_{44}D_3(x_1,0)=\frac{1}{Y_4(x_1)}\big\{x_1(\sigma_{33}^\infty+m_{44}D_3^\infty)+2(\sigma_0+m_{44}D_s)(\Phi_{41}(x_1)+\Phi_{42}(x_1))\big\}. \quad (28)$$

for the stresses and electrical displacements.

Furthermore, we employ the finite value conditions at the interface for $x_1\to c+0$ for the stresses and electrical displacements formulated in the first and third of equations (19)–(21). These conditions are satisfied if the equations

$$\begin{aligned}&c(\sigma_{33}^\infty+m_{14}D_3^\infty)(1-2i\varepsilon_1)+(1+\gamma_1)\big\{(\sigma_0+m_{14}D_s-im_{11}\tau_0)\Phi_{11}^c+(\sigma_0+m_{14}D_s+im_{11}\tau_0)\Phi_{12}^c\big\}=0,\\&c(\sigma_{33}^\infty+m_{44}D_3^\infty)+2(\sigma_0+m_{44}D_s)(\Phi_{41}^c+\Phi_{42}^c)=0,\end{aligned} \quad (29)$$

hold, where $\Phi_{jk}^c=\lim_{x_1\to c+0}\Phi_{jk}(x_1)$ $(j=1,4,\ k=1,2)$. The integrals Φ_{jk}^c can be calculated exactly:

$$\Phi_{11}^c=\frac{1}{2\pi i\gamma_1(1.5-i\varepsilon_1)}(a+c)^{-0.5+i\varepsilon_1}(a-c)^{1.5-i\varepsilon_1}{}_2F_1\Big(1,\frac{1}{2}-i\varepsilon_1,\frac{5}{2}-i\varepsilon_1,\frac{a-c}{a+c}\Big),$$
$$\Phi_{12}^c=-\frac{1}{2\pi i(0.5+i\varepsilon_1)}(a+c)^{0.5-i\varepsilon_1}(a-c)^{0.5+i\varepsilon_1}{}_2F_1\Big(1,-\frac{1}{2}+i\varepsilon_1,\frac{3}{2}+i\varepsilon_1,\frac{a-c}{a+c}\Big),$$
$$\Phi_{41}^c+\Phi_{42}^c=-\frac{c}{\pi}\cos^{-1}\Big(\frac{a}{c}\Big),$$

where

$${}_2F_1(\alpha,\beta,\gamma,z)=\sum_{m=0}^{\infty}\frac{(\alpha)_m(\beta)_m z^m}{(\gamma)_m m!}$$

is the Gauss hypergeometric function.

From the system (29), we derive the equality

$$\frac{c(\omega_{22}+2\varepsilon_1\omega_{12})(\sigma_{33}^{\infty}+m_{14}D_3^{\infty})}{(1+\gamma_1)(\omega_{11}\omega_{22}-\omega_{12}\omega_{21})}+\frac{\pi(\sigma_{33}^{\infty}+m_{44}D_3^{\infty})}{2\cos^{-1}(a/c)}-(m_{44}-m_{14})D_s=0, \tag{30}$$

where

$$\omega_{11}=\operatorname{Re}\Phi_{11}^c+\operatorname{Re}\Phi_{12}^c,\quad \omega_{12}=\operatorname{Im}\Phi_{11}^c-\operatorname{Im}\Phi_{12}^c,$$
$$\omega_{21}=\operatorname{Im}\Phi_{11}^c+\operatorname{Im}\Phi_{12}^c,\quad \omega_{22}=\operatorname{Re}\Phi_{12}^c-\operatorname{Re}\Phi_{11}^c;$$

solving (30) for c (which in general has to be done numerically) one obtains the value of c. After substituting this value into the system (29), one arrives the expressions

$$\tau_0=\frac{c(\omega_{21}+2\varepsilon_1\omega_{11})(\sigma_{33}^{\infty}+m_{14}D_3^{\infty})}{m_{11}(1+\gamma_1)(\omega_{11}\omega_{22}-\omega_{12}\omega_{21})},$$
$$\sigma_0=\frac{1}{m_{14}-m_{44}}\left\{\frac{cm_{44}(\omega_{22}+2\varepsilon_1\omega_{12})(\sigma_{33}^{\infty}+m_{14}D_3^{\infty})}{(1+\gamma_1)(\omega_{11}\omega_{22}-\omega_{12}\omega_{21})}+\frac{\pi m_{14}(\sigma_{33}^{\infty}+m_{44}D_3^{\infty})}{2\cos^{-1}(a/c)}\right\}$$

for the stresses in the prefracture zones.

When, for a given D_s, the prefracture zone length in terms of c and the appropriate values of σ_0 and τ_0 have been found, we are able to calculate the stresses and electrical displacement for $|x_1|>c$ from (27) and (28). In this case, the integrals $\Phi_{41}(x_1)$ and $\Phi_{42}(x_1)$ can be calculated analytically, while $\Phi_{11}(x_1)$ and $\Phi_{12}(x_1)$ can be represented via hypergeometric functions as

$$\Phi_{11}(x_1)=\frac{1}{4\pi ic\gamma_1(1.5-i\varepsilon_1)}(a+c)^{1.5+i\varepsilon_1}(a-c)^{1.5-i\varepsilon_1}\frac{1}{a+x_1}F_1\Big(1,2,1,\tfrac{5}{2}-i\varepsilon_1,\frac{c-a}{2c},\frac{(x_1-c)(c-a)}{2c(a+x_1)}\Big)$$
$$-\frac{1}{1+\gamma_1}\Big\{(x_1-c)^{0.5+i\varepsilon_1}(2c)^{0.5-i\varepsilon_1}{}_2F_1\Big(\tfrac{3}{2}+i\varepsilon_1,-\tfrac{1}{2}+i\varepsilon_1,\tfrac{3}{2}+i\varepsilon_1,\frac{c-x_1}{2c}\Big)-(x_1+c)^{0.5-i\varepsilon_1}(x_1-c)^{0.5+i\varepsilon_1}\Big\},$$
$$\Phi_{12}(x_1)=\frac{1}{2\pi i(1.5+i\varepsilon_1)}(a+c)^{0.5-i\varepsilon_1}(a-c)^{1.5+i\varepsilon_1}\frac{1}{x_1-a}F_1\Big(1,-\tfrac{1}{2}+i\varepsilon_1,1,\frac{5}{2}+i\varepsilon_1,\frac{a-c}{a+c},\frac{c-a}{x_1-a}\Big),$$
$$\Phi_{41}(x_1)+\Phi_{42}(x_1)=-\frac{x_1}{\pi}\left\{\cos^{-1}\Big(\frac{a}{c}\Big)-\frac{\sqrt{x_1^2-c^2}}{x_1}\cot^{-1}\Big(\frac{a}{x_1}\sqrt{\frac{x_1^2-c^2}{c^2-a^2}}\Big)\right\},$$

where

$$F_1(\alpha,\beta_1,\beta_2,\gamma,x,y)=\sum_{m,n}\frac{(\alpha)_{m+n}(\beta_1)_m(\beta_2)_n}{(\gamma)_{m+n}m!n!}x^my^n$$

is the Appell hypergeometric function.

Now consider the jumps in displacement and electrical potential at the crack faces. From (23) we have

$$F_1^-(x_1)=\frac{1}{\gamma_1}p_1(x_1)-F_1^+(x_1)\quad\text{and}\quad F_4^-(x_1)=p_4(x_1)-F_4^+(x_1)$$

for $|x_1|<c$. Substituting this into (16) and (17), one arrives at the equations

$$n_{11}[\![u_1'(x_1,0)]\!]+in_{13}[\![u_3'(x_1,0)]\!]+in_{14}[\![\varphi'(x_1,0)]\!]=\frac{1+\gamma_1}{\gamma_1}F_1^+(x_1)-\frac{1}{\gamma_1}p_1(x_1), \tag{31}$$
$$in_{43}[\![u_3'(x_1,0)]\!]+in_{44}[\![\varphi'(x_1,0)]\!]=2F_4^+(x_1)-p_4(x_1),\qquad |x_1|<c. \tag{32}$$

Then, substituting (25)–(26) into (31) and (32), we arrive at

$$
\begin{aligned}
n_{11}[\![u_1'(x_1,0)]\!] + in_{13}[\![u_3'(x_1,0)]\!] + in_{14}[\![\varphi'(x_1,0)]\!] \\
= \frac{1}{\gamma_1 Y_1^+(x_1)}\big\{(\sigma_{33}^\infty + m_{14}D_3^\infty)(x_1 - 2ic\varepsilon_1) + (1+\gamma_1)(\sigma_0 + m_{14}D_s - im_{11}\tau_0)\Phi_{11}^+(x_1) \\
+ (1+\gamma_1)(\sigma_0 + m_{14}D_s + im_{11}\tau_0)\Phi_{12}^+(x_1)\big\} \quad (33)
\end{aligned}
$$

and

$$
n_{43}[\![u_3'(x_1,0)]\!] + n_{44}[\![\varphi'(x_1,0)]\!] = \frac{\sigma_0 + m_{44}D_s}{2\pi}\left\{\Gamma(c,x_1,a) - \Gamma(c,x_1,-a)\right\}, |x_1| < c, \quad (34)
$$

where

$$
\Gamma(b,x_1,\xi) = \ln\frac{b^2 - x_1\xi - \sqrt{(b^2 - x_1^2)(b^2-\xi^2)}}{b^2 - x_1\xi + \sqrt{(b^2 - x_1^2)(b^2-\xi^2)}}.
$$

One can evaluate Appell's F_1 hypergeometric function using the approach of [Colavecchia et al. 2001], which is based on analytic continuations of F_1 outside the region of convergence of the series. Thus one can write for $|x_1| < c$

$$
\begin{aligned}
\Phi_{11}^+(x_1) = \frac{1}{4\pi ic\gamma_1(1.5 - i\varepsilon_1)}(a+c)^{1.5+i\varepsilon_1}(a-c)^{1,5-i\varepsilon_1}\frac{1}{x_1+a}F_1\Big(1,2,1,\tfrac{5}{2}-i\varepsilon_1,\frac{c-a}{2c},\frac{(x_1-c)(c-a)}{2c(a+x_1)}\Big) \\
-\frac{1}{1+\gamma_1}(x_1-c)^{0.5+i\varepsilon_1}\Big\{(2c)^{0.5-i\varepsilon_1}{}_2F_1\Big(\tfrac{3}{2}+i\varepsilon_1,-\tfrac{1}{2}+i\varepsilon_1,\tfrac{3}{2}+i\varepsilon_1,\frac{c-x_1}{2c}\Big) - (x_1+c)^{0.5-i\varepsilon_1}\Big\},
\end{aligned}
$$

while for $|x_1| < a$

$$
\Phi_{12}^+(x_1) = \frac{(2c)^{0.5-i\varepsilon_1}}{2\pi i(1.5+i\varepsilon_1)}(a-c)^{1.5+i\varepsilon_1}\frac{1}{x_1-c}F_1\Big(\tfrac{3}{2}+i\varepsilon_1,-\tfrac{1}{2}+i\varepsilon_1,1,\tfrac{5}{2}+i\varepsilon_1,\frac{c-a}{2c},\frac{c-a}{c-x_1}\Big),
$$

and for $a < |x_1| < c$

$$
\begin{aligned}
\Phi_{12}^+(x_1) = \frac{0.5+i\varepsilon_1}{i\exp(-\pi\varepsilon_1)(1+\gamma_1)}(x_1-c)^{0.5+i\varepsilon_1}(2c)^{0.5-i\varepsilon_1}{}_2F_1\left(\tfrac{3}{2}+i\varepsilon_1,-\tfrac{1}{2}+i\varepsilon_1,\tfrac{1}{2}+i\varepsilon_1,\frac{c+x_1}{2c}\right) \\
+\frac{(a-c)^{1.5+i\varepsilon_1}(a+c)^{1.5-i\varepsilon_1}}{4\pi ic(1.5-i\varepsilon_1)(a-x_1)}F_1\Big(1,2,1,\tfrac{5}{2}-i\varepsilon_1,\frac{c+a}{2c},\frac{(c+a)(c-x_1)}{2c(a-x_1)}\Big).
\end{aligned}
$$

Integrating (33) and (34) gives for the jumps in displacement and electrical potential the expressions

$$
\begin{aligned}
n_{11}[\![u_1(x_1,0)]\!] + in_{13}[\![u_3(x_1,0)]\!] + in_{14}[\![\varphi(x_1,0)]\!] \\
= \frac{1}{\gamma_1}(\sigma_{33}^\infty + m_{14}D_3^\infty)(x_1+c)^{0.5+i\varepsilon_1}(x_1-c)^{0.5-i\varepsilon_1} \\
+\frac{1+\gamma_1}{\gamma_1}\big\{(\sigma_0 + m_{14}D_s - im_{11}\tau_0)J_{11}(x_1) + (\sigma_0 + m_{14}D_s + im_{11}\tau_0)J_{12}(x_1)\big\} \quad (35)
\end{aligned}
$$

and, for $|x_1| < c$,

$$
n_{43}[\![u_3(x_1,0)]\!] + n_{44}[\![\varphi(x_1,0)]\!] = \frac{\sigma_0 + m_{44}D_s}{2\pi}\big\{(x_1-a)\Gamma(c,x_1,a) - (x_1+a)\Gamma(c,x_1,-a)\big\}, \quad (36)
$$

where

$$J_{11}(x_1) = \int_{-c}^{x_1} \frac{\Phi_{11}^{+}(t)\,dt}{Y_1^{+}(t)},\ J_{12}(x_1) = \int_{-c}^{x_1} \frac{\Phi_{12}^{+}(t)\,dt}{Y_1^{+}(t)}.$$

The most important quantities, namely the crack opening displacements and potential at the initial crack tips, are written

$$\delta_{u_1} = [\![u_1(a,0)]\!], \quad \delta_{u_3} = [\![u_3(a,0)]\!], \quad \delta_{\varphi} = [\![\varphi(a,0)]\!].$$

Because (35) and (36), we get among these quantities the relations

$$\begin{aligned} n_{11}\delta_{u_1}+in_{13}\delta_{u_3}+in_{14}\delta_{\varphi} &= \frac{1}{\gamma_1}(\sigma_{33}^{\infty}+m_{14}D_3^{\infty})(a+c)^{0.5+i\varepsilon_1}(a-c)^{0.5-i\varepsilon_1} \\ &\quad + \frac{1+\gamma_1}{\gamma_1}\big\{(\sigma_0+m_{14}D_s-im_{11}\tau_0)J_{11}(a)+(\sigma_0+m_{14}D_s+im_{11}\tau_0)J_{12}(a)\big\} \end{aligned} \tag{37}$$

and

$$n_{43}\delta_{u_3} + n_{44}\delta_{\varphi} = -\frac{a\,(\sigma_0 + m_{44}D_s)}{2\pi}\Gamma(c, a, -a). \tag{38}$$

The crack opening displacement (COD)

$$\delta = \sqrt{\left(\delta_{u_1}\right)^2 + \left(\delta_{u_3}\right)^2}$$

can be considered as a fracture criterion for crack growth. Following Gao et al. [1997], we use a contour Γ enclosing points a and c. The energy release rate at the crack tip, as the driving force of fracture, can then be calculated from the J-integral

$$J = \oint_{\Gamma} \big\{Wn_1 - \sigma_{ij}n_i u_{j,i} - D_i n_i \varphi_{,i}\big\}\,ds,$$

where W is the electric enthalpy. Using the property of path-independence of J, we reduce the contour Γ to the prefracture zone (a, c). Taking into account that the thickness of this zone tends to zero we arrive at the formula

$$G = \sigma_0\delta_{u_3} + \tau_0\delta_{u_1} + D_s\delta_{\varphi},$$

where G is the energy release rate, which here is equal to the J-integral.

As pointed out before, the solution constructed in this section corresponds to an electrical saturation zone where the stresses are constant. In addition, it is assumed that some relation

$$f\,(\sigma_s, \tau_s, \sigma_1) = 0$$

holds for the stresses in the prefracture zone, where $\sigma_{33} = \sigma_s$, $\sigma_{13} = \tau_s$, $\sigma_{11} = \sigma_1$. The function f, which can be interpreted as a law of interlayer material yielding or damage, may be determined experimentally or theoretically. For example, in the case of a von Mises yielding condition, we have

$$f(\sigma_s, \tau_s, \sigma_1) \equiv (\sigma_s - \sigma_1)^2 + 4\tau_s^2 - \tfrac{4}{3}\sigma_y^2 = 0,$$

where σ_y is the yield stress of the interface material. According to [Tvergaard and Hutchinson 1996], the stress σ_1 in front of the crack in such material is equal to $2\sigma_y$.

From the latter condition, we can now calculate for a given saturation value D_s the external loads such that the resultant stresses σ_s and τ_s reach a critical level for the onset of yielding in the prefracture zone. The conditions (29) of finite stresses and electrical displacement at the ends of prefracture zone lead to the system

$$\begin{aligned}(\sigma_{33}^{\infty})_s+m_{14}(D_3^{\infty})_s&=-\frac{1+\gamma_1}{c(1-2i\varepsilon_1)}\{(\sigma_s+m_{14}D_s-im_{11}\tau_s)\Phi_{11}^c+(\sigma_s+m_{14}D_s+im_{11}\tau_s)\Phi_{12}^c\},\\(\sigma_{33}^{\infty})_s+m_{44}(D_3^{\infty})_s&=\frac{2}{\pi}(\sigma_s+m_{44}D_s)\arccos\frac{a}{c},\end{aligned}\tag{39}$$

which suffices to determine the unknown external loads $(\sigma_{33}^{\infty})_s$ and $(D_3^{\infty})_s$, together with the unknown position of the point c.

After determining the value of c from the equation

$$\frac{\sigma_s+m_{14}D_s}{m_{14}}+\frac{(\omega_{22}+2\varepsilon_1\omega_{12})m_{11}\tau_s}{m_{14}(\omega_{21}+2\varepsilon_1\omega_{11})}=0,\tag{40}$$

which is derived from equation (30), the external load can be calculated from the system (39).

5. Numerical results and discussion

Consider an electrically impermeable interface crack of length $l=2a=2\,\mathrm{mm}$ perpendicular to the poling direction. Calculations have been performed for a bimaterial composed of materials PZT-5H (upper material) and $BaTiO_3$ (lower one). The parameters of these materials are given in [Pak 1992] and [Dunn and Taya 1994], respectively. The interface layer was assumed to be an elastic-perfectly plastic material with yield stress of $\sigma_y=50\,\mathrm{MPa}$.

In Table 1, the relative length $(c-a)/l$ of the prefracture zone in front of the right crack tip is listed together with the intensity of the external load. For this, the nonlinear equation (30) has been solved. It follows that the prefracture zone length is almost independent of σ_{33}^{∞}, but depends strongly and nonlinearly on D_3^{∞}.

The distributions of the normalized displacement jump $[\![u_3(x_1,0)]\!]/l$ along the material interface is shown in Figure 2, and that of the normal stress $\sigma_{33}(x_1,0)/\sigma_y$ in Figure 3. For the interface layer, the same material properties as before have been used. The remote normal stress was taken to be equal to $\sigma_{33}^{\infty}/\sigma_y=0.05$, while the remote electrical displacement has been varied. Curves 1, 2 and 3 correspond

D_3^{∞}/D_s	$(c-a)/l$	
	$\sigma_{33}^{\infty}/\sigma_y=0.05$	$\sigma_{33}^{\infty}/\sigma_y=0.4$
0.2	0.02574	0.02554
0.3	0.06119	0.06097
0.4	0.11808	0.11785
0.5	0.20717	0.20692
0.6	0.35075	0.35047

Table 1. Relative prefracture zone length $(c-a)/l$ as a function of the intensity of the external load.

to $D_3^\infty/D_s = 0.2$, 0.4 and 0.6, respectively. It can be seen that an increase in the remote electrical loading leads to an increase of the displacement jumps and, thus, to an increase of the prefracture zone length, while the stresses decrease in these zones.

Figure 4 shows the effect of the applied electric field E_3^∞ on the energy release rate and the COD δ. These Figures are obtained for the same material properties as before and various values of σ_{33}^∞. Curves 1, 2 and 3 are related to $\sigma_{33}^\infty/\sigma_y = 0.2$, 0.202 and 0.204, respectively. It can be seen from these numerical results that a fracture criterion based on the energy release rate differs essentially from one based on the COD δ. As for homogeneous piezoelectric materials [Gao et al. 1997], we can conclude that a fracture criterion based on energy release rate infers that the electric field should impede crack propagation independently of its sign. This contradicts experimental observations. This conclusion indicates that the energy release rate is not a reasonable basis for a fracture criterion and, rather, a fracture criterion

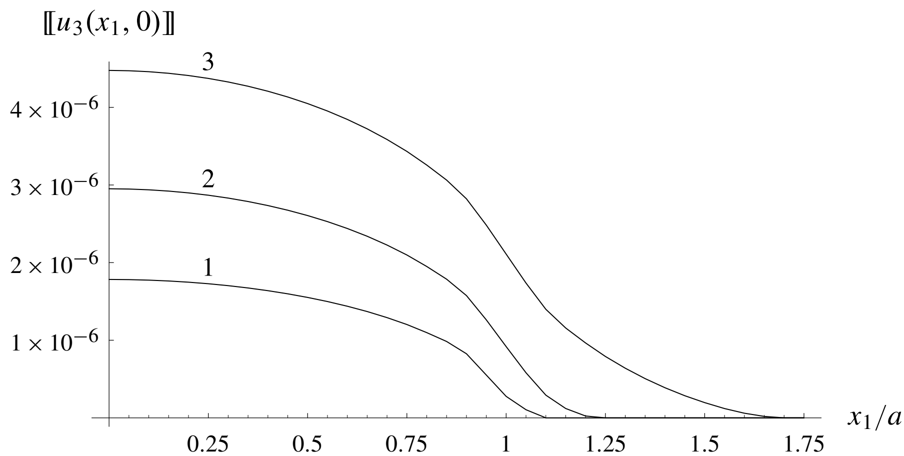

Figure 2. Distribution of normalized displacement jump $[\![u_3(x_1,0)]\!]/l$ along the material interface.

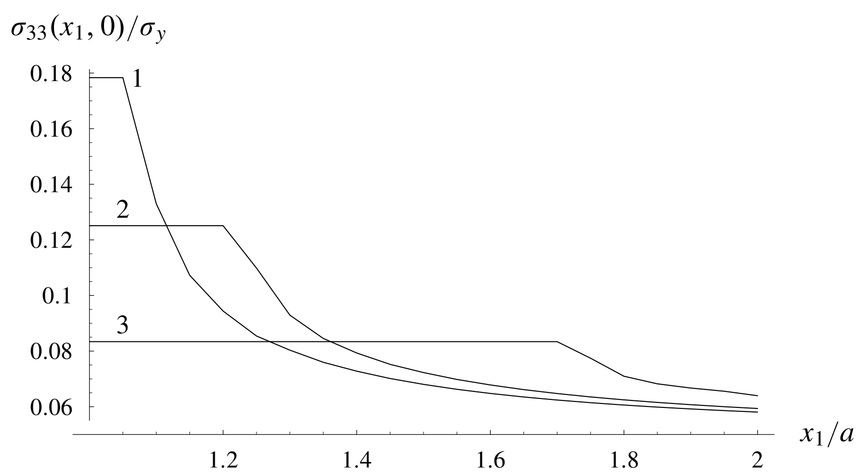

Figure 3. Distribution of normalized stress $\sigma_{33}(x_1,0)/\sigma_y$ along the material interface.

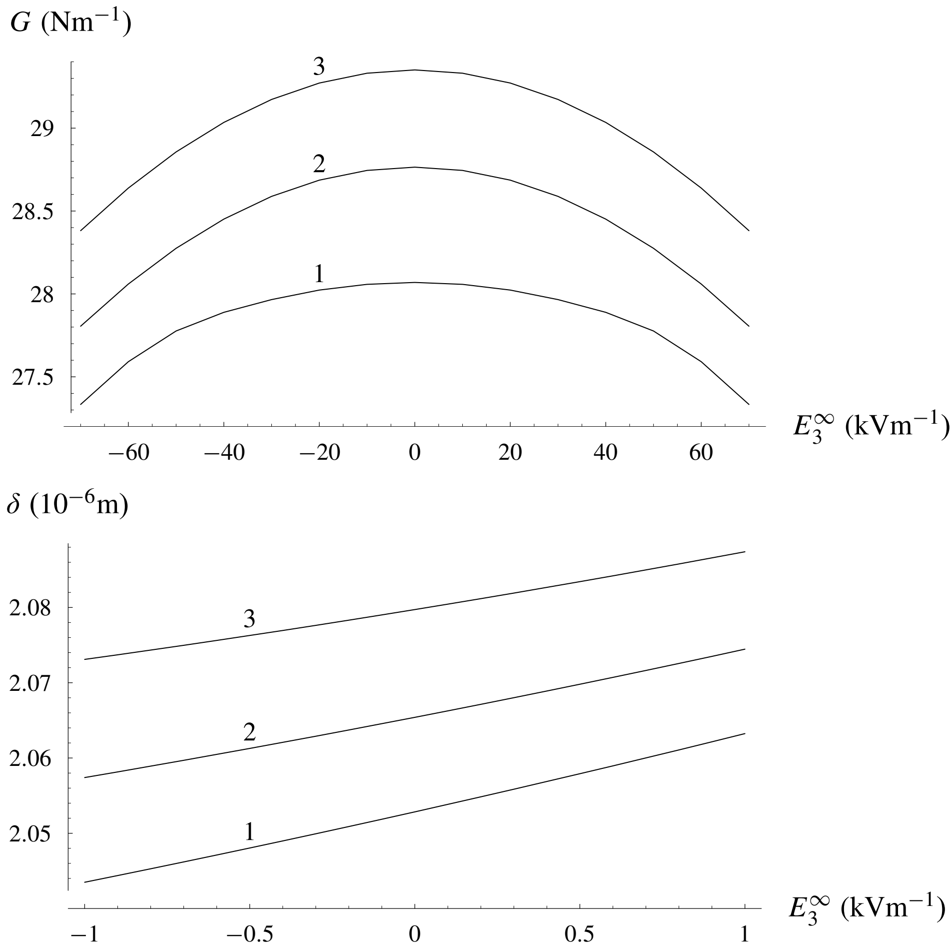

Figure 4. Energy release rate G (top) and crack opening displacement δ (bottom) as functions of the remote electric field E_3^∞.

based on the COD might suitable. From Figure 4, we find that the COD δ would predict that fracture is enhanced by a positive applied electric field and inhibited by a negative applied electric field. This is in qualitative agreement with experimental observations for homogeneous materials [Park and Sun 1995].

External loads $\left(\sigma_{33}^\infty\right)_s/\sigma_y$ and $\left(D_3^\infty\right)_s/D_s$ as well as corresponding prefracture zone length $(c-a)/l$ belonging to given values of the yield stress of the interface layer are listed in Table 2. To this end, the

σ_y/σ_s	$\left(\sigma_{33}^\infty\right)_s/\sigma_s$	$\left(D_3^\infty\right)_s/D_s$	$(c-a)/l$
0.3333	0.0551	0.0036	0.002568
0.5000	0.0849	0.0038	0.002764
0.6667	0.1148	0.0039	0.002843
0.8333	0.1446	0.0040	0.002889

Table 2. External load $(\sigma_{33}^\infty)_s/\sigma_s$, $(D_3^\infty)_s/D_s$ and the relative prefracture zone length $(c-a)/l$ resulting from corresponding values of the yield stress of the interface layer.

system of equations (39) has been solved and equation (40) was used for calculating the prefracture zone length. It follows from Table 2 that increasing the yield stress of the interface layer leads to a grow of both the external loading magnitude and the prefracture zone length.

6. Conclusion

A plane strain problem for two piezoelectric half-spaces adhered by means of some thin interlayer has been considered. This system is subject to the action of a symmetrical remote mechanical and electrical loading. An electrically impermeable crack, which may either be located between the interlayer and one of the piezoelectric materials or completely in the interlayer, is studied.

It is assumed that the piezoelectric bimaterial components are much stiffer than the intermediate layer. Therefore, prefracture zones may develop in the interlayer in front of the crack tip. The problem is reduced to one of linear fracture mechanics by neglecting the interlayer thickness and modeling the prefracture zones as continuations of the crack where electrical polarization reaches a saturation limit and stresses are constant with respect to position.

By assuming that the displacements and the electrical potential fields are independent of out of plane coordinate x_2, we were able to represent stresses and electrical displacements as well as the derivatives of the mechanical displacement and electrical potential jumps by a sectionally holomorphic vector function. This function is analytically continued across the mechanically and electrically bonded parts of the material interface. Furthermore, the problem is reduced to a Hilbert problem and solved exactly. From the condition of stress and electrical displacement to be finite at the end of the prefracture zone towards the undamaged interface layer, algebraic and transcendental equations have been formulated for the determination of the prefracture zone length and the stress magnitude in this zone. For the stresses and the electric displacement, the analytical relations (27) and (28) were derived. The electromechanical nonlinear effects on the structure of stress and electric displacement fields are investigated for different loading conditions. In addition, equations (35) and (36) for the crack opening at the crack tip were deduced.

In this paper, we focus on the special case, when the electrical saturation zone and mechanical yielding zone have the same length. The interface layer is assumed to be elastic perfectly-plastic according to the von Mises yield condition. For this situation, the prefracture zone length and the critical external loading corresponding to yielding are determined by system (39) and transcendental equation (40).

Numerical results for a bimaterial composed of piezoelectric materials PZT-5H and $BaTiO_3$ are obtained. The prefracture zone length, stresses in this zone and the crack opening at the crack tip corresponding to the respective remote loading are calculated. Note that, due to the suggested model, all mechanical and electrical quantities in the near-crack tip region are finite, i.e., all singularities connected with the crack are eliminated. The analysis of energy release rate and crack opening displacements indicates that a fracture criterion based on the crack opening displacements appears to be more appropriate from physical point of view.

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