CYCLIC APPROXIMATION OF THE HEAT EQUATION IN FINITE
STRAINS FOR THE HEAT BUILD-UP PROBLEM OF RUBBER

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It is well-known that rubber exhibits hysteretic mechanical behavior and has a low thermal conductivity. The main consequences are the heat generation and heat build-up phenomena which occur in a rubber component when subjected to repeated deformations. Estimating the heat build-up temperature implies the solution of a coupled thermomechanical problem. Due to the difference between the mechanical and the thermal diffusion characteristic times, a cyclic uncoupled approach is often used to solve the heat build-up problem.

In the uncoupled approach, the heat sources are first determined with a mechanical analysis, and the heat equation is then solved on a fixed geometry. At finite strains, the geometry of the body varies with the deformation but the foregoing method does not account for such changes in geometry. The exact solution would require describing the body deformation while solving the thermal problem, but this does not take advantage of the difference between the characteristic times of the thermal diffusion and the mechanical behaviour, respectively, and the exact numerical resolution is therefore unnecessarily time-consuming.

The purpose of the current work is to take into account kinematics in the thermal problem when using a cyclic uncoupled approach. The heat problem is written in the reference configuration. That implies that the problem is defined on a fixed domain: the initial configuration of the body. The changes in geometry in the reference heat equation are thus described by mechanical time-dependent variables. The cyclic assumption allows mean variables to be defined, for example the mean temperature. A time-integration method and an approximation of the heat equation are developed, leading to a simplified formulation with mechanical time-independent terms. This simplified heat problem is based on the mean variables.

1. Introduction

Heat build-up is the rise in temperature caused by heat generation in a component subjected to repeated deformation. It is characterized by a slow increase of the mean temperature until the steady-state temperature field is reached; see Figure 1. A high frequency oscillation in temperature is superimposed on the mean temperature, as seen on the second half of the figure. This oscillation is caused by thermoelastic effects: Gough [1805] first reported that the temperature of rubber increased when it was stretched. Measurements have shown that the temperature first decreases until reaching a minimum and then increases [Joule 1859; James and Guth 1943]. This thermoelastic inversion phenomenon has been studied and is due to the competition between the thermal expansion and the entropic aspect of rubber elasticity [Chadwick 1974; Treloar 1975]. This thermoelastic oscillation is not relevant in the
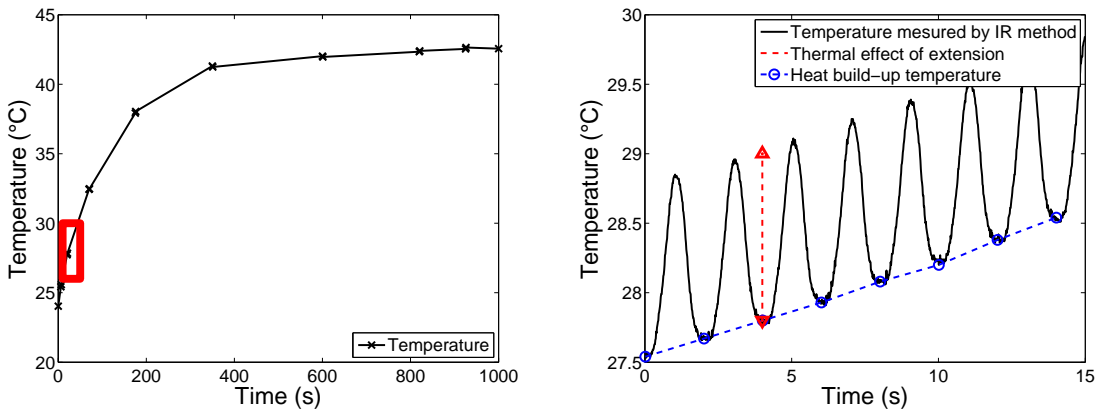
Figure 1. Temperature of a rubber component subjected to a cyclic deformation (0.5 Hz), measured with an infrared thermometer. The rectangle is enlarged on the right.

heat build-up problem; the heat build-up temperature is then defined as the actual temperature minus the thermoelastic effect.

Heat generation is the conversion of applied mechanical energy into heat due to the hysteretic behaviour of rubber. The causes and consequences of heat generation have been extensively explored in the literature [Medalia 1991; Meinecke 1991; Kar and Bhowmick1997a; 1997b; Park et al. 2000]. It is generally attributed to molecular friction, but at large strains in natural rubber much of the hysteresis is thought to be associated with strain-induced crystallisation [Toki et al. 2000; Trabelsi et al. 2003]. The main consequence of hysteresis is to increase the temperature.

Estimating the heat build-up in a rubber component requires the solution of a coupled thermomechanical problem, defined both by mechanical and thermal equations. Coupled or uncoupled methods are available to solve the thermomechanical problem

(1) Fully-coupled algorithm where both thermal and mechanical variables are calculated at the same time [Simo and Taylor 1991; Reese and Govindjee 1997].

(2) Staggered (fractional-step) methods that split the coupled thermomechanical problem into two simpler problems [Armero and Simo 1992].

(3) Uncoupling the thermal and mechanical problems under the assumption of cyclic loading [Whicker et al. 1981b; Sarkar et al. 1987; Ebbott et al. 1999]. Three analysis modules are needed: deformation, dissipation and thermal modules. First the mechanical problem is solved for one cycle. Then the dissipation is estimated and the thermal problem is solved for many cycles on one fixed geometry until the rise of temperature is significant, whereupon the mechanical problem is updated using the actual temperature.

The two first methods are exact and are indicated for solution of the thermoelastic problem over a limited number of cycles [Holzapfel and Simo 1996; Reese and Govindjee 1997]. The last method is often used in the literature for heat build-up problems since it offers a quicker solution.

The main difficulty of the thermomechanical problem lies in the coupling between the thermal and mechanical variables. Thermomechanical coupling can be classified into four categories:

(1) The dependence of stress on temperature [Anthony et al. 1942].

(2) Thermal effect of extension [James and Guth 1943; Treloar 1975].

(3) The heat generation produced by the hysteretic behaviour of rubber.

(4) The coupling between kinematics and the heat equation due to finite strains.

The dependence of stress on temperature is required for a very precise analysis of stress over a cycle but is not essential when the main issue is the determination of the heat build-up temperature. The contribution of the thermoelastic coupling to the dissipation over a stabilized cycle is actually zero [Lion 1997]. The consequence is that it is not necessary to represent thermoelastic effects to estimate the heat build-up temperature. The heat generation correspond to the heat sources: it is generally assumed that the loss of energy is nearly completely converted into heat. The last type of coupling arises from the solution of the thermal problem on a moving domain due to the cyclic deformation of the body. This coupling is neglected in uncoupled cyclic solutions since the heat equation is solved on one fixed geometry and not on the actual geometry. This coupling is negligible under the small strain condition, but it becomes essential to account for it at large strains. The purpose of our work is to account for this coupling within the cyclic uncoupled solving.

The paper is organized as follows: in Section 2 the cyclic uncoupled thermomechanical approach is presented. In Section 3 the heat equation is approximated over a cycle in order to take into account kinematics. Section 4 deals with numerical simulations of heat build-up of a specimen. Our conclusions are summarized in the final section.

2. Cyclic uncoupled thermomechanical approach

Let us assume a cyclic mechanical loading. Our analysis is based on the difference between mechanical and thermal diffusion characteristic times in heat build-up problems: the temperature increases slowly and many mechanical cycles are needed to reach the steady-state temperature. This difference between characteristic times cannot be emphasized by coupled thermomechanical algorithms since both thermal and mechanical variables are calculated at the same time. Only an uncoupled approach is able to take advantage of this difference in terms of time computation. The uncoupled cyclic algorithm is thus adopted [Whicker et al. 1981b; Sarkar et al. 1987].

This algorithm is the following (see Figure 2): the mechanical problem is first solved, then the loss of energy is estimated and put into the heat equation as heat sources. The thermal problem is solved on a fixed geometry and a loop is finally created. This loop allows us to update material data with the temperature. This algorithm uncouples the mechanical and the thermal solution in the sense that the heat equation is solved over a interval bigger than the mechanical characteristic time. The most interesting case is when the mechanical problem is solved for one cycle and the thermal problem is solved for many cycles. This feature quickens the solving of the heat build-up problem. The cyclic uncoupled thermomechanical algorithm is consistent with the definition of the heat build-up temperature given in the introduction that neglects the thermoelastic coupling.

The uncoupled approach is accurate for small or medium strains. The deformation of the body is small and the boundary value problem of heat equation can be solved on a fixed geometry and totally uncoupled from the mechanical analysis. But if the strain is large enough to modify the geometry of the component
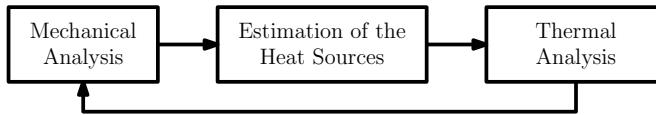
Figure 2. Simplified diagram of the thermomechanical uncoupled approach proposed in [Whicker et al. 1981a].

during the mechanical cycle, the solution of the thermal analysis will strongly depend on the choice of the fixed geometry, independently of the configuration. That implies that it would be necessary at large strains to follow the geometry deformation during the cycle to solve the thermomechanical problem. This coupled solution does not take advantage of the cyclic uncoupling algorithm in terms of time. In the next section a simplified heat equation, which accounts for the changes in geometry, is presented.

3. The simplified heat equation

In the case of periodic deformation, a method is developed to solve the heat equation with taking into account the cyclic geometry deformation. The scheme of the solving is the following: first write the heat equation in the reference configuration, then define mean variables such as temperature or thermal conductivity, and finally integrate with respect to time and approximate the heat equation and the boundary conditions.

3A. *Kinematic description.* A material point in actual (or Eulerian) configuration $\vec{x}$ is linked to its position in the reference (or Lagrangian) configuration $\vec{X}$ by the motion $\vec{x} = \Phi(\vec{X}, t)$. The deformation gradient $\boldsymbol{F}$ and the right Cauchy-Green strain tensor $\boldsymbol{C}$ are defined by

$$\boldsymbol{F} = \frac{\partial \Phi(\vec{X}, t)}{\partial \vec{X}} \quad \text{and} \quad \boldsymbol{C} = \boldsymbol{F}^T \boldsymbol{F}. \tag{3-1}$$

The transformation relations for the length, surface, and volume elements between the reference and the actual configuration are given by

$$\mathrm{d}\vec{x} = \boldsymbol{F}\mathrm{d}\vec{X}, \ \ \mathrm{d}\vec{a} = \mathrm{d}s\, \vec{n} = J\boldsymbol{F}^{-T}\mathrm{d}\vec{A}, \ \text{and}\ \mathrm{d}v = J\mathrm{d}V \ \text{with}\ \mathrm{d}\vec{A} = \mathrm{d}S\vec{N} \ \text{and}\ J = \det \boldsymbol{F}. \tag{3-2}$$

The density is obtained by $\rho_0(\vec{X}, t) = J\rho(\vec{x}, t)$.

3B. *Thermal conductivity.* The thermal conductivity of rubber is generally isotropic in the natural state, but it is sensible to think that it becomes anisotropic with the deformation. Previous models based on the network theory related the thermal conductivity to the deformation [van den Brule 1989; van den Brule and O'Brien 1990] . Measurements proved that the thermal conductivity of rubber actually changes with the deformation [Tautz 1959; Broerman et al. 1999]. The conductivity is higher in the stretched direction and lower in the compressed direction. The magnitude of this phenomenon depends on the material and varies from 10 to 100% at stretch $\lambda = 2$. In the following, the thermal conductivity tensor $\boldsymbol{k}(\boldsymbol{C}, T)$ is implicitly supposed to be dependent of the deformation.

3C. *The heat equation in the reference configuration.* The local equation of the energy balance in the actual reference and the boundary conditions are

$$\rho c \frac{\partial T}{\partial t} = \rho r - \operatorname{div} \vec{q} \text{ and } \vec{q} \cdot \mathrm{d}\vec{a} = h \mathrm{d}a(T - T_\infty) \tag{3-3}$$

where c is the heat capacity, $r(\vec{x}, t)$ represents the generation of heat supplied from outside, div is the divergence operator with respect to the coordinates $\vec{x}$ in the actual configuration and $\vec{q}(\vec{x}, t)$ is the heat flux. The latter is related to the temperature by Fourier's law

$$\vec{q}(\vec{x}, t) = -\boldsymbol{k}(\boldsymbol{C}, T)\operatorname{grad} T \tag{3-4}$$

where $\boldsymbol{k}(\boldsymbol{C}, T)$ is the thermal conductivity tensor and grad is the gradient operator with respect to the actual reference. Note that the boundary conditions represent either the natural convection with the convection coefficient h and the far-field air temperature T_∞, or the imposed temperature when $h \to \infty$. These equations are written in the reference configuration

$$\rho_0 c \frac{\partial T}{\partial t} = \rho_0 R - \operatorname{Div} \vec{Q}, \quad \vec{Q} \cdot \mathrm{d}\vec{A} = \underbrace{\frac{\mathrm{d}a}{\mathrm{d}A} h}_{H(\vec{X},t)} \mathrm{d}A(T - T_\infty), \tag{3-5}$$

where Div is the divergence operator with respect to the reference configuration, $R(\vec{X}, t) = r(\vec{x}, t)$ is the external heat production rate and $\vec{Q}$ is the transported heat flux given by

$$\vec{Q}(\vec{X}, t) = J(\vec{X}, t)\boldsymbol{F}^{-1}(\vec{X}, t)\vec{q}(\vec{x}, t). \tag{3-6}$$

Fourier's law and the relationship $\operatorname{grad} T = \boldsymbol{F}^{-T}\vec{\nabla}T$ between the Eulerian and Lagrangian gradient lead to the Lagrangian expression of the heat flux

$$\vec{Q}(\vec{X}, t) = -J(\vec{X}, t)\left(\boldsymbol{F}^{-1}\boldsymbol{k}(\boldsymbol{C}, T)\boldsymbol{F}^{-T}\right)(\vec{X}, t)\vec{\nabla}T(\vec{X}, t). \tag{3-7}$$

The reference thermal conductivity is given by

$$\boldsymbol{K}(\boldsymbol{C}, T) = J(\vec{X}, t)\left(\boldsymbol{F}^{-1}\boldsymbol{k}(\boldsymbol{C}, T)\boldsymbol{F}^{-T}\right)(\vec{X}, t). \tag{3-8}$$

The convection coefficient H in the reference configuration depends on time and takes into account the variation of the surface along the deformation. Its expression is given by

$$H(\vec{X}, t) = h \frac{\mathrm{d}a}{\mathrm{d}A} = h J \sqrt{\vec{N}^T \boldsymbol{C}^{-1} \vec{N}}. \tag{3-9}$$

3D. *Approximation of the heat equation.* Suppose that the deformation is periodic with period P. A mean temperature T_m function of time can be defined by

$$T_m(\vec{X}, t) = \frac{1}{P} \int_{-P/2}^{P/2} T(\vec{X}, t + \tau)\mathrm{d}\tau \tag{3-10}$$

and the temperature oscillation is

$$T_a(\vec{X}, t, \tau) = T(\vec{X}, t + \tau) - T_m(\vec{X}, t) \quad . \tag{3-11}$$

By extension, the mean and oscillatory variables are described by subscripts m and a. For the sake of readability, the space variable $\vec{X}$ will not be written anymore in the following. The mean conductivity tensor is given at a fixed temperature $T_m(t)$ by

$$\boldsymbol{K}_m(t) = \boldsymbol{K}_m(T_m(t)) = \frac{1}{P} \int_{-P/2}^{P/2} \boldsymbol{K}\left(\boldsymbol{C}(t+\tau), T_m(t)\right) \mathrm{d}\tau. \tag{3-12}$$

The oscillatory conductivity tensor is thus

$$\boldsymbol{K}_a(t,\tau) = \boldsymbol{K}(\boldsymbol{C},T)(t+\tau) - \boldsymbol{K}_m(t). \tag{3-13}$$

Given t, the heat equation becomes

$$\rho_0 c \frac{\partial T}{\partial \tau}(t+\tau) = \rho_0\,R(t+\tau) + \mathrm{Div}\left(\boldsymbol{K}_m(t) + \boldsymbol{K}_a(t,\tau)\right)\left(\vec{\nabla}T_m(t) + \vec{\nabla}T_a(t,\tau)\right). \tag{3-14}$$

The heat equation for T_m is obtained by the integration of (3-14) over one period

$$\rho_0 c \frac{\partial T_m}{\partial t} = \rho_0 R_m + \mathrm{Div}\left(\boldsymbol{K}_m(t)\vec{\nabla}T_m(t)\right)$$

$$+ \mathrm{Div}\underbrace{\left(\frac{1}{P}\int_{-P/2}^{P/2}\boldsymbol{K}_m(t)\vec{\nabla}T_a(t,\tau)\mathrm{d}\tau\right)}_{=0} + \mathrm{Div}\underbrace{\left(\frac{1}{P}\int_{-P/2}^{P/2}\boldsymbol{K}_a(t,\tau)\vec{\nabla}T(t+\tau)\mathrm{d}\tau\right)}_{A(t)}. \tag{3-15}$$

The left-hand term is developed as

$$\frac{\partial T_m}{\partial t} = \frac{\partial\left(\frac{1}{P}\int_{-P/2}^{P/2}T(t+\tau)\mathrm{d}\tau\right)}{\partial t} = \frac{T(t+P/2) - T(t-P/2)}{P} = \frac{1}{P}\int_{-P/2}^{P/2}\frac{\partial T(t+\tau)}{\partial \tau}\mathrm{d}\tau. \tag{3-16}$$

As regards the boundary conditions, the same considerations lead to

$$-\boldsymbol{K}_m(t)\vec{\nabla}T_m(t)\,\mathrm{d}\vec{A} = H_m(t)\mathrm{d}A\left(T_m(t) - T_\infty\right) + \underbrace{\frac{1}{P}\int_{-P/2}^{P/2}\boldsymbol{K}_m(t)\vec{\nabla}T_a(t,\tau)\mathrm{d}A\,\mathrm{d}\tau}_{=0}$$

$$+ \underbrace{\frac{1}{P}\int_{-P/2}^{P/2}\left(H_a(t,\tau)T_a(t,\tau)\mathrm{d}A + \boldsymbol{K}_a(t,\tau)\vec{\nabla}T(t+\tau).\mathrm{d}\vec{A}\right)\mathrm{d}\tau}_{B(t)}. \tag{3-17}$$

In the heat build-up problem of a rubber component, $A(t)$ and $B(t)$ are negligible. This approximation is justified by one assumption linking the two characteristic times: the mechanical characteristic time t_{mech} is small compared to the thermal diffusion time $t_{therm} = \rho c L^2/k$ where L is the characteristic dimension.

Note that for rubber the thermal diffusivity is of the order of $k/\rho c \simeq 1.10^{-7}$ (Table 1). Typical mechanical time for heat build-up problem ranges between $0.1\,\mathrm{s}$ and $1\,\mathrm{s}$. The characteristic thermal length corresponding to this mechanical time is then $\sqrt{kT_{\mathrm{meca}}/(\rho c)} \simeq 0.3\,\mathrm{mm} \ll L$. This means that a variation in the thermal problem during a mechanical cycle is limited at this scale. The conclusion to be drawn is that at the macroscopic scale (of the order of one millimeter), the temperature oscillation due

Parameter	Value
Heat generation	2×10^{-8} J m^{-3}
Frequency	10 Hz
Thermal conductivity	0.23 W m^{-1} K^{-1}
Convection coefficient	30 W m^{-2} K^{-1}
Temperature	23 °C
Density	1.1
Heat capacity	1.45×10^3 J Kg^{-1} K^{-1}(23 °C)
Thermal diffusivity	1.44×10^{-7} m^{2}s^{-1}

Table 1. Parameters used for the numerical simulation.

to the cyclic changes in the boundary conditions and in the conductivity is small compared to the mean temperature: $T_a \ll T_m$. The regularity of the heat equation leads to $\vec{\nabla} T_a \ll \vec{\nabla} T_m$. These two conditions imply that at the macroscopic scale, the terms $A(t)$ and $B(t)$ are negligible. The mean thermal problem is then properly defined on the reference configuration.

This approximation finally leads to this simplification of the heat equation

$$\rho_0 c \frac{\partial T_m}{\partial t} = \rho_0 R_m + \text{Div } \boldsymbol{K}_m(T_m)\vec{\nabla} T_m \tag{3-18}$$

with the boundary conditions

$$-\boldsymbol{K}_m(T_m)\vec{\nabla} T_m \cdot \vec{\text{d}A} = H_m \text{d}A(T_m - T_\infty), \tag{3-19}$$

where the heat flux is defined by $\vec{Q}_m = -\boldsymbol{K}_m(T_m)\vec{\nabla} T_m$.

This problem is defined on the reference configuration and the thermal parameters are constant. The thermal conductivity tensor is anisotropic and accounts for the mean path of the heat flux in the component over a cycle. The mean convection coefficient represents the mean of the exchange with ambient air. The resolution of this problem is also faster than the resolution of the coupled one, because the mechanical time does not appear anymore.

4. Numerical simulation

The simplified heat equation has been tested through FE modeling. The modeled specimen is a 5 mm radius cylinder. A sinusoidal deformation is imposed between stretches $\lambda = 1$ and $\lambda = 2$ (Figure 3). The heat source is homogeneous and temperature-independent. For the sake of simplicity, the thermal conductivity and the convection coefficient are supposed to be Eulerian constants. The value of the parameters are given in Table 1 and are typical for a carbon-black-filled rubber. In this example, the constitutive law has no importance. Four calculations have been undertaken:

- Fully-coupled calculation (50 increments per cycle).
- Uncoupled calculation on the maximal deformed geometry.
- Uncoupled calculation on the undeformed geometry.
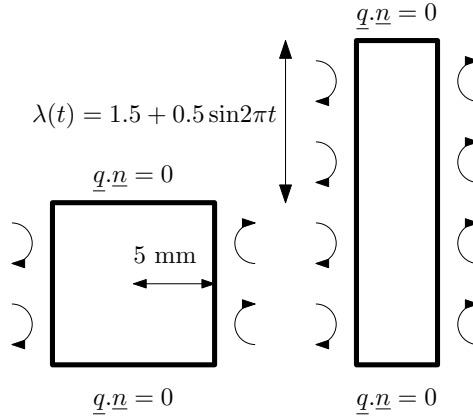- Uncoupled calculation with the simplified heat equation.

Figure 3. Undeformed and maximal deformed geometry of the 5 mm radius cylinder.

The results are presented in Figure 4. Whereas the uncoupled calculation on a fixed geometry implies an error of about 25% on the temperature compared to the fully-coupled solution, the error between the approximated solution and the coupled one is less than one percent and the two solutions can be considered to be equivalent. The time of calculation for the simplified problem is thus about three orders of magnitude smaller than for the fully-coupled problem for this simple specimen.
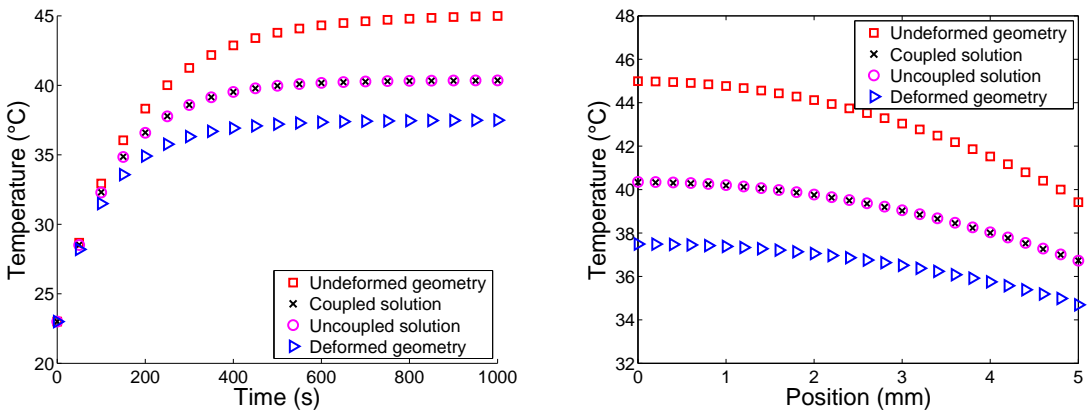

Figure 4. Left: Maximal temperature versus time for the four calculations. Right: Temperature versus radial position.

5. Conclusions

The heat build-up temperature has been defined as the actual temperature minus the temperature change due to thermoelastic effects. This heat build-up in rubber leads to a coupled thermomechanical problem. Due to the difference between the mechanical and thermal characteristic times, the quickest way to numerically solve this problem is to use a cyclic algorithm uncoupling the mechanical and the thermal problems. Unfortunately, the uncoupling approach does not account for the changes in geometry during the mechanical loading since the thermal problem is supposed to be solved on a fixed geometry. Under

the cyclic assumption, an approximation of the heat equation has been developed and allows the use of this algorithm at finite strains. This approximation is based on the integration of the heat equation and the boundary conditions over one cycle. These equations are written in the reference configuration so that the definition domain remains constant during one cycle. This approximation permits the use of the powerful uncoupling algorithm at finite strains: a numerical simulation illustrates the usefulness of this method.

Acknowledgements

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