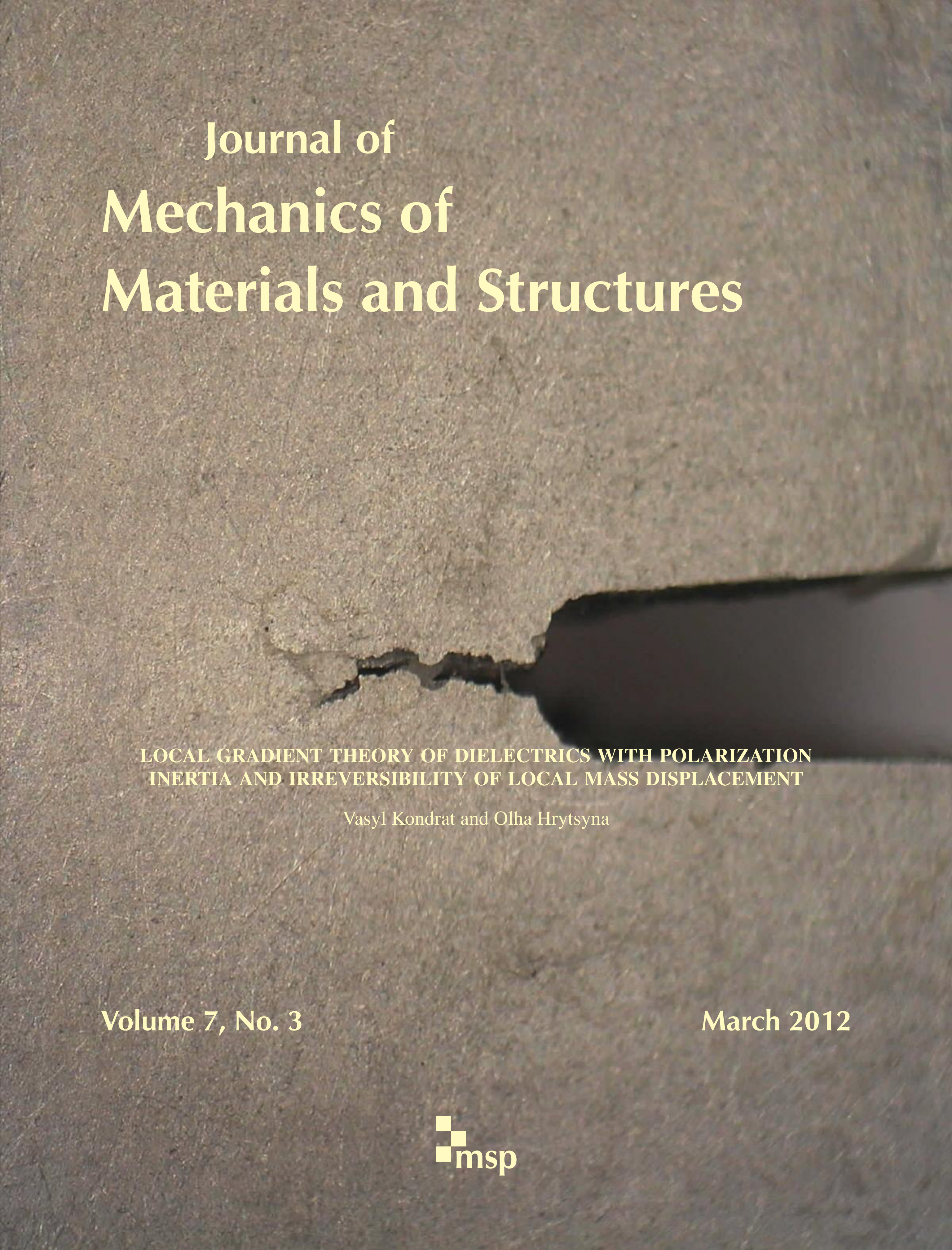

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LOCAL GRADIENT THEORY OF DIELECTRICS WITH POLARIZATION INERTIA AND IRREVERSIBILITY OF LOCAL MASS DISPLACEMENT

VASYL KONDRAT AND OLHA HRYTSYNA

A complete system of equations of the local gradient theory of electromagnetothermomechanics of polarized nonferromagnetic isotropic solids is obtained with regard to polarization inertia and the irreversibility of the processes of local mass displacement and polarization. It is shown that in this case the constitutive relations for specific vectors of the local mass displacement and polarization are rheological and contain time derivatives of the first and higher orders. A corresponding key system of equations for the isothermal approximation is obtained. This system is also written relatively to scalar and vector potentials of the displacement vector, vectors of the electromagnetic field, and a reduced energy measure μ'_π of the effect of the mass displacement on the internal energy. The Lorentz gauge is generalized in such a way that equations for the vector potential of the electromagnetic field and for the generalized scalar potential are not coupled and have similar structures. The effect of polarization inertia and the above-mentioned irreversibility of processes on the interaction of the fields is analyzed.

1. Introduction

In recent decades in the scientific literature there has been considerable interest in developing nonlocal theories of physical and mechanical processes in condensed matter. First of all, it is related to the need to describe some of the observed effects [Mead 1961; 1962; Ma and Cross 2003; Majdoub et al. 2009] and to the intensive introduction of composite materials in various technologies, including nanomaterials, [Tauchert and Guzelsu 1972; Buryachenko and Pagano 2003; Kuno 2004; Sharma et al. 2007; Majdoub 2010] where nonlocal effects are of crucial importance. These investigations also urged the development of the principles and methods of nonlocal thermodynamics [Ván 2003; Dolfin et al. 2004; Cimmelli and Ván 2005], thermomechanics and theories of heat conduction [Papenfuss and Forest 2006; Forest and Amestoy 2008], wave theory [Erofeyev 2003; Yerofeyev and Sheshenina 2005; Papargyri-Beskou et al. 2009], etc.

Nonlocal theories of deformable dielectrics are constructed by defining the functional constitutive equations of spatial type (strongly nonlocal theories) or by means of an expansion of the space of state parameter by gradients of certain physical quantities (weakly nonlocal theories) [Maugin 1988; Yang 2006; Kondrat and Hrytsyna 2009a]. In the scientific literature theories of both the first (see for example [Eringen 1984]) and second type are well known, the latter taking into account the dependence of the body state on the strain gradients [Tagantsev 1986; Majdoub et al. 2008; 2009; Majdoub 2010], the polarization gradient [Mindlin 1972], or the electric field gradients [Kafadar 1971; Yang and Yang 2004] corresponding to internal variables and their gradients [Ciancio 1989; Dolfin et al. 2004]. Another

Keywords: coupled electromagnetothermomechanical processes, local displacement of mass, dielectrics, nonlocal theory, irreversibility, inertia of polarization.

approach to the construction of the nonlocal theory of dielectrics, proposed in [Burak et al. 2008; Kondrat and Hrytsyna 2008a], is based on considering the *process of local mass displacement*. Burak [1987] was the first to pay attention to this process. He assumed that a mass flux has a component $\partial \mathbf{\Pi}_m/\partial t$ of nonconvective and nondiffusive nature. Burak related this flux to a process referred to as the process of local mass displacement and the vector $\mathbf{\Pi}_m$ was named the vector of this displacement [Burak 1987]. Later it was shown that the equations of the local gradient electromagnetothermomechanics of polarized nonferromagnetic solids describe surface inhomogeneity in the mechanical and electromagnetic fields [Burak et al. 2008], a high-frequency dispersion of elastic waves [Kondrat and Hrytsyna 2009b], and the Mead anomaly [Chapla et al. 2009]. It should be noted that the relations of the theory obtained in [Burak et al. 2008] are based on the assumption of reversibility of the polarization process; local mass displacement and polarization inertia is not taken into account. However, such an approximation may be insufficient and unacceptable for the study of transitional processes of the formation of near-surface inhomogeneities and the perturbation of electromagnetomechanical processes by shock loads, as well as for the description of acoustic and electromagnetic emission caused by the formation of surfaces, etc.

In this article a complete system of equations of the local gradient electromagnetothermomechanics of polarized nonferromagnetic solids is obtained taking into account the inertia of polarization and the irreversibility the of processes of polarization and local mass displacement. In order to consider the irreversibility of these processes we used the approach proposed in [Hrytsyna and Kondrat 2007; Kondrat and Hrytsyna 2008b]. Moreover, in the total energy balance equation we took into account the kinetic energy of polarization [Maugin 1988] that enables us to describe the inertia of polarization. The key system of equations is obtained in isothermal approximation for isotropic solids. This system is also written relative to the scalar and vector potentials of the displacement vector and the vectors of the electromagnetic field. In order to arrive at the potential description, the Lorentz gauge had to be generalized. Based on this, the interaction of the electromagnetic processes, deformation, and local mass displacement is discussed.

2. The balance equations

We consider an isotropic thermoelastic polarized nonferromagnetic body that occupies the domain (V) of Euclidean space and is bounded by the smooth surface (Σ) with unit exterior normal $\boldsymbol{n}$. The body is subjected to the action of an external load, which induces the mechanical, thermal, and electromagnetic processes, and the process of local displacement of mass. The polarization inertia as well as the irreversibility of processes of polarization and local mass displacement are taken into account.

Taking into account the process of local displacement of mass we represent the mass flux $\boldsymbol{J}_{m*}$ as the sum of the convective term $\boldsymbol{J}_{mc} = \rho \boldsymbol{v}_*$ and the term $\boldsymbol{J}_{ms} = \partial \mathbf{\Pi}_m/\partial t$, which is caused by structural changes of the fixed body element, namely $\boldsymbol{J}_{m*} = \boldsymbol{J}_{mc} + \boldsymbol{J}_{ms}$ [Burak et al. 2008]. Here $\boldsymbol{v}_*$ is a velocity of the convective displacement of the fixed body element and ρ is the mass density.

Let us define the vector of the velocity of the continuum center of mass $\boldsymbol{v}$ by the relation

$$\boldsymbol{v} = \boldsymbol{v}_* + \frac{1}{\rho}\frac{\partial \mathbf{\Pi}_m}{\partial t}.$$

Then, the equation of mass balance takes the standard form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0. \tag{2-1}$$

Let us introduce the quantity $\rho_{m\pi}$, which has dimension of mass density. We assume that, for an arbitrary body of finite size that occupies the domain (V), the vector of local displacement of mass and the density of the induced mass are such that [Burak et al. 2008]

$$\int_{(V)} \mathbf{\Pi}_m \, dV = \int_{(V)} \rho_{m\pi} \boldsymbol{r} \, dV. \tag{2-2}$$

Here $\boldsymbol{r}$ is the position vector. By analogy with an induced electric charge [Landau and Lifshitz 1982], we refer to the quantity $\rho_{m\pi}$ as a density of the induced mass [Burak et al. 2008].

The consequence of relation (2-2) is [Kondrat and Hrytsyna 2008a]

$$\rho_{m\pi} = -\nabla \cdot \mathbf{\Pi}_m. \tag{2-3}$$

By differentiating (2-3) with respect to time and taking into account that $\partial \mathbf{\Pi}_m / \partial t = \boldsymbol{J}_{ms}$, we obtain the equation

$$\frac{\partial \rho_{m\pi}}{\partial t} + \nabla \cdot \boldsymbol{J}_{ms} = 0,$$

which can be interpreted as the balance equation of induced mass.

We also write the entropy balance equation, which is of the form [de Groot and Mazur 1962]

$$\rho T \frac{ds}{dt} = -\nabla \cdot \boldsymbol{J}_q + \frac{1}{T} \boldsymbol{J}_q \cdot \nabla T + T \sigma_s + \rho \Re, \tag{2-4}$$

where s is the specific entropy, $\boldsymbol{J}_q$ is the density of heat flux, σ_s is the entropy production, T is the temperature, $\Re$ denotes the distributed thermal sources, and $d \ldots / dt = \partial \ldots / \partial t + \boldsymbol{v} \cdot \nabla \ldots$.

From Maxwell's equations,

$$\nabla \cdot \boldsymbol{B} = 0, \quad \nabla \cdot \boldsymbol{D} = \rho_e, \quad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \quad \nabla \times \boldsymbol{H} = \boldsymbol{J}_e + \frac{\partial \boldsymbol{D}}{\partial t}, \tag{2-5}$$

we obtain the electromagnetic field energy balance equation:

$$\frac{\partial U_e}{\partial t} + \nabla \cdot \boldsymbol{S}_e + \boldsymbol{J}_{e*} \cdot \boldsymbol{E}_* + \left[\rho_e \boldsymbol{E}_* + \left(\boldsymbol{J}_{e*} + \frac{\partial (\rho \boldsymbol{p})}{\partial t} \right) \times \boldsymbol{B} + \rho (\nabla \boldsymbol{E}_*) \cdot \boldsymbol{p} \right] \cdot \boldsymbol{v} \\ + \rho \boldsymbol{E}_* \cdot \frac{d\boldsymbol{p}}{dt} - \nabla \cdot [\rho (\boldsymbol{E}_* \cdot \boldsymbol{p}) \hat{\boldsymbol{I}} \cdot \boldsymbol{v}] = 0. \tag{2-6}$$

Here $U_e = (\varepsilon_0 \boldsymbol{E}^2 + \mu_0 \boldsymbol{H}^2)/2$ is the energy density of the electromagnetic field and $\boldsymbol{S}_e = \boldsymbol{E} \times \boldsymbol{H}$ is the flux density of its energy, $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic fields, $\boldsymbol{D}$ and $\boldsymbol{B}$ are the electric and magnetic inductions, $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \rho \boldsymbol{p}$, $\boldsymbol{p}$ is the specific vector of polarization, $\boldsymbol{J}_e$ is the density of electric current (the convection and conduction currents), ρ_e is the density of free electric charge, $\boldsymbol{E}_* = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}$, $\boldsymbol{J}_{e*} = \boldsymbol{J}_e - \rho_e \boldsymbol{v}$, ε_0 and μ_0 are the electric and magnetic constants, and $\hat{\boldsymbol{I}}$ is the unit tensor. For nonferromagnetic media $\boldsymbol{B} = \mu_0 \boldsymbol{H}$.

We assume that the total energy of the system "solid-electromagnetic field" is the sum of internal energy ρu, kinetic energy of mass center $\rho \boldsymbol{v}^2/2$, energy of the electromagnetic field U_e, and polarization kinetic energy $\frac{1}{2} \rho d_E (d\boldsymbol{p}/dt)^2$. Here d_E is the scalar related to the inertia of the polarization process [Maugin 1988]. The total energy of the system is changed due to the convective transport of energy through the surface, the flux of energy related to work $\hat{\boldsymbol{\sigma}} \cdot \boldsymbol{v}$ of internal forces, the heat flux $\boldsymbol{J}_q$, the electromagnetic energy flux $\boldsymbol{S}_e$, the flux of energy $\mu \boldsymbol{J}_m$ connected to the mass transport relative to the center of mass of the body, and the flux of energy $\mu_\pi(\partial \boldsymbol{\Pi}_m/\partial t)$ related to structural changes, as well as the action of mass forces $\boldsymbol{F}$ and distributed thermal sources $\Re$. Thus, the total energy balance equation in integral form looks like

$$\frac{d}{dt}\int_{(V)}\left(\rho u+U_e+\tfrac{1}{2}\rho \boldsymbol{v}^2+\tfrac{1}{2}\rho d_E\left(\frac{d\boldsymbol{p}}{dt}\right)^2\right)dV= \tag{2-7}$$
$$-\oint_{(\Sigma)}\left[\rho\left(u+\frac{1}{2}\boldsymbol{v}^2+\frac{1}{2}d_E\left(\frac{d\boldsymbol{p}}{dt}\right)^2\right)\boldsymbol{v}-\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{v}+\boldsymbol{S}_e+\boldsymbol{J}_q+\mu\boldsymbol{J}_m+\mu_\pi\frac{\partial\boldsymbol{\Pi}_m}{\partial t}\right]\cdot\boldsymbol{n}\,d\Sigma+\int_{(V)}(\rho\boldsymbol{F}\cdot\boldsymbol{v}+\rho\Re)\,dV.$$

Here $\hat{\boldsymbol{\sigma}}$ is the Cauchy stress tensor, μ is the chemical potential, μ_π is the energy measure of the effect of the mass displacement on the internal energy [Burak et al. 2008], and $\boldsymbol{J}_m=\rho(\boldsymbol{v}_*-\boldsymbol{v})$.

Taking into account the energy balance equation of the electromagnetic field (2-6), the balance equations of mass and entropy, (2-1) and (2-4), relation (2-3), and formula $\boldsymbol{J}_m=-\partial\boldsymbol{\Pi}_m/\partial t$ [Burak et al. 2008], as well as introducing the specific quantities $\boldsymbol{\pi}_m=\boldsymbol{\Pi}_m/\rho$ and $\rho_m=\rho_{m\pi}/\rho$, we finally obtain from (2-7) the following balance equation of internal energy in the local form:

$$\rho\frac{du}{dt}=\rho T\frac{ds}{dt}+\hat{\boldsymbol{\sigma}}_*:(\nabla\otimes\boldsymbol{v})+\rho\boldsymbol{E}_*\cdot\frac{d\boldsymbol{p}}{dt}+\rho\mu'_\pi\frac{d\rho_m}{dt}-\rho\nabla\mu'_\pi\cdot\frac{d\boldsymbol{\pi}_m}{dt}-\rho d_E\frac{d^2\boldsymbol{p}}{dt^2}\cdot\frac{d\boldsymbol{p}}{dt}+$$
$$+\boldsymbol{J}_{e*}\cdot\boldsymbol{E}_*-\boldsymbol{J}_q\cdot\frac{\nabla T}{T}-T\sigma_s+\boldsymbol{v}\cdot\left(-\rho\frac{d\boldsymbol{v}}{dt}+\nabla\cdot\hat{\boldsymbol{\sigma}}_*+\rho\boldsymbol{F}_*+\boldsymbol{F}_e\right). \tag{2-8}$$

Here $\mu'_\pi=\mu_\pi-\mu$, $\otimes$ denotes the tensor product, and

$$\hat{\boldsymbol{\sigma}}_*=\hat{\boldsymbol{\sigma}}-\rho(\boldsymbol{E}_*\cdot\boldsymbol{p}-\rho_m\mu'_\pi-\boldsymbol{\pi}_m\cdot\nabla\mu'_\pi)\hat{\boldsymbol{I}},\qquad \boldsymbol{F}_*=\boldsymbol{F}+\rho_m\nabla\mu'_\pi-\boldsymbol{\pi}_m\cdot\nabla\nabla\mu'_\pi,$$
$$\boldsymbol{F}_e=\rho_e\boldsymbol{E}_*+\left(\boldsymbol{J}_{e*}+\frac{\partial(\rho\boldsymbol{p})}{\partial t}\right)\times\boldsymbol{B}+\rho(\nabla\boldsymbol{E}_*)\cdot\boldsymbol{p}.$$

To take into account the irreversibility of the processes of local mass displacement and polarization we represent the vectors $\boldsymbol{E}_*$ and $\nabla\mu'_\pi$ as sums of a reversible component, $\boldsymbol{E}^r_*$ or $(\nabla\mu'_\pi)^r$, and an irreversible in, $\boldsymbol{E}^i_*$ or $(\nabla\mu'_\pi)^i$ [Hrytsyna and Kondrat 2007; Kondrat and Hrytsyna 2008b]:

$$\boldsymbol{E}_*=\boldsymbol{E}^r_*+\boldsymbol{E}^i_*,\qquad \nabla\mu'_\pi=(\nabla\mu'_\pi)^r+(\nabla\mu'_\pi)^i. \tag{2-9}$$

Taking into account (2-9) we rewrite (2-8) as follows:

$$\rho\frac{du}{dt}=\rho T\frac{ds}{dt}+\hat{\boldsymbol{\sigma}}_*:(\nabla\otimes\boldsymbol{v})+\rho\boldsymbol{E}^r_*\cdot\frac{d\boldsymbol{p}}{dt}+\rho\mu'_\pi\frac{d\rho_m}{dt}-\rho(\nabla\mu'_\pi)^r\cdot\frac{d\boldsymbol{\pi}_m}{dt}-\rho d_E\frac{d^2\boldsymbol{p}}{dt^2}\cdot\frac{d\boldsymbol{p}}{dt}+$$
$$+\boldsymbol{J}_{e*}\cdot\boldsymbol{E}_*-\boldsymbol{J}_q\cdot\frac{\nabla T}{T}+\rho\boldsymbol{E}^i_*\cdot\frac{d\boldsymbol{p}}{dt}-\rho(\nabla\mu'_\pi)^i\cdot\frac{d\boldsymbol{\pi}_m}{dt}-T\sigma_s+\boldsymbol{v}\cdot\left(-\rho\frac{d\boldsymbol{v}}{dt}+\nabla\cdot\hat{\boldsymbol{\sigma}}_*+\rho\boldsymbol{F}_*+\boldsymbol{F}_e\right).$$

Let us assume that the equilibrium part of the local electric field vector $\boldsymbol{E}^r_L$ is a state parameter [Maugin 1988]. This vector differs from term $\boldsymbol{E}^r_*$ of the macroscopic electric field $\boldsymbol{E}_*$. Therefore we rewrite the balance equation of internal energy as

$$\rho\frac{du}{dt}=\rho T\frac{ds}{dt}+\hat{\boldsymbol{\sigma}}_*:(\nabla\otimes\boldsymbol{v})+\rho\boldsymbol{E}_L^r\cdot\frac{d\boldsymbol{p}}{dt}+\rho\mu_\pi'\frac{d\rho_m}{dt}-\rho(\nabla\mu_\pi')^r\cdot\frac{d\boldsymbol{\pi}_m}{dt}+\rho\left(\boldsymbol{E}_*^r-\boldsymbol{E}_L^r-d_E\frac{d^2\boldsymbol{p}}{dt^2}\right)\cdot\frac{d\boldsymbol{p}}{dt}$$
$$+\boldsymbol{J}_{e*}\cdot\boldsymbol{E}_*-\boldsymbol{J}_q\cdot\frac{\nabla T}{T}+\rho\boldsymbol{E}_*^i\cdot\frac{d\boldsymbol{p}}{dt}-\rho(\nabla\mu_\pi')^i\cdot\frac{d\boldsymbol{\pi}_m}{dt}-T\sigma_s+\boldsymbol{v}\cdot\left(-\rho\frac{d\boldsymbol{v}}{dt}+\nabla\cdot\hat{\boldsymbol{\sigma}}_*+\rho\boldsymbol{F}_*+\boldsymbol{F}_e\right).\tag{2-10}$$

Using the Legendre transformation $f=u-Ts-\boldsymbol{E}_L^r\cdot\boldsymbol{p}+(\nabla\mu_\pi')^r\cdot\boldsymbol{\pi}_m$ we pass to a new thermodynamic function f, which is interpreted as a generalized Helmholtz free energy. Then, from the balance equation of internal energy (2-10) we obtain

$$\rho\frac{df}{dt}=-\rho s\frac{dT}{dt}+\hat{\boldsymbol{\sigma}}_*:(\nabla\otimes\boldsymbol{v})-\rho\boldsymbol{p}\cdot\frac{d\boldsymbol{E}_L^r}{dt}+\rho\mu_\pi'\frac{d\rho_m}{dt}+\rho\boldsymbol{\pi}_m\cdot\frac{d(\nabla\mu_\pi')^r}{dt}+\rho\left(\boldsymbol{E}_*^r-\boldsymbol{E}_L^r-d_E\frac{d^2\boldsymbol{p}}{dt^2}\right)\cdot\frac{d\boldsymbol{p}}{dt}$$
$$+\boldsymbol{J}_{e*}\cdot\boldsymbol{E}_*-\boldsymbol{J}_q\cdot\frac{\nabla T}{T}+\rho\boldsymbol{E}_*^i\cdot\frac{d\boldsymbol{p}}{dt}-\rho(\nabla\mu_\pi')^i\cdot\frac{d\boldsymbol{\pi}_m}{dt}-T\sigma_s+\boldsymbol{v}\cdot\left(-\rho\frac{d\boldsymbol{v}}{dt}+\nabla\cdot\hat{\boldsymbol{\sigma}}_*+\rho\boldsymbol{F}_*+\boldsymbol{F}_e\right).\tag{2-11}$$

The balance equation of free energy, (2-10), should be invariant relative to spatial translation, namely, it should not change if $\boldsymbol{v}\to\boldsymbol{v}+\boldsymbol{a}$, where $\boldsymbol{a}$ is a constant vector. As a consequence, from (2-11) we get

$$\rho\frac{d\boldsymbol{v}}{dt}=\nabla\cdot\hat{\boldsymbol{\sigma}}_*+\boldsymbol{F}_e+\rho\boldsymbol{F}_*\quad\text{for all }\ \boldsymbol{r}\in(V),\tag{2-12}$$

$$\rho\frac{df}{dt}=-\rho s\frac{dT}{dt}+\hat{\boldsymbol{\sigma}}_*:(\nabla\otimes\boldsymbol{v})-\rho\boldsymbol{p}\cdot\frac{d\boldsymbol{E}_L^r}{dt}+\rho\mu_\pi'\frac{d\rho_m}{dt}+\rho\boldsymbol{\pi}_m\cdot\frac{d(\nabla\mu_\pi')^r}{dt}$$
$$+\rho\left(\boldsymbol{E}_*^r-\boldsymbol{E}_L^r-d_E\frac{d^2\boldsymbol{p}}{dt^2}\right)\cdot\frac{d\boldsymbol{p}}{dt}+\boldsymbol{J}_{e*}\cdot\boldsymbol{E}_*-\boldsymbol{J}_q\cdot\frac{\nabla T}{T}+\rho\boldsymbol{E}_*^i\cdot\frac{d\boldsymbol{p}}{dt}-\rho(\nabla\mu_\pi')^i\cdot\frac{d\boldsymbol{\pi}_m}{dt}-T\sigma_s.\tag{2-13}$$

Relation (2-12) is the equation of motion. We see that the redefinition of stress tensor $\hat{\boldsymbol{\sigma}}_*$ and the emergence of additional mass force $\boldsymbol{F}_*$ are the result of the account of the process of the local mass displacement. Note also that the mass $\boldsymbol{F}_*$ and ponderomotive $\boldsymbol{F}_e$ forces have similar structure.

Equation (2-13) should remain unchanged if the body rotates with constant angular velocity $\boldsymbol{\Omega}$. In this case $\boldsymbol{v}\to\boldsymbol{v}+\boldsymbol{\Omega}\times\boldsymbol{r}$. As a consequence, we obtain that $\hat{\boldsymbol{\sigma}}_*$ is the symmetric tensor.

Let us represent the quantity $\nabla\otimes\boldsymbol{v}$ in the form $\nabla\otimes\boldsymbol{v}=d\hat{e}/dt+d\hat{\boldsymbol{\omega}}/dt$. Here $\boldsymbol{v}=d\boldsymbol{u}/dt$, $\boldsymbol{u}$ is the displacement vector, $\hat{e}$ is the symmetric strain tensor, and $\hat{\boldsymbol{\omega}}$ is the antisymmetric rotation tensor. These tensors are related to a displacement vector $\boldsymbol{u}$ by:

$$\hat{\boldsymbol{e}}=\tfrac{1}{2}[\nabla\otimes\boldsymbol{u}+(\nabla\otimes\boldsymbol{u})^T],\quad \hat{\boldsymbol{\omega}}=\tfrac{1}{2}[\nabla\otimes\boldsymbol{u}-(\nabla\otimes\boldsymbol{u})^T].\tag{2-14}$$

Since the convolution of the symmetric and antisymmetric tensors is equal to zero, then $\hat{\boldsymbol{\sigma}}_*:(d\hat{\boldsymbol{\omega}}/dt)=0$. Thus, the free energy balance equation (2-13) takes the form

$$\rho\frac{df}{dt}=-\rho s\frac{dT}{dt}+\hat{\boldsymbol{\sigma}}_*:\frac{d\hat{\boldsymbol{e}}}{dt}-\rho\boldsymbol{p}\cdot\frac{d\boldsymbol{E}_L^r}{dt}+\rho\mu_\pi'\frac{d\rho_m}{dt}+\rho\boldsymbol{\pi}_m\cdot\frac{d(\nabla\mu_\pi')^r}{dt}$$
$$+\rho\left(\boldsymbol{E}_*^r-\boldsymbol{E}_L^r-d_E\frac{d^2\boldsymbol{p}}{dt^2}\right)\cdot\frac{d\boldsymbol{p}}{dt}+\boldsymbol{J}_{e*}\cdot\boldsymbol{E}_*-\boldsymbol{J}_q\cdot\frac{\nabla T}{T}+\rho\boldsymbol{E}_*^i\cdot\frac{d\boldsymbol{p}}{dt}-\rho(\nabla\mu_\pi')^i\cdot\frac{d\boldsymbol{\pi}_m}{dt}-T\sigma_s.\tag{2-15}$$

Since the summands in the second line of (2-15) do not depend on the velocities

$$\frac{dT}{dt},\quad \frac{d\hat{\boldsymbol{e}}}{dt},\quad \frac{d\boldsymbol{E}_L^r}{dt},\quad \frac{d\rho_m}{dt},\quad \frac{d(\nabla\mu'_\pi)^r}{dt},$$

we get the generalized Gibbs equation

$$df = -s\,dT + \rho^{-1}\hat{\boldsymbol{\sigma}}_* : d\hat{\boldsymbol{e}} - \boldsymbol{p}\cdot d\boldsymbol{E}_L^r + \mu'_\pi\, d\rho_m + \boldsymbol{\pi}_m\cdot d(\nabla\mu'_\pi)^r, \tag{2-16}$$

the expression for the entropy production

$$\sigma_s = \boldsymbol{J}_{e*}\cdot\frac{\boldsymbol{E}_*}{T} - \boldsymbol{J}_q\cdot\frac{\nabla T}{T^2} + \rho\frac{d\boldsymbol{p}}{dt}\cdot\frac{\boldsymbol{E}_*^i}{T} - \rho\frac{d\boldsymbol{\pi}_m}{dt}\cdot\frac{(\nabla\mu'_\pi)^i}{T}, \tag{2-17}$$

and the relation

$$\boldsymbol{E}_*^r - \boldsymbol{E}_L^r = d_E\frac{d^2\boldsymbol{p}}{dt^2}, \tag{2-18}$$

which is sometimes referred to as "the equation of intramolecular force balance" [Maugin 1988].

3. The constitutive relations

Since parameters T, ρ_m, $\boldsymbol{E}_L^r$, $(\nabla\mu'_\pi)^r$, and $\hat{\boldsymbol{e}}$ are independent, the Gibbs equation (2-18) yields the state equations

$$\begin{gathered} s = -\frac{\partial f}{\partial T}\bigg|_{\rho_m,(\nabla\mu'_\pi)^r,\boldsymbol{E}_L^r,\hat{\boldsymbol{e}},},\quad \hat{\boldsymbol{\sigma}}_* = \rho\frac{\partial f}{\partial\hat{\boldsymbol{e}}}\bigg|_{T,\rho_m,(\nabla\mu'_\pi)^r,\boldsymbol{E}_L^r},\quad \mu'_\pi = \frac{\partial f}{\partial\rho_m}\bigg|_{T,(\nabla\mu'_\pi)^r,\boldsymbol{E}_L^r,\hat{\boldsymbol{e}}}, \\ \boldsymbol{p} = -\frac{\partial f}{\partial\boldsymbol{E}_L^r}\bigg|_{T,\rho_m,(\nabla\mu'_\pi)^r,\hat{\boldsymbol{e}}},\quad \boldsymbol{\pi}_m = \frac{\partial f}{\partial(\nabla\mu'_\pi)^r}\bigg|_{T,\rho_m,\boldsymbol{E}_L^r,\hat{\boldsymbol{e}}}. \end{gathered} \tag{3-1}$$

Let us decompose the free energy density f into a Taylor series in perturbations of the state parameters with respect to the original state of unlimited homogeneous medium with $\hat{\boldsymbol{e}} = 0$, $\hat{\boldsymbol{\sigma}}_* = 0$, $\boldsymbol{E}_L^r = 0$, $(\nabla\mu'_\pi)^r = 0$, $\boldsymbol{p} = 0$, $\boldsymbol{\pi}_m = 0$, $T = T_0$, $s = s_0$, $\rho = \rho_0$, $\rho_m = 0$, and $\mu'_\pi = \mu'_{\pi 0}$. For small perturbations, we retain quadratic terms in this decomposition which enables us to get the linear state equations. Therefore, the free energy density for isotropic material has the form

$$\begin{aligned} f = f_0 - s_0(T-T_0) + \mu'_{\pi 0}\rho_m - \frac{C_V}{2T_0}(T-T_0)^2 + \frac{1}{2\rho_0}\big(K-\tfrac{2}{3}G\big)I_1^2 + \frac{1}{\rho_0}GI_2 + \tfrac{1}{2}d_\rho\rho_m^2 - \frac{1}{\rho_0}K\alpha_t I_1(T-T_0) \\ - \frac{1}{\rho_0}K\alpha_\rho I_1\rho_m - \beta_{T\rho}\rho_m(T-T_0) - \frac{1}{2}\chi_m(\nabla\mu'_\pi)^r\cdot(\nabla\mu'_\pi)^r - \tfrac{1}{2}\chi_E\boldsymbol{E}_L^r\cdot\boldsymbol{E}_L^r + \chi_{Em}\boldsymbol{E}_L^r\cdot(\nabla\mu'_\pi)^r. \end{aligned}$$

Here $I_1 = \hat{\boldsymbol{e}} : \hat{\boldsymbol{I}} \equiv e$ and $I_2 = \hat{\boldsymbol{e}} : \hat{\boldsymbol{e}}$ are the first and second invariants of strain tensor, respectively, K is the modulus of volume elasticity at constant temperature and specific density of the induced mass, G is the shear modulus, α_t is the temperature coefficient of volume dilatation at uniform specific density of the induced mass, α_ρ is the coefficient of volume dilatation caused by the local displacement of mass at uniform temperature, C_V is the specific heat at constant deformation and specific density of the induced mass, χ_E is the dielectric susceptibility, $\beta_{T\rho}$ and d_ρ are the isothermal-isochoric coefficients of dependency of entropy and potential μ'_π on a specific density of the induced mass, and χ_m and χ_{Em} are the coefficients that characterize the local displacement of mass and body polarization due to the gradient of potential μ'_π, respectively.

If the potential f is known, the constitutive equations (3-1) take on the form

$$s = s_0 + \frac{C_V}{T_0}(T - T_0) + \frac{1}{\rho_0} K\alpha_t e + \beta_{T\rho}\rho_m, \quad \hat{\boldsymbol{\sigma}}_* = 2G\hat{\boldsymbol{e}} + \left\{\left(K - \tfrac{2}{3}G\right)e - K[\alpha_t(T - T_0) + \alpha_\rho \rho_m]\right\}\hat{\boldsymbol{I}},$$

$$\mu'_\pi = \mu'_{\pi 0} + d_\rho \rho_m - \beta_{T\rho}(T - T_0) - \frac{1}{\rho_0} K\alpha_\rho e, \tag{3-2}$$

$$\boldsymbol{p} = \chi_E \boldsymbol{E}^r_L - \chi_{Em}(\nabla\mu'_\pi)^r, \quad \boldsymbol{\pi}_m = -\chi_m(\nabla\mu'_\pi)^r + \chi_{Em}\boldsymbol{E}^r_L. \tag{3-3}$$

Taking into account (2-18) we rewrite the state equations (3-3) as follows:

$$\boldsymbol{p} = \chi_E\left(\boldsymbol{E}^r - d_E\frac{d^2\boldsymbol{p}}{dt^2}\right) - \chi_{Em}(\nabla\mu'_\pi)^r, \quad \boldsymbol{\pi}_m = -\chi_m(\nabla\mu'_\pi)^r + \chi_{Em}\left(\boldsymbol{E}^r - d_E\frac{d^2\boldsymbol{p}}{dt^2}\right). \tag{3-4}$$

We obtain kinetic relations based on (2-17) for entropy production. Let us assume that thermodynamic fluxes $\boldsymbol{j}_1 = \boldsymbol{J}_q$, $\boldsymbol{j}_2 = \boldsymbol{J}_{e*}$, $\boldsymbol{j}_3 = \rho(d\boldsymbol{p}/dt)$, and $\boldsymbol{j}_4 = \rho(d\boldsymbol{\pi}_m/dt)$ are linear functions of the thermodynamic forces $\boldsymbol{X}_1 = -\nabla T/T^2$, $\boldsymbol{X}_2 = \boldsymbol{E}_*/T$, $\boldsymbol{X}_3 = \boldsymbol{E}^i_*/T$, and $\boldsymbol{X}_4 = -(\nabla\mu'_\pi)^i/T$:

$$\boldsymbol{j}_i = \sum_{j=1}^{4} \mathscr{L}_{ij}\boldsymbol{X}_j, \quad (i = \overline{1,4}). \tag{3-5}$$

Here $\mathscr{L}_{ij}$ $(i, j = \overline{1,4})$ are constant kinetic coefficients. In general, the relations (3-5) are nonlinear. If we take into account formulas (2-9) and (3-4) and exclude irreversible terms $\boldsymbol{E}^i$ and $(\nabla\mu'_\pi)^i$ and reversible terms $\boldsymbol{E}^r$ and $(\nabla\mu'_\pi)^r$ of vectors $\boldsymbol{E}$ and $\nabla\mu'_\pi$, in the linearized approximation we obtain from (3-5) the following relations for the vectors of heat flux and electric current density:

$$\begin{aligned}\boldsymbol{J}_q &= -L^T_1\nabla T + L^E_1\boldsymbol{E} - L^\mu_1\nabla\mu'_\pi - L^d_1\frac{d^2\boldsymbol{p}}{dt^2} + L^p_1\boldsymbol{p} + L^\pi_1\boldsymbol{\pi}_m,\\ \boldsymbol{J}_e &= -L^T_2\nabla T + L^E_2\boldsymbol{E} - L^\mu_2\nabla\mu'_\pi - L^d_2\frac{d^2\boldsymbol{p}}{dt^2} + L^p_2\boldsymbol{p} + L^\pi_2\boldsymbol{\pi}_m,\end{aligned} \tag{3-6}$$

and the rheological constitutive relations for vectors of polarization and local mass displacement:

$$\begin{aligned}L^d_3\frac{d^2\boldsymbol{p}}{dt^2} + \rho_0\frac{d\boldsymbol{p}}{dt} - L^p_3\boldsymbol{p} - L^\pi_3\boldsymbol{\pi}_m &= -L^T_3\nabla T + L^E_3\boldsymbol{E} - L^\mu_3\nabla\mu'_\pi,\\ L^d_4\frac{d^2\boldsymbol{p}}{dt^2} + \rho_0\frac{d\boldsymbol{\pi}_m}{dt} - L^p_4\boldsymbol{p} - L^\pi_4\boldsymbol{\pi}_m &= -L^T_4\nabla T + L^E_4\boldsymbol{E} - L^\mu_4\nabla\mu'_\pi.\end{aligned} \tag{3-7}$$

Here

$$L^d_i = d_E\frac{\mathscr{L}_{i3}}{T_0},\ L^T_i = \frac{\mathscr{L}_{i1}}{T_0^2},\ L^E_i = \frac{\mathscr{L}_{i2}+\mathscr{L}_{i3}}{T_0},\ L^\mu_i = \frac{\mathscr{L}_{i4}}{T_0},\ L^\pi_i = \frac{\chi_{Em}\mathscr{L}_{i3} - \chi_E\mathscr{L}_{i4}}{T_0(\chi_E\chi_m - \chi^2_{Em})},\ L^p_i = \frac{\chi_{Em}\mathscr{L}_{i4} - \chi_m\mathscr{L}_{i3}}{T_0(\chi_E\chi_m - \chi^2_{Em})},$$

where $i = \overline{1,4}$. Note that summands $L^d_3(d^2\boldsymbol{p}/dt^2)$ and $L^d_4(d^2\boldsymbol{p}/dt^2)$ in the left hand parts of (3-7) appear due to polarization inertia being taken into account while the summands $\rho_0(d\boldsymbol{p}/dt)$, $\rho_0(d\boldsymbol{\pi}_m/dt)$, $L^\pi_3\boldsymbol{\pi}_m$, $L^p_4\boldsymbol{p}$, $L^T_3\nabla T$, and $L^T_4\nabla T$ appear due to considering the irreversibility of the processes of local mass displacement and polarization. According to (3-7), a body can be theoretically polarized not only in the electric field but also in a gradient of temperature as well as in a gradient of potential μ'_π. Therefore, the rheological relations (3-7) should describe both the surface polarization (the gradient of potential μ'_π may be significant in near-surface regions [Burak et al. 2008]) and the thermopolarization effect which consists in the linear response of the dielectric polarization to the temperature gradient.

Due to simple transformations, we can rewrite relations (3-7) as follows:

$$\boldsymbol{L}(\boldsymbol{p})=\boldsymbol{L}_{pT}(\nabla T)-\boldsymbol{L}_{pE}(\boldsymbol{E})+\boldsymbol{L}_{p\mu}(\nabla\mu'_\pi),\quad \boldsymbol{L}(\boldsymbol{\pi}_m)=\boldsymbol{L}_{\pi T}(\nabla T)-\boldsymbol{L}_{\pi E}(\boldsymbol{E})+\boldsymbol{L}_{\pi\mu}(\nabla\mu'_\pi). \quad (3\text{-}8)$$

Here we introduce the operators

$$\boldsymbol{L}=\frac{L_3^\pi}{\rho_0}\left(L_4^p-L_4^d\frac{d^2}{dt^2}\right)-\left(L_3^d\frac{d^2}{dt^2}+\rho_0\boldsymbol{L}_p\right)\boldsymbol{L}_\pi,\quad \boldsymbol{L}_\pi=\frac{d}{dt}-\frac{1}{\tau_\pi},\quad \boldsymbol{L}_p=\frac{d}{dt}-\frac{1}{\tau_p},$$

$$\boldsymbol{L}_{p\alpha}=L_3^\alpha\boldsymbol{L}_\pi+L_3^\pi\frac{L_4^\alpha}{\rho_0},\quad \boldsymbol{L}_{\pi\alpha}=\frac{1}{\rho_0}(L_3^\alpha\boldsymbol{L}_{p2}+L_4^\alpha\boldsymbol{L}_{p1}),\quad \boldsymbol{L}_{p1}=L_3^d\frac{d^2}{dt^2}+\rho_0\boldsymbol{L}_p,\quad \boldsymbol{L}_{p2}=L_4^p-L_4^d\frac{d^2}{dt^2},$$

where $\alpha=\{T,E,\mu\}$, $\tau_p=\rho_0/L_3^p$, and $\tau_\pi=\rho_0/L_4^\pi$.

The constitutive equations (3-2), (3-6), and (3-8), the linearized balance equations (2-1), (2-4), and (2-12), Maxwell's equations (2-5), the strain-displacement relation (2-14), and the balance equation of induced mass, which in linearized approximation looks like

$$\rho_m=-\nabla\cdot\boldsymbol{\pi}_m, \quad (3\text{-}9)$$

comprise a fundamental system of equations of linear augmented local gradient theory of deformable nonferromagnetic polarized isotropic solids in which account is taken of the polarization inertia and the irreversibility of both local mass displacement and polarization.

4. The key system of equations and a potential description

We shall study the ideal dielectrics for which $\rho_e=0$ and $\boldsymbol{J}_e=0$ and assume the isothermal approximation. We write the key equations in the linearized approximation for perturbations of the following functions: $\boldsymbol{u}$, $\boldsymbol{E}$, $\boldsymbol{B}$, and $\tilde{\mu}'_\pi=\mu'_\pi-\mu'_{\pi0}$. Thus, we get

$$\rho_0\frac{\partial^2\boldsymbol{u}}{\partial t^2}=\left(K+\frac{1}{3}G-\frac{K^2\alpha_\rho^2}{\rho_0 d_\rho}\right)\nabla(\nabla\cdot\boldsymbol{u})+G\Delta\boldsymbol{u}-K\frac{\alpha_\rho}{d_\rho}\nabla\tilde{\mu}'_\pi+\rho_0\boldsymbol{F}, \quad (4\text{-}1)$$

$$\boldsymbol{L}_{\pi\mu}(\Delta\tilde{\mu}'_\pi)+\frac{1}{d_\rho}\boldsymbol{L}(\tilde{\mu}'_\pi)=-\frac{K\alpha_\rho}{\rho_0 d_\rho}\boldsymbol{L}(\nabla\cdot\boldsymbol{u})+\boldsymbol{L}_{\pi E}(\nabla\cdot\boldsymbol{E}), \quad (4\text{-}2)$$

$$\nabla\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial t},\quad \nabla\cdot\boldsymbol{B}=0,\quad \boldsymbol{L}_\varepsilon(\nabla\cdot\boldsymbol{E})+\rho_0\boldsymbol{L}_{p\mu}(\Delta\tilde{\mu}'_\pi)=0,$$

$$\boldsymbol{L}(\nabla\times\boldsymbol{B})=\mu_0\left[\boldsymbol{L}_\varepsilon\left(\frac{\partial\boldsymbol{E}}{\partial t}\right)+\rho_0\boldsymbol{L}_{p\mu}\left(\frac{\partial(\nabla\tilde{\mu}'_\pi)}{\partial t}\right)\right]. \quad (4\text{-}3)$$

Here, $\boldsymbol{L}_\varepsilon=\varepsilon_0\boldsymbol{L}-\rho_0\boldsymbol{L}_{pE}$ and Δ is the Laplace operator. As may be seen in (4-3) the electrodynamics equations contain time derivatives of higher orders. The increase in the orders of these equations is due to accounting for the irreversibility of processes as well as the polarization inertia. Moreover, for the potential $\tilde{\mu}'_\pi$ we obtain an equation which includes dynamic terms (for comparison, see [Burak et al. 2008]).

We see that (4-1)–(4-3) are interrelated. Therefore, this theory accommodates an electromechanical interaction even for isotropic materials.

Let us represent the displacement $\boldsymbol{u}$ and mass force $\boldsymbol{F}$ as sums of their potential and vortex components:

$$\boldsymbol{u} = \nabla\phi_u + \nabla\times\boldsymbol{\psi}_u, \quad \boldsymbol{F} = \nabla\Phi + \nabla\times\boldsymbol{\Psi}, \quad \nabla\cdot\boldsymbol{\psi}_u = 0, \quad \nabla\cdot\boldsymbol{\Psi} = 0, \tag{4-4}$$

Similarly, we represent the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ in terms of the scalar (electrical) potential ϕ_e and the vector potential $\boldsymbol{A}$:

$$\boldsymbol{E} = -\nabla\phi_e - \frac{\partial\boldsymbol{A}}{\partial t}, \quad \boldsymbol{B} = \nabla\times\boldsymbol{A}. \tag{4-5}$$

With the relationship

$$\phi_{em} = \varepsilon^{-1}[\boldsymbol{L}_\varepsilon\phi_e - \rho_0\boldsymbol{L}_{p\mu}\tilde{\mu}'_\pi], \tag{4-6}$$

we introduce the generalized potential ϕ_{em}, where ε is the dielectric permittivity of the medium. By neglecting the inertia of polarization and its irreversibility we have: $\varepsilon = \varepsilon_0 + \rho_0\chi_E$.

If the Lorentz gauge condition is modified in such a way:

$$\boldsymbol{L}(\nabla\cdot\boldsymbol{A}) + \varepsilon\mu_0\frac{\partial\phi_{em}}{\partial t} = 0, \tag{4-7}$$

then the electrodynamics equations (4-3) are reduced to two unrelated similar differential relations for vector $\boldsymbol{A}$ and scalar ϕ_{em} potentials:

$$\boldsymbol{L}(\Delta\boldsymbol{A}) - \mu_0\boldsymbol{L}_\varepsilon\left(\frac{\partial^2\boldsymbol{A}}{\partial t^2}\right) = 0, \quad \boldsymbol{L}(\Delta\phi_{em}) - \mu_0\boldsymbol{L}_\varepsilon\left(\frac{\partial^2\phi_{em}}{\partial t^2}\right) = 0. \tag{4-8}$$

Note that the presence of operators $\boldsymbol{L}$ and $\boldsymbol{L}_{pE}$ in (4-8) leads to the dispersion of electromagnetic waves in an infinite medium. This dispersion was discussed in [Kondrat and Hrytsyna 2010] for the case of reversible processes of local mass displacement and polarization.

Substitution of (4-4) into (4-1) and (4-2) gives the equations:

$$G\Delta\boldsymbol{\psi}_u + \rho_0\boldsymbol{\Psi} - \rho_0\frac{\partial^2\boldsymbol{\psi}_u}{\partial t^2} = 0, \tag{4-9}$$

$$\left(K + \frac{4}{3}G - \frac{K^2\alpha_\rho^2}{\rho_0 d_\rho}\right)\Delta\phi_u + \rho_0\Phi - \rho_0\frac{\partial^2\phi_u}{\partial t^2} = K\frac{\alpha_\rho}{d_\rho}\tilde{\mu}'_\pi, \tag{4-10}$$

$$\boldsymbol{L}\left([\boldsymbol{L}_\varepsilon\boldsymbol{L}_{\pi\mu} + \rho_0\boldsymbol{L}_{\pi E}\boldsymbol{L}_{\rho\mu}](\Delta\tilde{\mu}'_\pi) + \frac{1}{d_\rho}\boldsymbol{L}_\varepsilon\boldsymbol{L}(\tilde{\mu}'_\pi) + \frac{K\alpha_\rho}{\rho_0 d_\rho}\boldsymbol{L}_\varepsilon\boldsymbol{L}(\Delta\varphi_u) - \varepsilon(1 - \boldsymbol{L}_{\pi E})(\Delta\varphi_{em})\right) = 0. \tag{4-11}$$

The system (4-8)–(4-11) can be solved consistently. First we find the potentials $\boldsymbol{A}$ and ϕ_{em} from the homogeneous equations (4-8). Then, in the next step, the functions $\tilde{\mu}'_\pi$ and ϕ_u can be found based on (4-10) and (4-11). If the potentials ϕ_{em} and $\tilde{\mu}'_\pi$ are found, then in order to find ϕ_e we use differential equation (4-6). To determine the vector potentials $\boldsymbol{\psi}_u$ and $\boldsymbol{A}$ homogeneous unrelated equations are obtained, which are also unrelated to the remaining equations of this system. However, to determine the potential $\boldsymbol{\psi}_u$ we use a relation identical to the ones obtained earlier in [Kondrat and Hrytsyna 2009b], whereas for the potential $\boldsymbol{A}$ we get the modified equation due to accounting for the polarization inertia. Formula (4-11), unlike its analog obtained in [Kondrat and Hrytsyna 2009b], contains the generalized potential ϕ_{em} and dynamical terms, caused by accounting for the polarization inertia and the irreversibility of both the local mass displacement and polarization. From (4-8)–(4-11) it follows that in linear approximation the local mass displacement is not associated with the change of body shape. This process is related to the change of the body volume (the processes of compression-tension) and the electric scalar potential. The local displacement of mass is the cause of perturbation of the electromagnetic field.

5. Conclusions

We obtained a complete system of equations of the local gradient theory of deformable nonferromagnetic polarized isotropic solids in which the polarization inertia and the irreversibility of local mass displacement and polarization are taken into account. With the assumption that the body state is defined by the vector of the local electric field, we obtain the so-called equation of intramolecular force balance as well as the corresponding constitutive equations. It is shown that by accounting for the inertia of polarization and the above-mentioned irreversibility for the specific vector of polarization and the specific vector local mass displacement we got the rheological constitutive relations which include time derivatives of first and higher orders. The key system of equations is obtained for an isothermal approximation. This system is also written down relative to the scalar ϕ_u and vector $\boldsymbol{\psi}_u$ potentials of the vector displacement, the potential $\tilde{\mu}'_\pi$, the vector potential $\boldsymbol{A}$, and the scalar generalized potential ϕ_{em}. Function ϕ_{em} is related to the scalar electric potential ϕ_e and the potential $\tilde{\mu}'_\pi$ by differential relation (4-6). The Lorentz gauge is generalized in such a way that equations for the vector potential of the electromagnetic field and for the generalized scalar potential are not interrelated and have similar structures. The effects of the mentioned irreversibility and polarization inertia on the interaction of the investigated fields is discussed.

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