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NUMERICAL AND EXPERIMENTAL INVESTIGATION OF THE DYNAMIC CHARACTERISTICS OF CABLE-SUPPORTED BARREL VAULT STRUCTURES

SUN GUO-JUN, CHEN ZHI-HUA AND RICHARD W. LONGMAN

The cable-supported barrel vault (CSBV) structure system is a new style hybrid spatial steel structure, based on beam string structures (or truss string structures) and cylindrical latticed shell structures. A numerical investigation of the dynamic characteristics of a CSBV structure is presented, and an experimental test was created to validate the model's ability to obtain good predictions of the dynamic behavior. In order to simulate the construction process of a CSBV structure, the numerical investigation progresses in three phases: the first phase models the barrel vault without struts and cables, the second includes the influence of the cables, and the third introduces the added mass of a roof. The first nine vibration modes were obtained. The cables and struts in the CSBV improve the seismic behavior. The experimental results validate the numerical model, allowing us to study the influence of the rise-span and sag-span ratios.

1. Introduction

The cable-supported barrel vault (CSBV) is a new type of space structure that introduces tensegrity into thick double-layer or multilayer barrel vaults [Chen et al. 2008], as shown in Figure 1. It is composed of a single-layer or thin double-layer barrel vault, with struts and cables. On the one hand, due to the action of struts and cables, the rigidity of the whole structure is improved, as is the out-of-plane stability of the structure. Therefore, producing large spans is facilitated. On the other hand, by adopting the single-layer or the thin double-layer barrel vault, less steel is used, reducing costs. The difficulty of construction is also reduced, and the horizontal arch thrust is effectively reduced by the prestressed cables, so the heavy burden on the lower structures supporting the barrel vault is substantially reduced.

At present, the static and dynamic performance of barrel vault structures has been studied [Dong and Yao 1994; Wang and Li 1999; Langbecker and Albermani 2001; He et al. 2004; Cao et al. 2009; Kumagai et al. 2009], but only the static performance has been addressed [Chen et al. 2010]. The dynamic response of a cable-supported structure under dynamic loads, such as wind, earthquakes, or traffic, is very complex and requires special study. The dynamic characteristics of cable-supported spherical shells have been studied experimentally in [Tatemichi et al. 1997; Chen et al. 2004; Zhang et al. 2007]. The structural response under dynamic loads not only depends on the load, but also on the dynamic characteristics of the structure. The natural frequency of vibration is extremely significant, as it influences directly the

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Keywords: cable-supported barrel vault, dynamic characteristics, natural frequency, rise-span ratio, sag-span ratio.

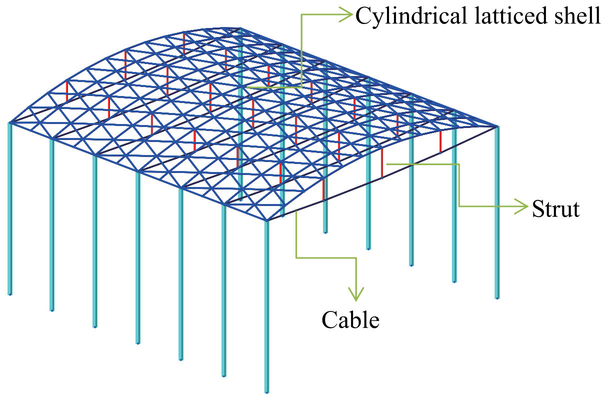

Figure 1. Cable-supported barrel vault (CSBV) structure.

dynamic response of the structure. Therefore, for CSBV structures, the natural frequencies of vibration and the mode shapes need further study. In this paper, a model of a CSBV structure was designed, and the natural frequencies of vibration and the mode shapes were found by numerical analysis followed by experiments on a test facility created for this purpose. The results of the numerical analysis and experiments are analyzed and compared. A parametric study of the dynamic characteristics of the CSBV structure was also conducted. From the numerical and experimental results, some conclusions are drawn for practical engineering applications.

2. Numerical model

For this paper, the cable-supported barrel vault (CSBV) structure shown in Figure 2 was designed. It has a span of 3.333 m, length of 3.587 m, rise of 0.3 m, and sag of 0.05 m. The upper barrel vault model is composed of three kinds of steel tube sections, $\varnothing 8 \times 1$, $\varnothing 10 \times 1$, and $\varnothing 12 \times 1$. The sections of all struts are steel tube $\varnothing 12 \times 1$. The cables are all steel bar $\varnothing 6$. The sections of the columns are all steel tube $\square 50 \times 2$ with a length of 2.1 m. All the steel tubes are Q235B and the steel bar's ultimate strength is 1860 MPa. The design of the cable-supported structure is different from other steel structures not only in that the sections are modified, but also because of the need to determine the cable force. Some

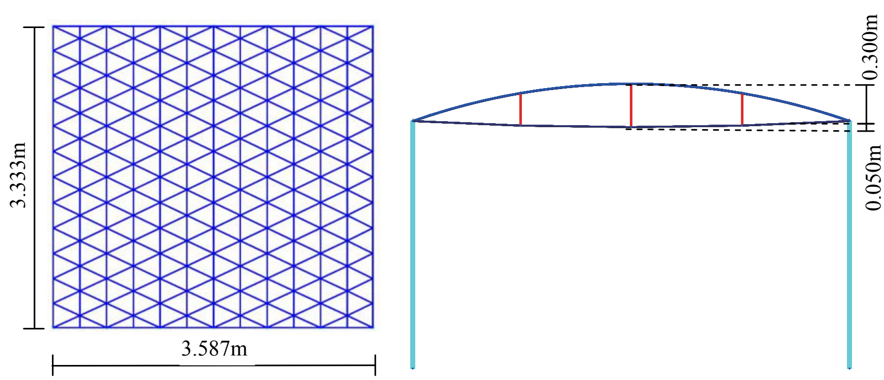

Figure 2. Elevation drawing and plane graph.

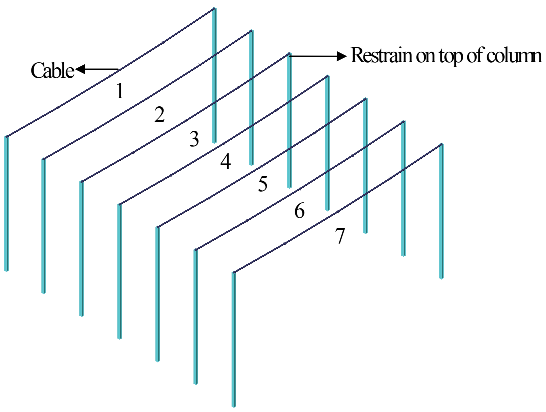

Figure 3. Cable numbering.

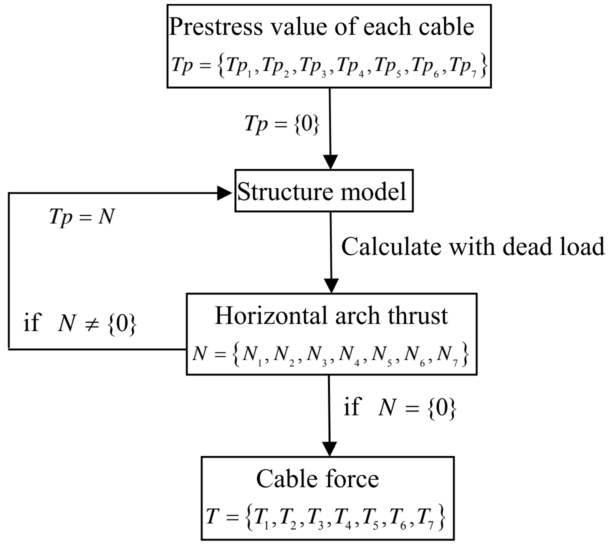

Figure 4. Iteration flow chart.

design principles and optimizations of cable-supported structures have been presented in the literature [Kawaguchi et al. 1999; Kang et al. 2003; Chen and Li 2005; Xue and Liu 2009].

In this paper, the cable prestress design principle for CSBV structures, in which the peripheral restriction is usually the spatial pin, is that the horizontal arch thrust is zero. Figure 3 labels the seven cables in the CSBV, and the cable prestress is set to zero, $Tp = \{Tp_1, Tp_2, Tp_3, Tp_4, Tp_5, Tp_6, Tp_7\} = \{0\}$. Being subjected to the dead weight of the structure, the horizontal arch thrusts can be calculated, and are denoted by $\boldsymbol{N} = \{N_1, N_2, N_3, N_4, N_5, N_6, N_7\}$. Then, the structure is reanalyzed; this time $\boldsymbol{N}$ is considered to be the cable force, and is applied on each cable correspondingly, and the horizontal arch thrust is recalculated. This process is repeated until the horizontal arch thrust is zero or close to zero, resulting in the target values of the cable prestress, $T = \{T_1, T_2, T_3, T_4, T_5, T_6, T_7\}$. It should be noted that the nonlinearity of the geometry of the structure must be considered in the calculation process above. The computation can be accomplished using general finite element analysis software such as ANSYS. The iteration calculation flow chart is shown in Figure 4. The iterative process for the research model (see Figure 2) converged within three iteration steps; see Figure 5.

3. Numerical analysis

Following the finite element software package ANSYS 12.0 user manual [ANSYS 2010], the element BEAM188 was adopted to simulate the steel members in the upper vault structure, while the struts and the cables were simulated by LINK8 and LINK10, respectively. The finite element model of the CSBV is shown in Figure 6. The subspace iteration method [Jung et al. 1999] for the dynamic characteristic was used. The research process can be divided into three phases: in the first phase the barrel vault without struts and cables is modeled; in the second, the CSBV structure is modeled; and in the third, the CSBV structure is modeled, with an added mass of 15 kg to simulate the case with a roof. The added mass distribution is shown in Figure 7. The difference in rigidity between the barrel vault and

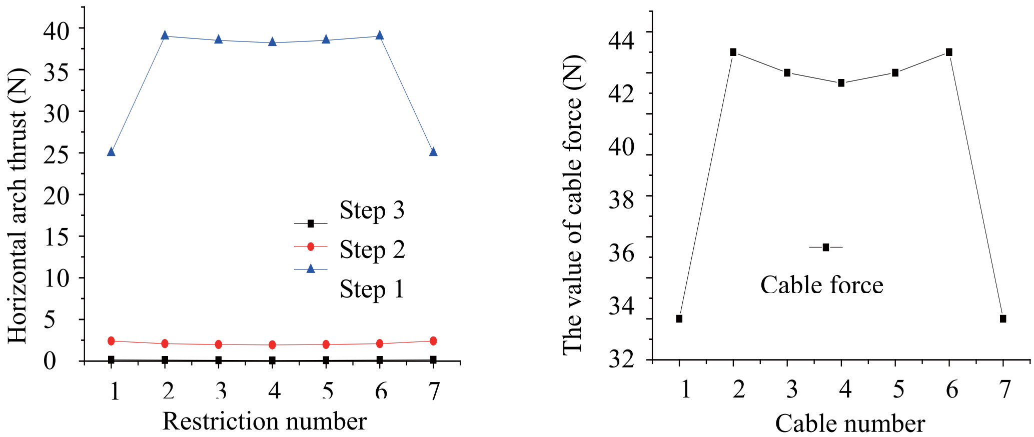

Figure 5. Values of cable forces: three iterations to convergence (left), and final values (right).

CSBV structures can be obtained from the first and second phases. The difference in rigidity between the construction and operational phases can be obtained from the second and third phases.

The first nine mode frequencies for each phase of the above numerical analysis are shown in Table 1 and the corresponding mode shapes for each phase are shown in Figures 8–10. Comparing the first and second phases, the frequencies of the CSBV are higher than those of the barrel vault structure, that is, the rigidity of the CSBV structure is greater than that of the barrel vault structure. Comparing the second

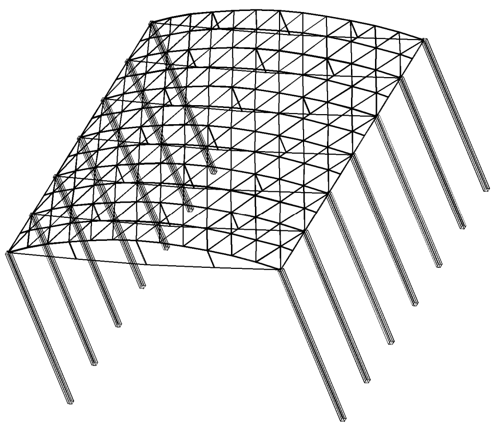

Figure 6. Finite element model of the CSBV.

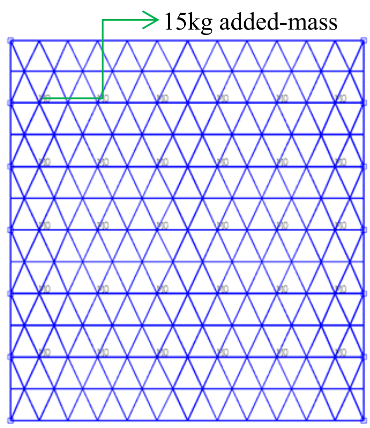

Figure 7. Added mass distribution.

Mode number	1	2	3	4	5	6	7	8	9
First phase	6.33	7.24	8.88	10.10	10.98	13.01	14.21	25.25	26.81
Second phase	8.72	11.27	13.62	14.35	15.83	24.82	26.05	28.43	33.26
Third phase	2.44	3.03	3.41	3.81	4.76	4.94	5.68	7.04	7.38

Table 1. The first nine mode frequencies, in Hz, for each phase of the analysis.

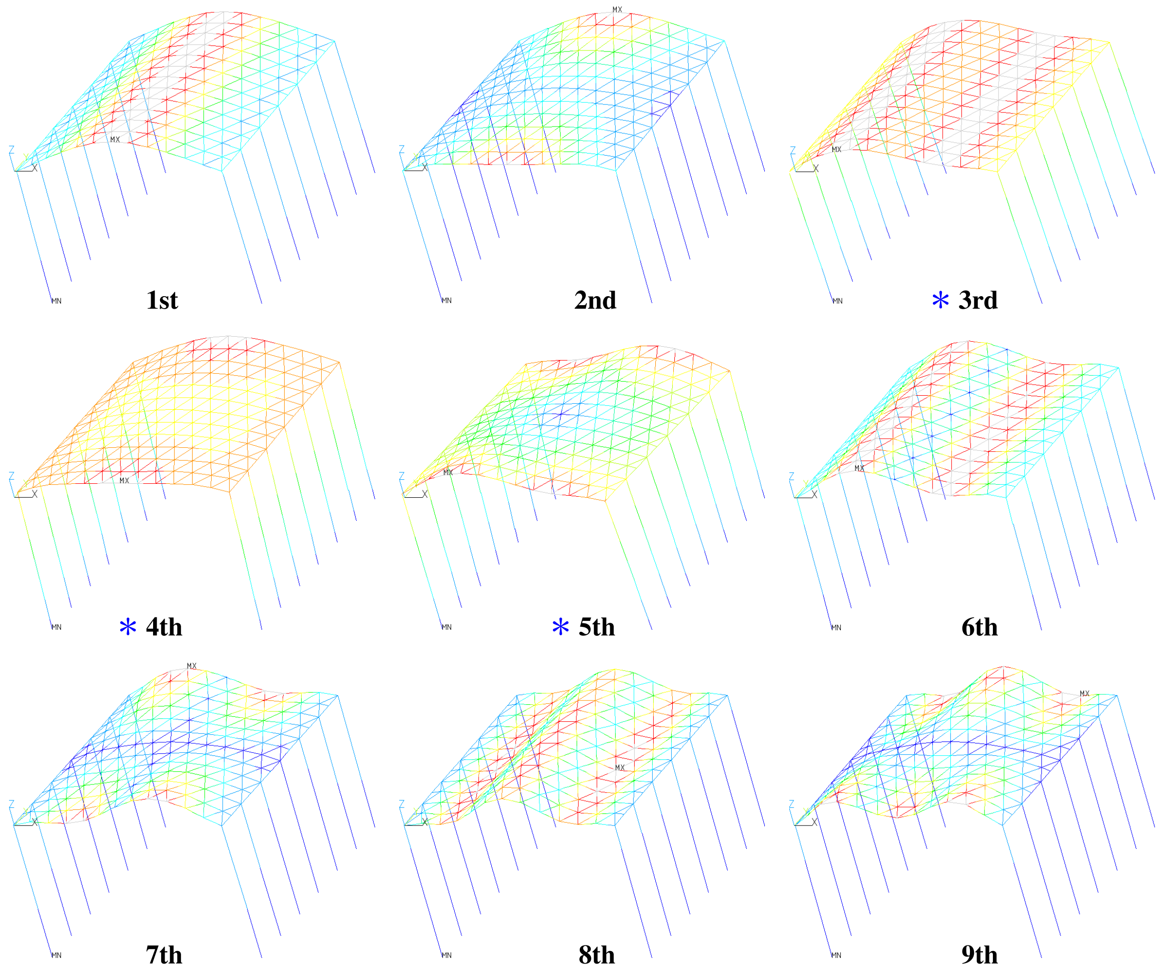

Figure 8. Mode shapes of the first nine mode frequencies for the first phase.

and third phases, with the CSBV having added mass, the frequency of the CSBV is reduced. It can be observed from the mode shapes for each phase that each mode involves vibration of the whole structure, and is not a local mode. The barrel vault structure vibrates with smaller wave numbers than the CSBV, because of the cables and struts. In the first phase, the natural vibration property of the barrel vault is not only related to the roof, but also the stiffness of the bottom structure, as illustrated in the 3rd to 5th modes in Figure 8, marked with $\ast$. The same conclusion applies to the parts of Figures 9 and 10 marked with $\ast$. This indicates that global analysis is necessary for the CSBV.

4. Experimental work

Test model. In order to validate the above numerical modeling of CSBV structures, an experimental test facility was created. All the bars of the upper barrel vault in Figure 1 were connected with welded hollow pipe joints. The ends of the upper barrel vault were welded to the tops of the columns. The tops of the

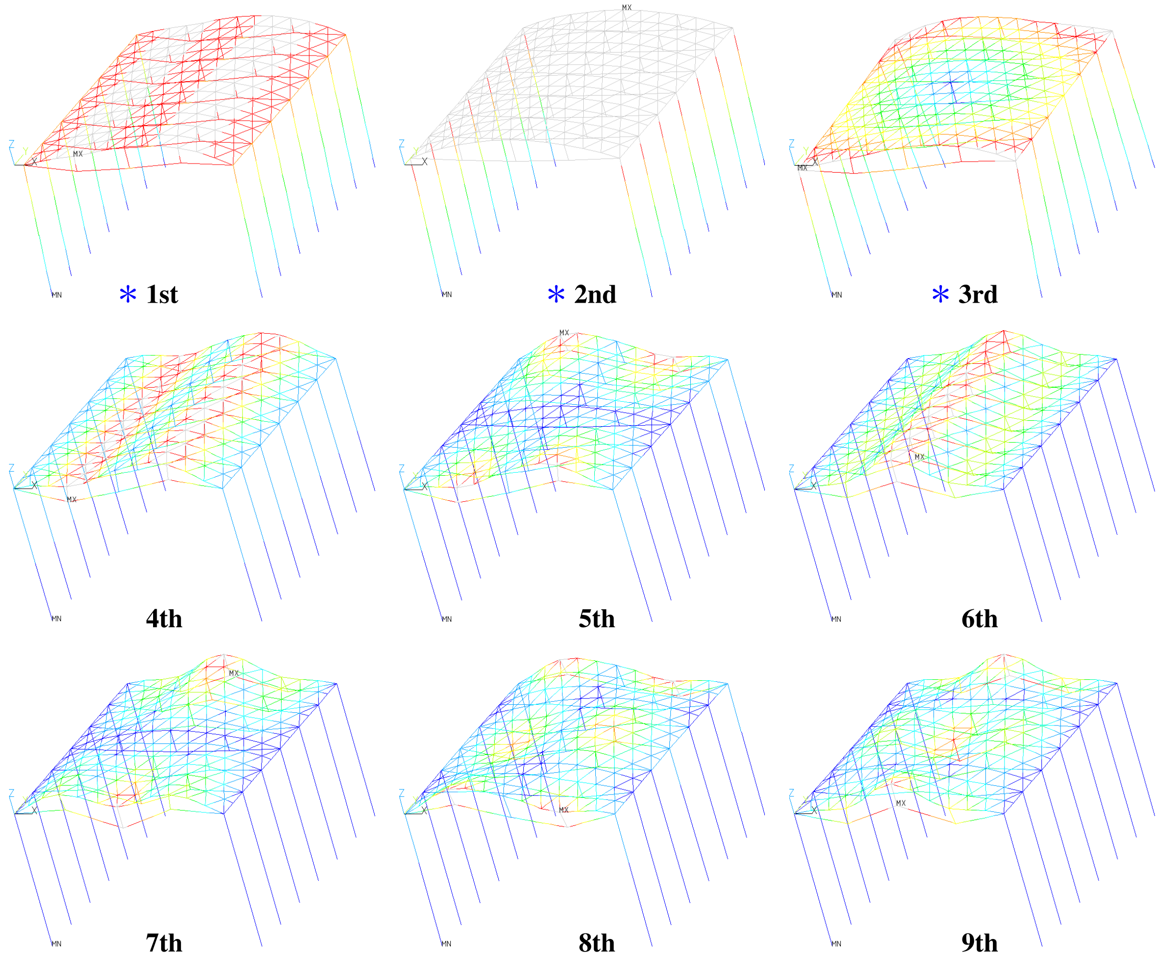

Figure 9. Mode shapes of the first nine mode frequencies for the second phase. (See page 7 for the asterisks.)

struts were connected to the upper barrel vault with one-way hinge bolt joints. The cable groove was used as a connection at the bottom end of the struts while bolts were adopted to link the end of the cable with the top of column. By twisting the bolts, the cable forces could be adjusted to the design values mentioned above. Then the cables were fixed by cable buttons on each side of the struts to avoid slipping.

Test program. The purpose of the test was to obtain the fundamental natural frequencies and vibration modes. In order to avoid wind disturbances, the model was tested in a tent. Vibration data was generated by applying impacts to the structure to obtain the frequency response function. Piezoelectric acceleration sensors were used to collect data at chosen measurement points. There were three phases of experimental tests. In the first phase, 35 measurement locations were chosen on the structure so that accurate mode parameters could be obtained. In the second and third phases, 56 measurement points were chosen to obtain accurate mode parameters. The chosen locations are shown in Figure 11. Because the stiffness

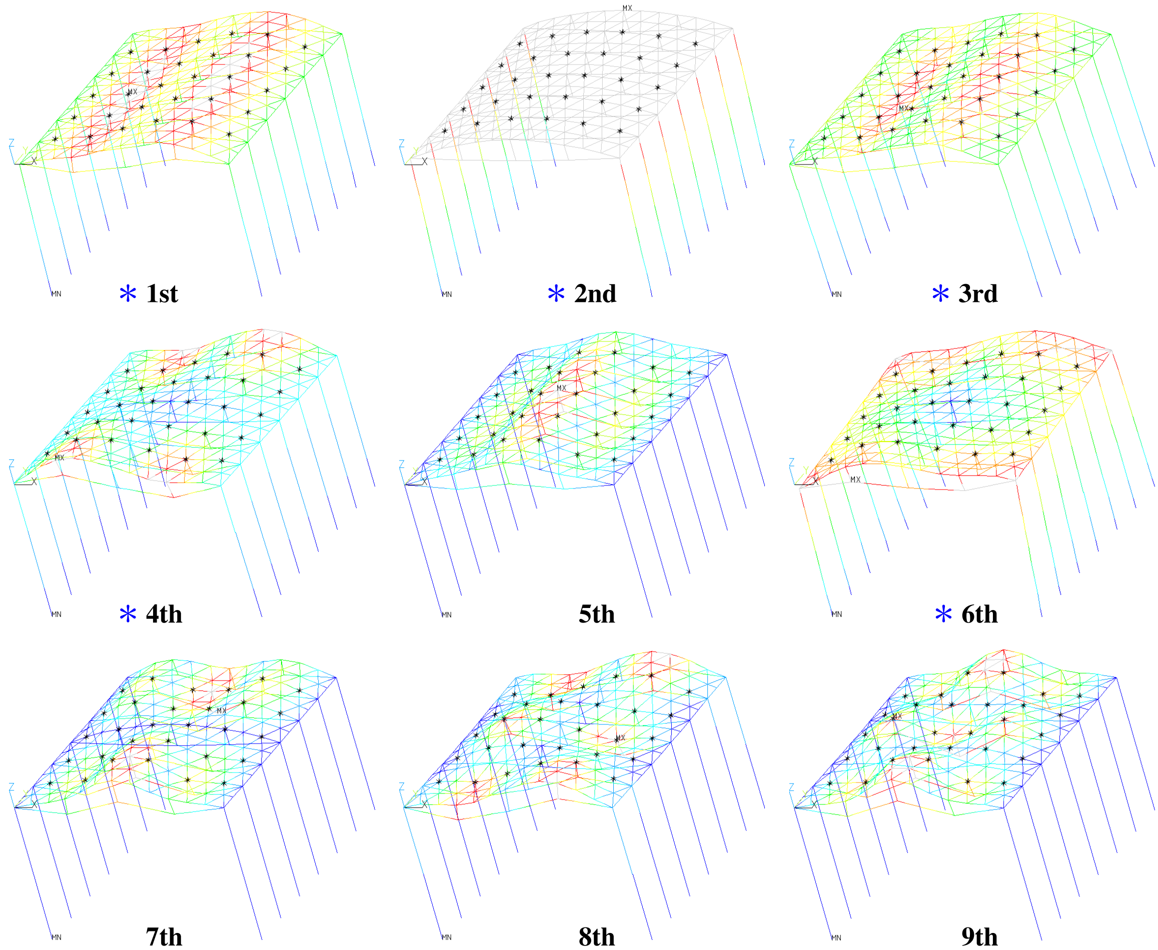

Figure 10. Mode shapes of the first nine mode frequencies for the third phase. (See page 7 for the asterisks.)

at different locations can differ greatly, impulses were applied at one corner of the structure, and the responses at multiple locations were recorded. The force applied by the force hammer was recorded using the piezoelectric force sensor in the hammer. The piezoelectric acceleration sensors gave three-axis acceleration information at each measurement point. The piezoelectric force and acceleration signals were amplified and passed through an antialiasing low-pass filter, and were stored in a data acquisition and signal processing (DASP) system. Figure 12 shows the signal-collection process.

To ensure that the test data were accurate and reliable, pretests were conducted as follows:

(1) The linearity of the test object was evaluated by applying different impacts.

(2) Appropriate sample rates were chosen for the impact force signal and for the accelerometer sensor signals in order to obtain good-fidelity information.

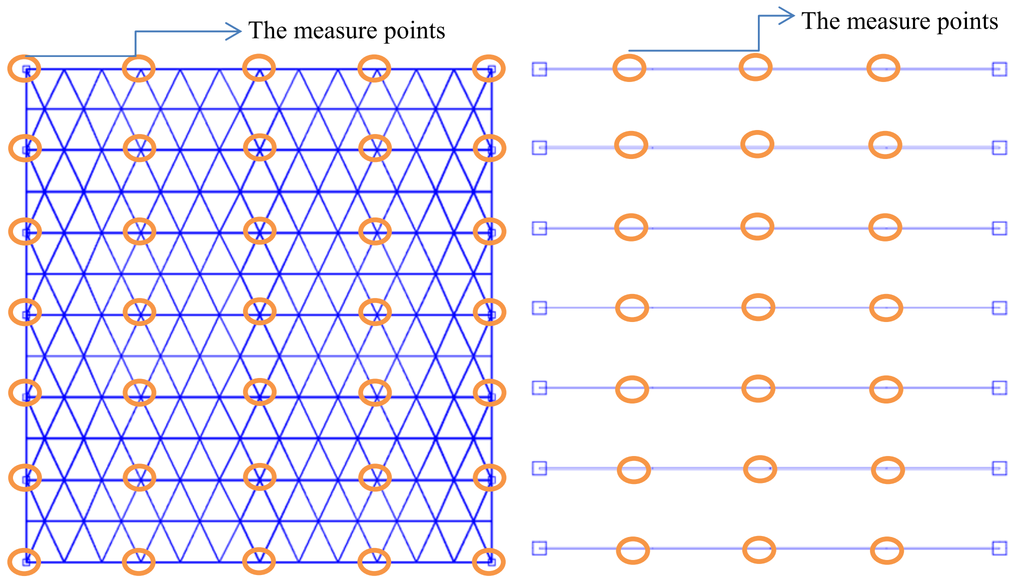

Figure 11. Accelerometer sensor locations (measurement points) on the barrel vault (left) and along the cables (right).

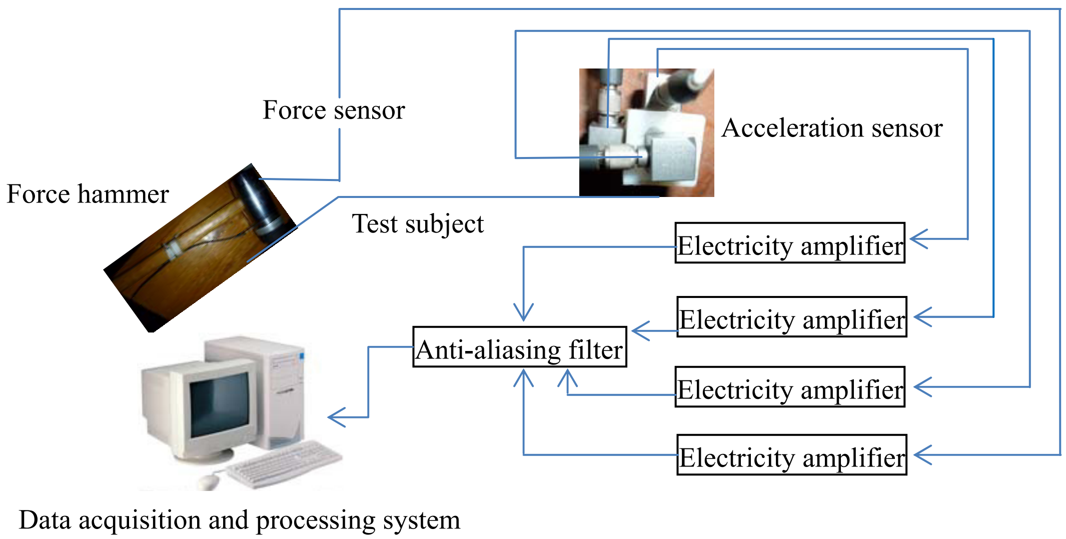

Figure 12. Data-collection hardware configuration.

(3) The symmetry of the matrix of frequency response functions was tested by interchanging the points of the input impact and the resulting response.

(4) Tests were conducted to determine the best location for applying the impulse input. It was determined that a top corner of the structure gave the best excitation of modes, amplitude responses, and signal-to-noise ratio, as shown in Figure 13.

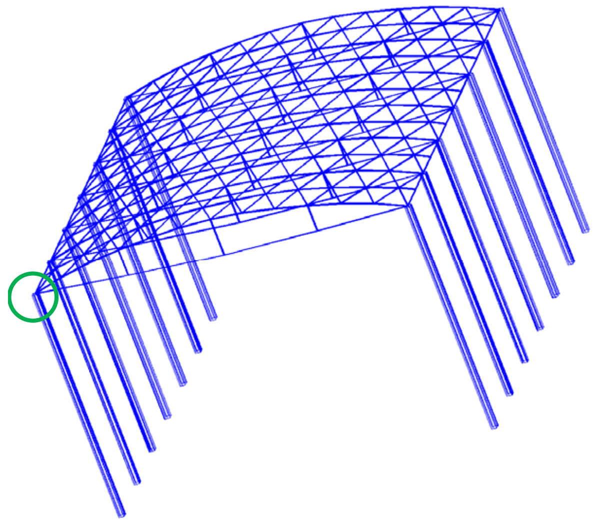

Figure 13. Location of impact using the force hammer (circled).

The bandwidth of the formal test is 100 Hz. Fast Fourier transforms were used on the impact force signal and acceleration response signals to obtain the frequency response functions. The "set total average" and "point spread function" in the DASP modal analysis software were used to identify the mode frequencies and shapes. By comparing the modal fits, the mode of vibration was obtained by the mass-normalized method.

Test results. The test research contents can be divided into three phases: first, the barrel vault (without struts and cables); second, the CSBV structure; and third, the CSBV structure with added masses of 15 kg to simulate the roof. Experimental and numerical values of the natural frequencies for each phase can be compared in Table 2.

The vibration mode shapes of the CSBV are close to each other and their vibration directions are mainly in the vertical direction. The mode shapes are mostly symmetric or antisymmetric. The vibration mode shows a trend of the symmetry number increasing as the modal order increases. The frequency spectrum is very concentrated, making the dynamic characteristics of the CSBV extremely complex. Furthermore, the rigidity of the CSBV is greater than that of the traditional shell structure. Due to the existence of the cable and strut, the vibration modes of the CSBV are changed and the seismic behavior is improved. The dynamic analysis models, used during the process of seismic analysis of the three stages of construction and forming, have greater differences. The changes in the dynamic response of the structure in the construction process cannot be neglected.

5. Parameters research

The finite element model of the CSBV from before is used in the following subsections to study the influence of rise-span ratio and sag-span ratio.

Influence of rise-span ratio. The rise-span ratio used above was 1/100. The influence of the rise-span ratio on the mode frequencies of the structure was analyzed by considering two additional ratios, 1/50

First phase			Second phase			Third phase		
Mode number	Test result	Numerical result	Mode number	Test result	Numerical result	Mode number	Test result	Numerical result
1	6.78	6.33	1	8.87	8.72	1	3.12	2.44
2	7.42	7.24	2	12.12	11.27	2	3.68	3.03
3	8.97	8.88	3	14.02	13.62	3	4.24	3.41
4	10.68	10.1	4	14.29	14.35	4	4.67	3.81
5	11.26	10.98	5	15.62	15.83	5	5.28	4.76
6	13.36	13.01	6	23.43	24.82	6	5.93	4.94
7	14.53	14.21	7	25.42	26.05	7	6.79	5.68
8	25.49	25.25	8	27.43	28.43	8	8.11	7.04
9	27.28	26.81	9	32.14	33.26	9	8.45	7.38

Table 2. Comparison between the experimental and numerical values, in Hz, for natural frequencies. "First phase" refers to the barrel vault without struts and cables; "second phase" to the CSBV structure; and "third phase" to, the CSBV structure with added masses of 15 kg to simulate the roof.

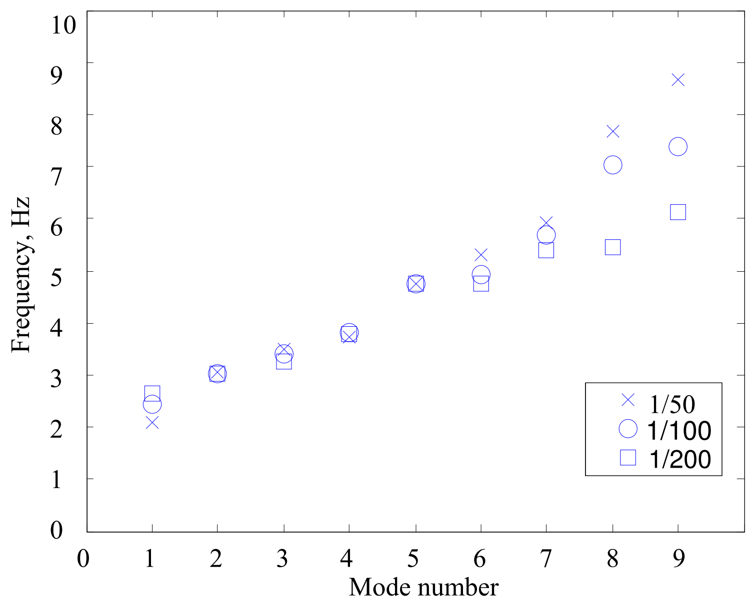

Figure 14. Influence of rise-span ratio on the natural frequency of the CSBV.

and 1/200. The modified models were subjected to an added mass to simulate the third phase. The natural frequency results are shown in Figure 14.

The natural frequency of the CSBV decreases as the rise-span ratio decreases, while the period increases as the rise-span ratio decreases. The variation in the natural frequency is small for small order numbers and becomes larger as the order number increases.

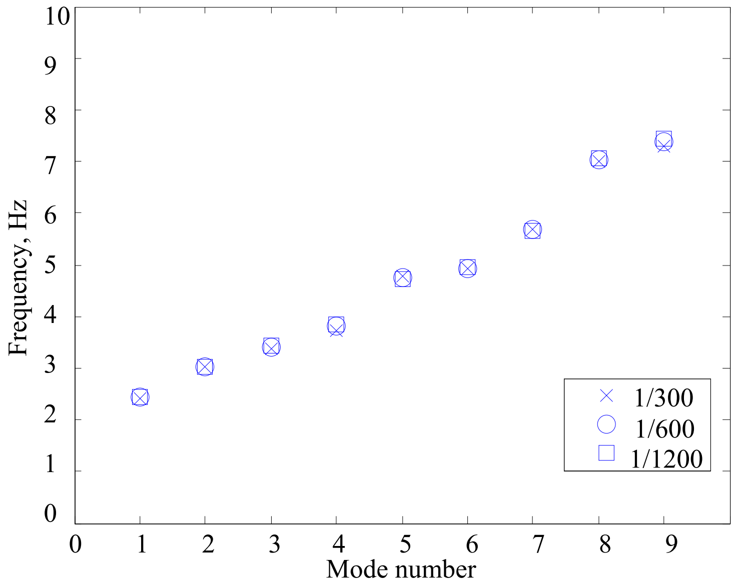

Figure 15. Influence of sag-span ratio on the natural frequencies of the CSBV.

Influence of sag-span ratio. The sag-span ratio used above was $1/600$. The influence of the sag-span ratio on the mode frequencies of the structure was analyzed by considering two additional ratios, $1/300$ and $1/1200$. The modified models were subjected to an added mass to simulate the third phase. The natural frequency results are shown in Figure 15.

The natural frequency of the CSBV increases as the sag-span ratio decreases, while the period decreases as the sag-span ratio decreases. However, the natural frequency does not change significantly as the sag-span ratio changes.

6. Conclusions

The cable-supported barrel vault (CSBV) structure, based on beam string structures (or truss string structures) and cylindrical latticed shell structures, is a new-style hybrid spatial steel structure. The design of the cable-supported structure is different from other steel structures. The process of designing the cable-supported structure is complicated and includes section design and cable-force determination. In this paper, a finite element model of the CSBV with the cable force determined by an iterative calculation was created. In order to simulate the construction process of a CSBV structure, the research was divided into three phases: the first phase considered the barrel vault without struts and cables, the second the CSBV structure, and the third the CSBV structure with added mass to simulate the influence of a roof, considered here to consist of 30 added masses of 15 kg each. Research was conducted using numerical investigation by generating a finite element model, and a CSBV structure was created for experimental verification of the finite element results.

The mode shapes of the CSBV structure are similar, mostly symmetric and antisymmetric, and their vibrations are mostly in the vertical direction. The frequency spectrum is concentrated, creating dynamic characteristics that are very complex. The vibration modes show a trend of the symmetry number increasing as the modal order increasing. The rigidity of the CSBV is greater than that of the traditional shell structure, and the cables and struts change the vibration modes and improve the seismic behavior. The changes in the dynamic response of the structure in the construction process cannot be neglected. The vibration modes of the CSBV are influenced by the roof, as well as by the stiffness of the bottom structure, indicating that global analysis is necessary.

From investigation of the influence of rise-span and sag-span ratios we make the following conclusions: the natural frequencies of the CSBV decrease with the increase of the rise-span ratio, while the period increases with the increase of the rise-span ratio. The variation in the natural frequency is small for small order numbers and becomes larger as the order number increases. The natural frequency of the CSBV increases as the sag-span ratio decreases, while the period decreases as the sag-span ratio decreases. However, the natural frequency is not significantly changed as the sag-span ratio changes.

References

[ANSYS 2010] *ANSYS 12.0 users' manual*, Swanson Analysis Systems, 2010.

[Cao et al. 2009] Z.-G. Cao, F. Fan, Y. Sun, and H.-H. Ma, "Elasto-plastic stability of single-layer cylindrical reticulated shells", pp. 1832–1840 in *Evolution and trends in design, analysis and construction of shell and spatial structures: 50th Anniversary Symposium of the IASS* (Valencia, 2009), edited by A. Domingo and C. Lazaro, Editorial de la Universitat Politécnica de Valencia, Valencia, 2009.

[Chen and Li 2005] Z.-H. Chen and Y. Li, "Parameter analysis on stability of a suspendome", *Int. J. Space Struct.* **20**:2 (2005), 115–124.

[Chen et al. 2004] Z.-H. Chen, Y. Guo, and Y. Li, "Experimental research and analysis of the prestress and dynamic behavior for suspendom structure system", *Build. Struct.* **34**:5 (2004), 42–45. In Chinese.

[Chen et al. 2008] Z.-H. Chen, X.-Y. Yan, W.-T. Qiao, X.-L. Liu, Z.-X. Sun, Q. Li, J. Meng, and X.-Y. Tang, "Structure system of cable supported barrel vault structure", patent ZL200810053722.8, China, 2008, http://www.soopat.com/Patent/200810053722. In Chinese.

[Chen et al. 2010] Z.-H. Chen, W.-T. Qiao, and X.-Y. Yan, "Cable supported barrel vault structure system and research on mechanics feature", *Adv. Steel Constr.* (IJASC, ISSN 1816-112X) **6**:3 (2010), 867–878.

[Dong and Yao 1994] S.-L. Dong and J. Yao, "The future and prospect of latticed shell structures", *Spat. Struct.* **1**:1 (1994), 3–10. In Chinese.

[He et al. 2004] Y.-J. He, X.-H. Zhou, and S.-L. Dong, "Research on static and stability properties of single layer latticed intersected cylindrical shell", *J. Hunan Univ.* (*Nat. Sci.*) **31**:4 (2004), 45–50. In Chinese.

[Jung et al. 1999] H.-J. Jung, M.-C. Kim, and I.-W. Lee, "An improved subspace iteration method with shifting", *Comput. Struct.* **70**:6 (1999), 625–633.

[Kang et al. 2003] W.-J. Kang, Z.-H. Chen, H.-F. Lam, and C.-R. Zuo, "Analysis and design of the general and outmost-ring stiffed suspen-dome structures", *Eng. Struct.* **25**:13 (2003), 1685–1695.

[Kawaguchi et al. 1999] M. Kawaguchi, M. Abe, and I. Tatemichi, "Design, tests and realization of 'suspen-dome' system", *J. IASS* **40**:3/131 (1999), 179–192.

[Kumagai et al. 2009] T. Kumagai, T. Takeuchi, and I. Suzuki, "Dynamic responses of cylindrical lattice shell roofs under horizontal earthquake motions with arbitrary direction by shaking table tests", pp. 409–419 in *Evolution and trends in design, analysis and construction of shell and spatial structures: 50th Anniversary Symposium of the IASS* (Valencia, 2009), edited by A. Domingo and C. Lazaro, Editorial de la Universitat Politécnica de Valencia, Valencia, 2009.

[Langbecker and Albermani 2001] T. Langbecker and F. Albermani, "Kinematic and non-linear analysis of foldable barrel vaults", *Eng. Struct.* **23**:2 (2001), 158–171.

[Tatemichi et al. 1997] I. Tatemichi, T. Hatato, and Y. Anma, "Vibration tests on a full-size suspen-dome structure", *Int. J. Space Struct.* **12**:31 (1997), 143–161.

[Wang and Li 1999] B.-B. Wang and Y.-Y. Li, "A theoretical study of super-span latticed shells", *J. Constr. Steel Res.* **51**:3 (1999), 287–296.

[Xue and Liu 2009] W.-C. Xue and S. Liu, "Design optimization and experimental study on beam string structures", *J. Constr. Steel Res.* **65**:1 (2009), 70–80.

[Zhang et al. 2007] A.-L. Zhang, D.-M. Wang, and X.-C. Liu, "Dynamic performance experiment and theoretical analysis on the suspend-dome structure model of the badminton gymnasium for 2008 Olympic Games", *J. Build. Struct.* **28**:6 (2007), 68–75. In Chinese.

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