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PLANE WAVES AT THE BOUNDARY OF TWO MICROPOLAR THERMOELASTIC SOLIDS WITH DISTINCT CONDUCTIVE AND THERMODYNAMIC TEMPERATURES

RAJNEESH KUMAR, MANDEEP KAUR AND SATISH C. RAJVANSHI

The present investigation is concerned with wave propagation at an interface of two different micropolar thermoelastic solid half-spaces with distinct conductive and thermodynamic temperatures. Reflection and transmission phenomena of plane waves impinging obliquely at a plane interface between two different micropolar thermoelastic solid half-spaces with two temperatures are investigated. The incident wave is assumed to be striking at the plane interface after propagating through one of the micropolar generalized thermoelastic solids with two temperatures. Amplitude ratios of the various reflected and transmitted waves are obtained in closed form and it is found that these are functions of the angle of incidence and frequency, and are affected by the elastic properties of the media. Micropolarity and two-temperature effects are shown on these amplitude ratios for a specific model. Results of some earlier workers have also been deduced from the present investigation.

1. Introduction

The theory of micropolar elasticity introduced and developed by Eringen [1966] has aroused much interest in recent years because of its possible utility in investigating the deformation properties of solids for which classical theory is inadequate. Micropolar theory is believed to be particularly useful in investigating material consisting of bar-like molecules, which exhibit microrotational effects and can support body and surface couples. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies undergoing both translational and rotational motions. The force at a point of the surface element of a body is completely characterized by the force stress vector and couple stress vector at that point.

The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effects. A comprehensive review of the subject was given in [Eringen 1970; 1999; Nowacki 1981]. Tauchert et al. [1968] also derived the basic equations of the linear theory of micropolar coupled thermoelasticity. Dost and Tabarrok [1978] presented the generalized thermoelasticity by using Green and Lindsay theory. Chandrasekharaiah [1986] developed a heat flux-dependent micropolar thermoelasticity. Boschi and Ieşan [1973] extended a generalized theory of micropolar thermoelasticity that permits the transmission of heat as thermal waves at finite speeds.

Thermoelasticity with two temperatures is one of the nonclassical theories of the thermoelasticity of elastic solids. The main difference between this theory and the classical theory is the thermal dependence.

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Chen and Williams [1968] and Chen et al. [1969] formulated a theory of heat conduction in deformable bodies. This depends on two distinct temperatures, the conductive temperature Φ and thermodynamic temperature T. Chen et al. [1969] suggested that the difference between these two temperatures is proportional to the heat supply. These two temperatures may be equal under certain conditions for time-independent situations. However, for time-dependent problems relating to wave propagation, these two temperatures are, in general, different, regardless of the presence of a heat supply. The two temperatures and the strain are found to have representation in the form of a traveling wave pulse, a response which occurs instantaneously throughout the body [Boley and Tolins 1962]. Warren and Chen [1973] investigated wave propagation in the two-temperature theory of thermoelasticity.

Youssef [2006] presented a new theory of generalized thermoelasticity by taking into account the theory of heat conduction in deformable bodies, which depends on distinct conductive and thermodynamic temperatures. He also established a uniqueness theorem for the equation of two-temperature generalized linear thermoelasticity for a homogeneous and isotropic body. Recently, Puri and Jordan [2006] studied the propagation of plane waves under two temperatures. Youssef and Al-Lehaibi [2007] and Youssef and Al-Harby [2007] investigated various problems on the basis of two-temperature thermoelasticity with a relaxation time and showed that the obtained results are qualitatively different when compared to those in the case of one-temperature thermoelasticity. Magaña and Quintanilla [2009] investigated the uniqueness and growth of the solution in two-temperature generalized thermoelastic theories. Mukhopadhyay and Kumar [2009] studied thermoelastic interaction in two-temperature generalized thermoelasticity in an infinite medium with a cylindrical cavity. Various investigators have studied problems in two temperatures, for example, [Kaushal et al. 2010; Kumar and Mukhopadhyay 2010; El-Karamany 2011; El-Karamany and Ezzat 2011; Kaushal et al. 2011].

Various authors have investigated the problems of reflection and transmission at the boundary surface of micropolar elastic solid half-spaces, for example, [Tomar and Gogna 1992; 1995a; 1995b; Hsia and Cheng 2006; Hsia et al. 2007; Kumar and Barak 2007; Kumar et al. 2008a; 2008b].

In this paper, we study the problem of reflection and transmission of plane waves at an interface of two different micropolar generalized thermoelastic solid half-spaces with two temperatures. Micropolarity and two-temperature effects are depicted graphically on the amplitude ratios for the incidence of various plane waves, that is, longitudinal displacement waves (LD waves), thermal waves (T waves), and transverse displacement waves coupled with transverse microrotational waves (CD-I and CD-II waves).

2. Basic equations

Following [Eringen 1966; Ezzat and Awad 2010], the field equations in an isotropic, homogeneous, micropolar elastic medium in the context of the generalized theory of thermoelasticity with two temperatures, without body forces, body couples, or heat sources, are given by

$$(\lambda+2\mu+K)\nabla(\nabla\cdot\vec{u})-(\mu+K)\nabla\times(\nabla\times\vec{u})+K(\nabla\times\vec{\phi})-\nu\nabla T=\rho\frac{\partial^2\vec{u}}{\partial t^2}, \tag{1}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla\cdot\vec{\phi})-\gamma\nabla\times(\nabla\times\vec{\phi})+K(\nabla\times\vec{u})-2K\vec{\phi}=\rho j\frac{\partial^2\vec{\phi}}{\partial t^2}, \tag{2}$$

$$K^*\nabla^2\Phi=\rho c^*\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)+\nu T_0\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)(\nabla\cdot\vec{u}), \tag{3}$$

where

$$T = (1 - a\nabla^2)\Phi,$$

and the constitutive relations are

$$t_{ij} = \lambda u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \epsilon_{ijr}\phi_r) - \nu(1 - a\nabla^2)\Phi\delta_{ij}, \tag{4}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \quad i, j, r = 1, 2, 3, \tag{5}$$

where λ and μ are Lamé's constants; K, α, β, and γ are micropolar constants; t_{ij} are the components of the stress tensor; m_{ij} are the components of couple stress tensor; $\vec{u}$ and $\vec{\phi}$ are the displacement and microrotation vectors; δ_{ij} is the Kronecker delta; ρ is the density; ϵ_{ijr} is the alternating symbol; j is the microinertia; K^* is the thermal conductivity; c^* is the specific heat at constant strain; τ_0 is the relaxation time; T is the deviation of the thermodynamic temperature from the reference temperature; Φ is the deviation of the conductive temperature from the reference temperature; T_0 is the reference temperature; a is the two-temperature parameter; and $\nu = (3\lambda + 2\mu + K)\alpha_T$, where α_T is the coefficient of linear thermal expansion.

The necessary and sufficient conditions for the internal energy to be nonnegative as given in [Eringen 1970] are

$$0 \le (3\lambda + 2\mu + K), \quad 0 \le \mu, \quad 0 \le K, \quad 0 \le 3\alpha + 2\gamma, \quad -\gamma \le \beta \le \gamma, \quad 0 \le \gamma.$$

3. Formulation of the problem

We consider a homogeneous, isotropic, micropolar, thermoelastic solid half-space with two temperatures (medium M_2) lying over another homogeneous, isotropic, micropolar, thermoelastic solid half-space with two temperatures (medium M_1). The rectangular Cartesian coordinate system $Ox_1x_2x_3$ having origin on the surface $x_3 = 0$ with the x_3-axis pointing vertically into the medium M_1 is introduced. Quantities in medium M_2 are denoted with a bar, while those in medium M_1 have no bar.

We consider the two-dimensional problem in the x_1x_3-plane, so that the displacement vector $\vec{u}$ and microrotation vector $\vec{\phi}$ for the solid medium M_1 are taken as

$$\vec{u} = (u_1(x_1, x_3), 0, u_3(x_1, x_3)), \quad \vec{\phi} = (0, \phi_2(x_1, x_3), 0). \tag{6}$$

For convenience, the following nondimensional quantities are introduced:

$$\begin{aligned} &x_1' = \frac{\omega^* x_1}{c_1}, \quad x_3' = \frac{\omega^* x_3}{c_1}, \quad u_1' = \frac{\rho\omega^* c_1}{\nu T_0}u_1, \quad u_3' = \frac{\rho\omega^* c_1}{\nu T_0}u_3, \quad \phi_2' = \frac{\rho c_1^2}{\nu T_0}\phi_2, \\ &t' = \omega^* t, \quad T' = \frac{T}{T_0}, \quad \Phi' = \frac{\Phi}{T_0}, \quad t_{ij}' = \frac{1}{\nu T_0}t_{ij}, \quad m_{ij}' = \frac{\omega^*}{c_1\nu T_0}m_{ij}, \\ &\tau_0' = \omega^*\tau_0, \quad a' = \frac{\omega^{*2}}{c_1^2}a, \end{aligned} \tag{7}$$

where

$$\omega^* = \frac{\rho c^* c_1^2}{K^*}, \quad c_1^2 = \frac{\lambda+2\mu+K}{\rho}.$$

The expressions relating the displacement components u_1 and u_3 to the potential functions ϕ and ψ in dimensionless form are taken as

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1}. \tag{8}$$

Making use of (8) in (1)–(2) and with the aid of (6) and (7) (after suppressing the primes), we obtain

$$\nabla^2 \phi - (1 - a\nabla^2)\Phi - \frac{\partial^2 \phi}{\partial t^2} = 0, \tag{9}$$

$$\nabla^2 \psi + a_1 \phi_2 - a_2 \frac{\partial^2 \psi}{\partial t^2} = 0, \tag{10}$$

$$\nabla^2 \phi_2 - a_3 \nabla^2 \psi - a_4 \phi_2 - a_5 \frac{\partial^2 \phi_2}{\partial t^2} = 0, \tag{11}$$

$$\nabla^2 \Phi = a_6 \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial}{\partial t}(1 - a\nabla^2)\Phi + a_7 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) \nabla^2 \phi, \tag{12}$$

where

$$a_1 = \frac{K}{\mu + K}, \quad a_2 = \frac{\rho c_1^2}{\mu + K}, \quad a_3 = \frac{K c_1^2}{\gamma \omega^{*2}}, \quad a_4 = 2a_3, \quad a_5 = \frac{\rho \hat{j} c_1^2}{\gamma}, \quad a_6 = \frac{\rho c^* c_1^2}{K^* \omega^*}, \quad a_7 = \frac{\nu^2 T_0}{\rho K^* \omega^*},$$

and $\nabla^2 = \dfrac{\partial^2}{\partial x_1^2} + \dfrac{\partial^2}{\partial x_3^2}$ is the Laplacian operator.

4. Boundary conditions

The boundary conditions at the interface $x_3 = 0$ are requirements of the continuity of the normal stress component, the tangential stress component, the tangential couple stress component, the tangential displacement component, the normal displacement component, the microrotation component, and of the thermodynamic temperature and normal component of the heat flux. Mathematically these can be written as

$$t_{33} = \bar{t}_{33}, \quad t_{31} = \bar{t}_{31}, \quad m_{32} = \bar{m}_{32}, \quad u_1 = \bar{u}_1, \quad u_3 = \bar{u}_3, \quad \phi_2 = \bar{\phi}_2, \quad T = \bar{T}, \quad K^* \frac{\partial T}{\partial x_3} = \bar{K}^* \frac{\partial \bar{T}}{\partial x_3}. \tag{13}$$

5. Reflection and transmission

We consider LD waves, T waves, CD-I, and CD-II waves propagating through medium M_1, which we designate as the region $x_3 > 0$, and incident at the plane $x_3 = 0$ with direction of propagation at angle θ_0 normal to the surface. Corresponding to each incident wave, we get reflected LD, T, CD-I, and CD-II waves in medium M_1 and transmitted LD, T, CD-I, and CD-II waves in medium M_2, as shown in Figure 1. In order to solve (9)–(12), we assume solutions of the form

$$\{\phi, \Phi, \psi, \phi_2\} = \{\tilde{\phi}, \tilde{\Phi}, \tilde{\psi}, \tilde{\phi}_2\} e^{\iota\{k(x_1 \sin\theta - x_3 \cos\theta) - \omega t\}}, \tag{14}$$

where k is the wave number, ω the angular frequency, θ the angle of incidence, and ϕ, Φ, ψ, and ϕ_2 arbitrary constants.

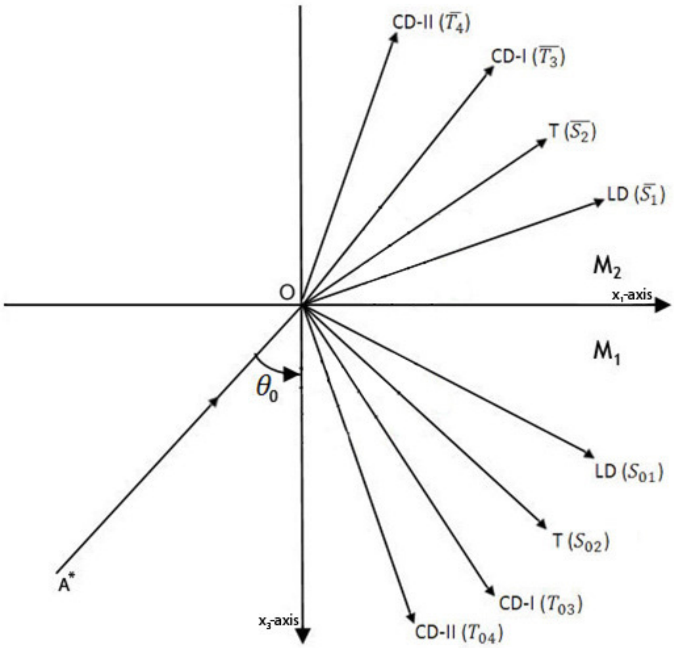

Figure 1. Geometry of the problem.

Making use of (14) in (9)–(12) yields

$$V^4 + D_1 V^2 + E_1 = 0, \tag{15}$$

$$V^4 + D_2 V^2 + E_2 = 0, \tag{16}$$

where

$$D_1 = \frac{-1 + (a - 1/\omega^2) a_6 \omega^2 (\iota/\omega + \tau_0) - a_7(\iota/\omega + \tau_0)}{a_6(\iota/\omega + \tau_0)}, \quad E_1 = \frac{1 - a\omega^2 [a_7(\iota/\omega + \tau_0) + a_6(\iota/\omega + \tau_0)]}{a_6(\iota/\omega + \tau_0)},$$

$$D_2 = \left(\frac{a_1 a_3}{\omega^2 a_2} + 1\right) \frac{1}{(a_4/\omega^2 - a_5)} - \frac{1}{a_2}, \qquad E_2 = \frac{1}{(a_5 - a_4/\omega^2) a_2},$$

and $V^2 = \omega^2/k^2$.

Equations (15) and (16) are quadratic in V^2, therefore the roots of these equations give four values of V^2. Corresponding to each value of V^2 in (15), there exist two types of waves in medium M_1 which are, in decreasing order of their velocities, a LD and a T wave. Similarly, corresponding to each value of V^2 in (16), there exist two types of waves in medium M_1, a CD-I and a CD-II wave. Let V_1 and V_2 be the velocities of the reflected LD and T waves, respectively, and V_3 and V_4 be the velocities of the reflected CD-I and CD-II waves in medium M_1, respectively.

In view of (14), the appropriate solutions of (9)–(12) for mediums M_1 and M_2 are assumed in the following forms.

For medium M_1:

$$\{\phi,\,\Phi\} = \sum_{i=1}^{2}\{1,\,f_i\}[S_{0i}e^{\iota\{k_i(x_1\sin\theta_{0i}-x_3\cos\theta_{0i})-\omega_i t\}} + P_i], \tag{17}$$

$$\{\psi,\,\phi_2\} = \sum_{j=3}^{4}\{1,\,f_j\}[T_{0j}e^{\iota\{k_j(x_1\sin\theta_{0j}-x_3\cos\theta_{0j})-\omega_j t\}} + P_j]. \tag{18}$$

Medium M_2:

$$\{\bar{\phi},\,\bar{\Phi}\} = \sum_{i=1}^{2}\{1,\,\bar{f}_i\}[\bar{S}_i e^{\iota\{\bar{k}_i(x_1\sin\bar{\theta}_i-x_3\cos\bar{\theta}_i)-\bar{\omega}_i t\}}], \tag{19}$$

$$\{\bar{\psi},\,\bar{\phi}_2\} = \sum_{j=3}^{4}\{1,\,\bar{f}_j\}[\bar{T}_j e^{\iota\{\bar{k}_j(x_1\sin\bar{\theta}_j-x_3\cos\bar{\theta}_j)-\bar{\omega}_j t\}}], \tag{20}$$

where

$$P_i = S_i e^{\iota\{k_i(x_1\sin\theta_i+x_3\cos\theta_i)-\omega_i t\}},\qquad P_j = T_j e^{\iota\{k_j(x_1\sin\theta_j+x_3\cos\theta_j)-\omega_j t\}},$$

$$f_i = \frac{\omega^2(1-1/V_i^2)}{1+a\omega^2/V_i^2},\qquad f_j = \frac{-\omega^2(a_2-1/V_j^2)}{a_1},$$

and S_{0i} and T_{0j} are the amplitudes of the incident LD and T waves, and CD-I and CD-II waves, respectively. S_i and T_j are the amplitudes of the reflected LD and T waves, and CD-I and CD-II waves, respectively, and $\bar{S}_i$ and $\bar{T}_j$ are the amplitudes of the transmitted LD and T waves, and CD-I and CD-II waves, respectively.

In order to satisfy the boundary conditions, we use the following extension of Snell's law:

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\theta_3}{V_3} = \frac{\sin\theta_4}{V_4} = \frac{\sin\bar{\theta}_1}{\bar{V}_1} = \frac{\sin\bar{\theta}_2}{\bar{V}_2} = \frac{\sin\bar{\theta}_3}{\bar{V}_3} = \frac{\sin\bar{\theta}_4}{\bar{V}_4}, \tag{21}$$

where

$$k_1V_1 = k_2V_2 = k_3V_3 = k_4V_4 = \bar{k}_1\bar{V}_1 = \bar{k}_2\bar{V}_2 = \bar{k}_3\bar{V}_3 = \bar{k}_4\bar{V}_4 = \omega,\qquad \text{at } x_3 = 0. \tag{22}$$

Making use of the values of ϕ, ψ, Φ, and ϕ_2 from (17)–(20) in boundary conditions (13), and with the aid of (4)–(8), using (21) and (22), we obtain a system of eight nonhomogeneous equations which can be written as

$$\sum_{j=1}^{8} a_{ij}Z_j = Y_i\quad (i = 1, 2, 3, 4, 5, 6, 7, 8), \tag{23}$$

where

$$a_{1i} = \left(d_1 + d_2\left(1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{V_i^2} + (1-\tau_1\iota\omega)\left(1 + a\frac{\omega^2}{V_i^2}\right)f_i,$$

$$a_{1j} = d_2\frac{\omega^2}{V_jV_0}\sin\theta_0\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0},$$

$$a_{15} = -\left[\left(\bar{d}_1 + \bar{d}_2\left(1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{\bar{V}_1^2} + \bar{p}_0(1 - \bar{\tau}_1\iota\omega)\left(1 + \bar{a}\frac{\omega^2}{\bar{V}_1^2}\right)\bar{f}_1\right],$$

$$a_{16} = -\left[\left(\bar{d}_1 + \bar{d}_2\left(1 - \frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{\bar{V}_2^2} + \bar{p}_0(1 - \bar{\tau}_1\iota\omega)\left(1 + \bar{a}\frac{\omega^2}{\bar{V}_2^2}\right)\bar{f}_2\right],$$

$$a_{17} = -\bar{d}_2\frac{\omega^2}{\bar{V}_3 V_0}\sin\theta_0\sqrt{1 - \frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0}, \quad a_{18} = -\bar{d}_2\frac{\omega^2}{\bar{V}_4 V_0}\sin\theta_0\sqrt{1 - \frac{\bar{V}_4^2}{V_0^2}\sin^2\theta_0},$$

$$a_{2i} = -(2d_4 + d_5)\frac{\omega^2}{V_i V_0}\sin\theta_0\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0},$$

$$a_{2j} = \left(d_4\frac{\omega^2}{V_j^2}\left(1 - 2\frac{V_j^2}{V_0^2}\sin^2\theta_0\right) + d_5\frac{\omega^2}{V_j^2}\left(1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0\right)\right) - d_5 f_j,$$

$$a_{25} = (2\bar{d}_4 + \bar{d}_5)\frac{\omega^2}{\bar{V}_1 V_0}\sin\theta_0\sqrt{1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0}, \quad a_{26} = (2\bar{d}_4 + \bar{d}_5)\frac{\omega^2}{\bar{V}_2 V_0}\sin\theta_0\sqrt{1 - \frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0},$$

$$a_{27} = -\left[\bar{d}_4\frac{\omega^2}{\bar{V}_3^2}\left(1 - 2\frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0\right) + \bar{d}_5\left(\frac{\omega^2}{\bar{V}_3^2}\left(1 - \frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0\right) - \bar{f}_3\right)\right],$$

$$a_{28} = -\left[\bar{d}_4\frac{\omega^2}{\bar{V}_4^2}\left(1 - 2\frac{\bar{V}_4^2}{V_0^2}\sin^2\theta_0\right) + \bar{d}_5\left(\frac{\omega^2}{\bar{V}_4^2}\left(1 - \frac{\bar{V}_4^2}{V_0^2}\sin^2\theta_0\right) - \bar{f}_4\right)\right],$$

$$a_{3i} = 0, \quad a_{3j} = \iota\frac{\omega}{V_j}\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0}\, f_j, \quad a_{35} = a_{36} = 0, \quad a_{37} = \iota\frac{\omega}{\bar{V}_3}p_1\sqrt{1 - \frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0}\,\bar{f}_3,$$

$$a_{38} = \iota\frac{\omega}{\bar{V}_4}p_1\sqrt{1 - \frac{\bar{V}_4^2}{V_0^2}\sin^2\theta_0}\,\bar{f}_4,$$

$$a_{4i} = \iota\frac{\omega}{V_0}\sin\theta_0, \quad a_{4j} = -\iota\frac{\omega}{V_j}\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0},$$

$$a_{45} = a_{46} = -\iota\frac{\omega}{V_0}\sin\theta_0, \quad a_{47} = -\iota\frac{\omega}{\bar{V}_3}\sqrt{1 - \frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0}, \quad a_{48} = -\iota\frac{\omega}{\bar{V}_4}\sqrt{1 - \frac{\bar{V}_4^2}{V_0^2}\sin^2\theta_0},$$

$$a_{5i} = \iota\frac{\omega}{V_i}\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0}, \quad a_{5j} = \iota\frac{\omega}{V_0}\sin\theta_0,$$

$$a_{55} = \iota\frac{\omega}{\bar{V}_1}\sqrt{1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0}, \quad a_{56} = \iota\frac{\omega}{\bar{V}_2}\sqrt{1 - \frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0}, \quad a_{57} = a_{58} = -\iota\frac{\omega}{V_0}\sin\theta_0,$$

$$a_{6i} = 0, \quad a_{6j} = f_j, \quad a_{65} = a_{66} = 0, \quad a_{67} = -\bar{f}_3, \quad a_{68} = -\bar{f}_4,$$

$$a_{7i} = \left(1 + a\frac{\omega^2}{V_i^2}\right)f_i, \quad a_{7j} = 0, \quad a_{75} = -\left(1 + \bar{a}\frac{\omega^2}{\bar{V}_1^2}\right)\bar{f}_1, \quad a_{76} = -\left(1 + \bar{a}\frac{\omega^2}{\bar{V}_2^2}\right)\bar{f}_2, \quad a_{77} = a_{78} = 0,$$

$$
\begin{aligned}
&a_{8i} = \iota \frac{\omega}{V_i}\left(1 + a\frac{\omega^2}{V_i^2}\right)\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0}\, f_i, \quad a_{8j} = 0, \\
&a_{85} = \iota p_2 \frac{\omega}{\bar{V}_1}\left(1 + \bar{a}\frac{\omega^2}{\bar{V}_1^2}\right)\sqrt{1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0}\, \bar{f}_1, \\
&a_{86} = \iota p_2 \frac{\omega}{\bar{V}_2}\left(1 + \bar{a}\frac{\omega^2}{\bar{V}_2^2}\right)\sqrt{1 - \frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0}\, \bar{f}_2, \quad a_{87} = a_{88} = 0, \\
&\bar{p}_0 = \frac{\bar{\nu}}{\nu}, \quad d_1 = \frac{\lambda}{\rho c_1^2}, \quad d_2 = \frac{(2\mu + K)}{\rho c_1^2}, \quad d_4 = \frac{\mu}{\rho c_1^2}, \quad d_5 = \frac{K}{\rho c_1^2}, \quad p_1 = \frac{\bar{\gamma}}{\gamma}, \quad p_2 = \frac{K_1^*}{K^*}, \\
&\bar{d}_1 = \frac{\bar{\lambda}}{\rho c_1^2}, \quad \bar{d}_2 = \frac{(2\bar{\mu} + \bar{K})}{\rho c_1^2}, \quad \bar{d}_4 = \frac{\bar{\mu}}{\rho c_1^2}, \quad \bar{d}_5 = \frac{\bar{K}}{\rho c_1^2}.
\end{aligned}
\tag{24}
$$

In (24), $i = 1, 2$ and $j = 3, 4$, and

$$
Z_1 = \frac{S_1}{A^*}, \quad Z_2 = \frac{S_2}{A^*}, \quad Z_3 = \frac{T_3}{A^*}, \quad Z_4 = \frac{T_4}{A^*}, \quad Z_5 = \frac{\bar{S}_1}{A^*}, \quad Z_6 = \frac{\bar{S}_2}{A^*}, \quad Z_7 = \frac{\bar{T}_3}{A^*}, \quad Z_8 = \frac{\bar{T}_4}{A^*}. \tag{25}
$$

(1) For an incident LD wave:

$$
\begin{gathered}
A^* = S_{01}, \quad S_{02} = T_{03} = T_{04} = 0, \quad Y_1 = -a_{11}, \quad Y_2 = a_{21}, \quad Y_3 = a_{31} = 0, \\
Y_4 = -a_{41}, \quad Y_5 = a_{51}, \quad Y_6 = a_{61} = 0, \quad Y_7 = -a_{71}, \quad Y_8 = a_{81}.
\end{gathered}
$$

(2) For an incident T wave:

$$
\begin{gathered}
A^* = S_{02}, \quad S_{01} = T_{03} = T_{04} = 0, \quad Y_1 = -a_{12}, \quad Y_2 = a_{22}, \quad Y_3 = a_{32} = 0, \\
Y_4 = -a_{42}, \quad Y_5 = a_{52}, \quad Y_6 = a_{62} = 0, \quad Y_7 = -a_{72}, \quad Y_8 = a_{82}.
\end{gathered}
$$

(3) For an incident CD-I wave:

$$
\begin{gathered}
A^* = T_{03}, \quad S_{01} = S_{02} = T_{04} = 0, \quad Y_1 = a_{13}, \quad Y_2 = -a_{23}, \quad Y_3 = a_{33}, \\
Y_4 = a_{43}, \quad Y_5 = -a_{53}, \quad Y_6 = -a_{63}, \quad Y_7 = a_{73} = 0, \quad Y_8 = a_{83} = 0.
\end{gathered}
$$

(4) For an incident CD-II wave:

$$
\begin{gathered}
A^* = T_{04}, \quad S_{01} = S_{02} = T_{03} = 0, \quad Y_1 = a_{14}, \quad Y_2 = -a_{24}, \quad Y_3 = a_{34}, \\
Y_4 = a_{44}, \quad Y_5 = -a_{54}, \quad Y_6 = -a_{64}, \quad Y_7 = a_{74} = 0, \quad Y_8 = a_{84} = 0,
\end{gathered}
$$

where Z_1, Z_2, Z_3, and Z_4 are the amplitude ratios of the reflected LD, T, and coupled CD-I and CD-II waves in the medium M_1, and Z_5, Z_6, Z_7, and Z_8 are the amplitude ratios of the transmitted LD, T, and coupled CD-I and CD-II waves in medium M_2.

6. Particular cases

Case I. If the two-temperature parameters vanish, that is, $a = 0$ and $\bar{a} = 0$ in (23), then we obtain the amplitude ratios at the interface of the two micropolar thermoelastic solid half-spaces with the following changed values of a_{ij}:

$$\begin{aligned}
a_{1i} &= \left(d_1 + d_2\left(1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{V_i^2} + (1 - \tau_1 \iota\omega) f_i,\\
a_{15} &= -\left[\left(\bar{d}_1 + \bar{d}_2\left(1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{\bar{V}_2^2} + \bar{p}_0(1 - \bar{\tau}_1 \iota\bar{\omega}_1)\bar{f}_1\right],\\
a_{16} &= -\left[\left(\bar{d}_1 + \bar{d}_2\left(1 - \frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{\bar{V}_2^2} + \bar{p}_0(1 - \bar{\tau}_1 \iota\bar{\omega}_2)\bar{f}_2\right],\\
a_{7i} &= f_i, \quad a_{75} = -\bar{f}_1, \quad a_{76} = -\bar{f}_2, \quad a_{8i} = \iota\frac{\omega}{V_i}\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0}\, f_i,\\
a_{85} &= \iota p_2 \frac{\omega}{\bar{V}_1}\sqrt{1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0}\,\bar{f}_1 \quad a_{86} = \iota p_2 \frac{\omega}{\bar{V}_2}\sqrt{1 - \frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0}\,\bar{f}_2.
\end{aligned} \tag{26}$$

Case II. By neglecting the thermal effect and the two-temperature effect in (23), the amplitude ratios at the interface of the two micropolar elastic solid half-spaces are given by

$$\sum_{j=1}^{6} a_{ij} Z_j = Y_i \quad (i = 1, 2, 3, 4, 5, 6),$$

where the values of a_{ij} are given by

$$\begin{aligned}
&a_{1i} = 0, \quad a_{11} = \left(d_1 + d_2\left(1 - \frac{V_1^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{V_1^2}, \quad a_{12} = d_2\frac{\omega^2}{V_3 V_0}\sin\theta_0\sqrt{1 - \frac{V_3^2}{V_0^2}\sin^2\theta_0},\\
&a_{13} = d_2\frac{\omega^2}{V_4 V_0}\sin\theta_0\sqrt{1 - \frac{V_4^2}{V_0^2}\sin^2\theta_0}, \quad a_{14} = -\left[\left(\bar{d}_1 + \bar{d}_2\left(1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{\bar{V}_1^2}\right],\\
&a_{15} = -\bar{d}_2\frac{\omega^2}{\bar{V}_3 V_0}\sin\theta_0\sqrt{1 - \frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0}, \quad a_{16} = -\bar{d}_2\frac{\omega^2}{\bar{V}_4 V_0}\sin\theta_0\sqrt{1 - \frac{\bar{V}_4^2}{V_0^2}\sin^2\theta_0},\\
&a_{21} = -(2d_4 + d_5)\frac{\omega^2}{V_1 V_0}\sin\theta_0\sqrt{1 - \frac{V_1^2}{V_0^2}\sin^2\theta_0},\\
&a_{22} = \left(d_4\frac{\omega^2}{V_3^2}\left(1 - 2\frac{V_3^2}{V_0^2}\sin^2\theta_0\right) + d_5\frac{\omega^2}{V_3^2}\left(1 - \frac{V_3^2}{V_0^2}\sin^2\theta_0\right)\right) - d_5 f_3,\\
&a_{23} = \left(d_4\frac{\omega^2}{V_4^2}\left(1 - 2\frac{V_4^2}{V_0^2}\sin^2\theta_0\right) + d_5\frac{\omega^2}{V_4^2}\left(1 - \frac{V_4^2}{V_0^2}\sin^2\theta_0\right)\right) - d_5 f_4,\\
&a_{24} = (2\bar{d}_4 + \bar{d}_5)\frac{\omega^2}{\bar{V}_1 V_0}\sin\theta_0\sqrt{1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0},
\end{aligned}$$

$$
\begin{aligned}
a_{25} &= -\left[\bar{d}_4 \frac{\omega^2}{\bar{V}_2^2}\left(1-2\frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0\right)+\bar{d}_5\left(\frac{\omega^2}{\bar{V}_2^2}\left(1-\frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0\right)-\bar{f}_3\right)\right],\\
a_{26} &= -\left[\bar{d}_4 \frac{\omega^2}{\bar{V}_3^2}\left(1-2\frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0\right)+\bar{d}_5\left(\frac{\omega^2}{\bar{V}_3^2}\left(1-\frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0\right)-\bar{f}_4\right)\right],\\
a_{31} &= 0, \quad a_{32} = \iota\frac{\omega}{V_3}\sqrt{1-\frac{V_3^2}{V_0^2}\sin^2\theta_0}\, f_3, \quad a_{33} = \iota\frac{\omega}{V_4}\sqrt{1-\frac{V_4^2}{V_0^2}\sin^2\theta_0}\, f_4,\\
a_{34} &= 0, \quad a_{35} = \iota\frac{\omega}{\bar{V}_3}p_1\sqrt{1-\frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0}\,\bar{f}_3, \quad a_{36} = \iota\frac{\omega}{\bar{V}_4}p_1\sqrt{1-\frac{\bar{V}_4^2}{V_0^2}\sin^2\theta_0}\,\bar{f}_4,\\
a_{41} &= \iota\frac{\omega}{V_0}\sin\theta_0, \quad a_{42} = -\iota\frac{\omega}{V_3}\sqrt{1-\frac{V_3^2}{V_0^2}\sin^2\theta_0}, \quad a_{43} = -\iota\frac{\omega}{V_4}\sqrt{1-\frac{V_4^2}{V_0^2}\sin^2\theta_0},\\
a_{44} &= -\iota\frac{\omega}{V_0}\sin\theta_0, \quad a_{45} = -\iota\frac{\omega}{\bar{V}_3}\sqrt{1-\frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0}, \quad a_{46} = -\iota\frac{\omega}{\bar{V}_4}\sqrt{1-\frac{\bar{V}_4^2}{V_0^2}\sin^2\theta_0},\\
a_{51} &= \iota\frac{\omega}{V_1}\sqrt{1-\frac{V_1^2}{V_0^2}\sin^2\theta_0}, \quad a_{52} = a_{53} = \iota\frac{\omega}{V_0}\sin\theta_0, \quad a_{54} = \iota\frac{\omega}{\bar{V}_1}\sqrt{1-\frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0},\\
a_{55} &= a_{56} = -\iota\frac{\omega}{V_0}\sin\theta_0,\\
a_{61} &= 0, \quad a_{62} = f_3, \quad a_{63} = f_4, \quad a_{64} = 0, \quad a_{65} = -\bar{f}_3, \quad a_{66} = -\bar{f}_4.
\end{aligned} \tag{27}
$$

V_1 is the velocity of the reflected P wave and

$$
Z_1 = \frac{S_1}{A^*}, \quad Z_2 = \frac{T_3}{A^*}, \quad Z_3 = \frac{T_4}{A^*}, \quad Z_4 = \frac{\bar{S}_1}{A^*}, \quad Z_5 = \frac{\bar{T}_3}{A^*}, \quad Z_6 = \frac{\bar{T}_4}{A^*}, \tag{28}
$$

where Z_1, Z_2, and Z_3 are the amplitude ratios of the reflected P and coupled CD-I and CD-II waves in medium M_1, and Z_5, Z_6, and Z_7 are the amplitude ratios of the transmitted P and coupled CD-I and CD-II waves in medium M_2.

The above results are similar to those obtained by Tomar and Gogna [1995a; 1995b], changing the dimensionless quantities into physical quantities.

Case III. By neglecting the micropolarity effect in medium M_2, we obtain amplitude ratios at the interface of the micropolar thermoelastic solid with two temperatures and the thermoelastic solid with two temperatures as

$$
\sum_{j=1}^{7} a_{ij} Z_j = Y_i \quad (i = 1, 2, 3, 4, 5, 6, 7),
$$

where

$$a_{1i} = \left(d_1 + d_2\left(1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{V_i^2} + (1-\tau_1\iota\omega)\left(1 + a\frac{\omega^2}{V_i^2}\right)f_i,$$

$$a_{1j} = d_2\frac{\omega^2}{V_j V_0}\sin\theta_0\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0},$$

$$a_{15} = -\left[\left(\bar{d}_1 + \bar{d}_2\left(1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{\bar{V}_1^2} + \bar{p}_0(1-\bar{\tau}_1\iota\omega)\left(1 + \bar{a}\frac{\omega^2}{\bar{V}_1^2}\right)\bar{f}_1\right],$$

$$a_{16} = -\left[\left(\bar{d}_1 + \bar{d}_2\left(1 - \frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{\bar{V}_2^2} + \bar{p}_0(1-\bar{\tau}_1\iota\omega)\left(1 + \bar{a}\frac{\omega^2}{\bar{V}_2^2}\right)\bar{f}_2\right],$$

$$a_{17} = -\bar{d}_2\frac{\omega^2}{\bar{V}_3 V_0}\sin\theta_0\sqrt{1 - \frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0},$$

$$a_{2i} = -(2d_4 + d_5)\frac{\omega^2}{V_i V_0}\sin\theta_0\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0},$$

$$a_{2j} = \left(d_4\frac{\omega^2}{V_j^2}\left(1 - 2\frac{V_j^2}{V_0^2}\sin^2\theta_0\right) + d_5\frac{\omega^2}{V_j^2}\left(1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0\right)\right) - d_5 f_j,$$

$$a_{25} = (2\bar{d}_4 + \bar{d}_5)\frac{\omega^2}{\bar{V}_1 V_0}\sin\theta_0\sqrt{1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0},$$

$$a_{26} = (2\bar{d}_4 + \bar{d}_5)\frac{\omega^2}{\bar{V}_2 V_0}\sin\theta_0\sqrt{1 - \frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0},$$

$$a_{27} = -\bar{d}_4\frac{\omega^2}{\bar{V}_3^2}\left(1 - 2\frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0\right),$$

$$a_{3i} = 0, \quad a_{3j} = \iota\frac{\omega}{V_j}\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0}\, f_j, \quad a_{35} = a_{36} = a_{37} = 0, \quad ,$$

$$a_{4i} = \iota\frac{\omega}{V_0}\sin\theta_0, \quad a_{4j} = -\iota\frac{\omega}{V_i}\sqrt{1 - \frac{V_j^2}{V_0^2}\sin^2\theta_0}, \quad a_{45} = a_{46} = -\iota\frac{\omega}{V_0}\sin\theta_0,$$

$$a_{47} = -\iota\frac{\omega}{\bar{V}_3}\sqrt{1 - \frac{\bar{V}_3^2}{V_0^2}\sin^2\theta_0}, \quad a_{5i} = \iota\frac{\omega}{V_i}\sqrt{1 - \frac{V_i^2}{V_0^2}\sin^2\theta_0},$$

$$a_{5j} = \iota\frac{\omega}{V_0}\sin\theta_0, \quad a_{55} = \iota\frac{\omega}{\bar{V}_1}\sqrt{1 - \frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0}, \quad a_{56} = \iota\frac{\omega}{\bar{V}_2}\sqrt{1 - \frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0}, \quad a_{57} = -\iota\frac{\omega}{V_0}\sin\theta_0,$$

$$a_{6i} = \left(1 + a\frac{\omega^2}{V_i^2}\right)f_i, \quad a_{6j} = 0, \quad a_{65} = -\left(1 + \bar{a}\frac{\omega^2}{\bar{V}_1^2}\right)\bar{f}_1, \quad a_{66} = -\left(1 + \bar{a}\frac{\omega^2}{\bar{V}_2^2}\right)\bar{f}_2, \quad a_{67} = 0,$$

$$a_{7i}=\iota\frac{\omega}{V_i}\left(1+a\frac{\omega^2}{V_i^2}\right)\sqrt{1-\frac{V_i^2}{V_0^2}\sin^2\theta_0 f_i},\quad a_{7j}=0,\quad a_{75}=\iota p_2\frac{\omega}{\bar{V}_1}\left(1+\bar{a}\frac{\omega^2}{\bar{V}_1^2}\right)\sqrt{1-\frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0 \bar{f}_1},$$
$$a_{76}=\iota p_2\frac{\omega}{\bar{V}_2}\left(1+\bar{a}\frac{\omega^2}{\bar{V}_2^2}\right)\sqrt{1-\frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0 \bar{f}_2},\quad a_{77}=0, \tag{29}$$

and

$$Z_1=\frac{S_1}{A^*},\quad Z_2=\frac{S_2}{A^*},\quad Z_3=\frac{T_3}{A^*},\quad Z_4=\frac{T_4}{A^*},\quad Z_5=\frac{\bar{S}_1}{A^*},\quad Z_6=\frac{\bar{S}_2}{A^*},\quad Z_7=\frac{\bar{T}_3}{A^*},$$

where Z_1, Z_2, Z_3, and Z_4 are the amplitude ratios of the reflected LD, T, and coupled CD-I and CD-II waves in medium M_1, and Z_5, Z_6, and Z_7 are the amplitude ratios of the transmitted LD, T, and SV (transverse) waves in medium M_2.

Subcase (a). By taking $\bar{a}=0$, we obtain amplitude ratios at the interface of the micropolar thermoelastic solid with two temperatures and the thermoelastic solid. The values of a_{ij} are from (29), with the following changes:

$$a_{15}=-\left[\left(\bar{d}_1+\bar{d}_2\left(1-\frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{\bar{V}_1^2}+\bar{p}_0(1-\bar{\tau}_1\iota\omega)\bar{f}_1\right],$$
$$a_{16}=-\left[\left(\bar{d}_1+\bar{d}_2\left(1-\frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{\bar{V}_2^2}+\bar{p}_0(1-\bar{\tau}_1\iota\omega)\bar{f}_2\right],$$
$$a_{65}=-\bar{f}_1,\quad a_{66}=-\bar{f}_2,$$
$$a_{75}=\iota p_2\frac{\omega}{\bar{V}_1}\sqrt{1-\frac{\bar{V}_1^2}{V_0^2}\sin^2\theta_0 \bar{f}_1},\quad a_{76}=\iota p_2\frac{\omega}{\bar{V}_2}\sqrt{1-\frac{\bar{V}_2^2}{V_0^2}\sin^2\theta_0 \bar{f}_2}.$$

Case IV. If the upper medium M_2 is neglected and in the absence of two-temperature effect, we obtain the amplitude ratios at the free surface of micropolar generalized thermoelastic solid half-space as

$$\sum_{j=1}^{4}a_{ij}Z_j=Y_i\quad(i=1,2,3,4), \tag{30}$$

where the values of a_{ij} are given by

$$a_{1i}=\left(d_1+d_2\left(1-\frac{V_i^2}{V_0^2}\sin^2\theta_0\right)\right)\frac{\omega^2}{V_i^2}+(1-\tau_1\iota\omega)f_i,\quad a_{1j}=d_2\frac{\omega^2}{V_jV_0}\sin\theta_0\sqrt{1-\frac{V_j^2}{V_0^2}\sin^2\theta_0},$$
$$a_{2i}=-(2d_4+d_5)\frac{\omega^2}{V_iV_0}\sin\theta_0\sqrt{1-\frac{V_i^2}{V_0^2}\sin^2\theta_0},$$
$$a_{2j}=\left(d_4\frac{\omega^2}{V_j^2}\left(1-2\frac{V_j^2}{V_0^2}\sin^2\theta_0\right)+d_5\frac{\omega^2}{V_j^2}\left(1-\frac{V_j^2}{V_0^2}\sin^2\theta_0\right)\right)-d_5f_j,$$

$$a_{3i} = 0, \quad a_{3j} = \iota \frac{\omega}{V_j} \sqrt{1 - \frac{V_j^2}{V_0^2} \sin^2\theta_0 f_j}, \quad a_{4i} = \iota \frac{\omega}{V_i} \sqrt{1 - \frac{V_i^2}{V_0^2} \sin^2\theta_0 f_i}, \quad a_{4j} = 0,$$

and

$$Z_1 = \frac{S_1}{A^*}, \quad Z_2 = \frac{S_2}{A^*}, \quad Z_3 = \frac{T_3}{A^*}, \quad Z_4 = \frac{T_4}{A^*}, \tag{31}$$

where Z_1, Z_2, Z_3, and Z_4 are the amplitude ratios of the reflected LD, T, and coupled CD-I and CD-II waves in medium M_1.

The above results are in agreement with those obtained by Singh and Kumar [1998], changing the dimensionless quantities into physical quantities.

7. Numerical results and discussion

For numerical computations, we take the following values of the relevant parameters for both the half-spaces.

Following Eringen [1984], the values of the micropolar constants for medium M_1 are taken as

$$\lambda = 9.4 \times 10^{10}\,\mathrm{Nm^{-2}}, \quad \mu = 4.0 \times 10^{10}\,\mathrm{Nm^{-2}}, \quad K = 1.0 \times 10^{10}\,\mathrm{Nm^{-2}},$$
$$\gamma = 7.79 \times 10^{-10}\,\mathrm{N}, \quad j = 0.002 \times 10^{-17}\,\mathrm{m^2}, \quad \rho = 1.74 \times 10^3\,\mathrm{Kgm^{-3}},$$

and the thermal parameters for medium M_1 are taken as

$$T_0 = 0.298\,\mathrm{K}, \quad \nu = 0.268 \times 10^8\,\mathrm{Nm^{-2}K^{-1}}, \quad c^* = 0.104 \times 10^4\,\mathrm{Jkg^{-1}K^{-1}}, \quad a = 0.3\,\mathrm{m^2},$$
$$K^* = 1.7 \times 10^2\,\mathrm{Jm^{-1}s^{-1}K^{-1}}, \quad \omega = 1, \quad \tau_0 = 0.813 \times 10^{-12}\,\mathrm{s}.$$

Following Gauthier [1982], the values of the micropolar constants for medium M_2 are taken as

$$\bar{\lambda} = 7.59 \times 10^{10}\,\mathrm{Nm^{-2}}, \quad \bar{\mu} = 0.00189 \times 10^{13}\,\mathrm{Nm^{-2}}, \quad \bar{j} = 0.00196 \times 10^{-16}\,\mathrm{m^2},$$
$$\bar{K} = 0.0149 \times 10^{10}\,\mathrm{Nm^{-2}}, \quad \bar{\gamma} = 2.68 \times 10^{-7}\,\mathrm{N}, \quad \bar{\rho} = 2.19 \times 10^3\,\mathrm{Kgm^{-3}}.$$

The thermal parameters for medium M_2 are taken to be of comparable magnitudes:

$$\bar{T}_0 = 0.0296\,\mathrm{K}, \quad \bar{\nu} = 0.2603 \times 10^7\,\mathrm{Nm^{-2}K^{-1}}, \quad \bar{c}^* = 0.921 \times 10^4\,\mathrm{Jkg^{-1}K^{-1}},$$
$$\bar{K}^* = 2.04 \times 10^2\,\mathrm{Jm^{-1}s^{-1}K^{-1}}, \quad \bar{a} = 0.1\,\mathrm{m^2}, \quad \bar{\tau}_0 = 0.713 \times 10^{-12}\,\mathrm{s}.$$

In Figures 2–25, we represent with the solid line the incident wave for the micropolar thermoelastic solid with one relaxation time and two temperatures (NL1), with the small-dashed line the incident wave for the micropolar thermoelastic solid with one relaxation time (ZL1), with the dash-dot-dash line the incident wave for the thermoelastic solid with one relaxation time (TZL1), and with the large-dashed line the incident wave for the thermoelastic solid with one relaxation time and two temperatures (TL1).

7.1. *Incident LD wave.* Variations of amplitude ratios $|Z_i|$, $1 \leq i \leq 8$, with angle of incidence θ_0, for incident LD waves are shown in Figures 2–9.

Figures 2 and 3 show that the values of the amplitude ratios $|Z_1|$ and $|Z_2|$ for ZL1 remain greater than the values for NL1 and that the values for TL1 remain greater than the values for TZL1 in the whole range. The values of $|Z_2|$ for NL1 are magnified by a factor of 10^2. Figures 4 and 5 show that the values

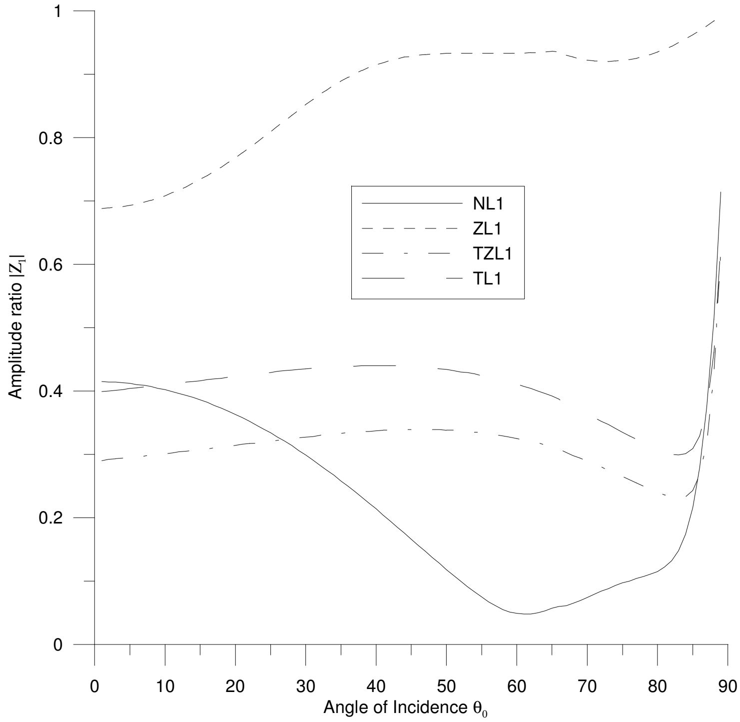

Figure 2. Variation of amplitude ratio with angle of incidence for LD wave.

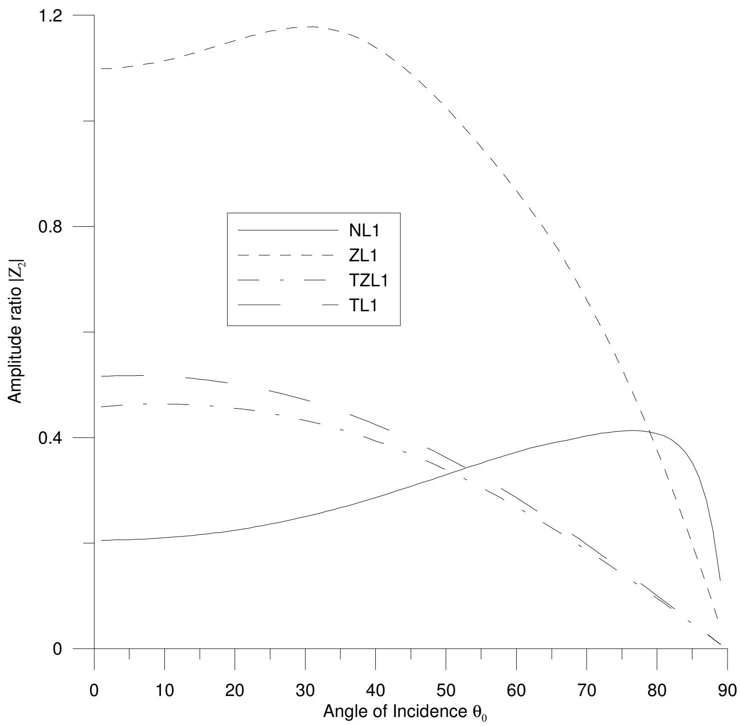

Figure 3. Variation of amplitude ratio with angle of incidence for LD wave.

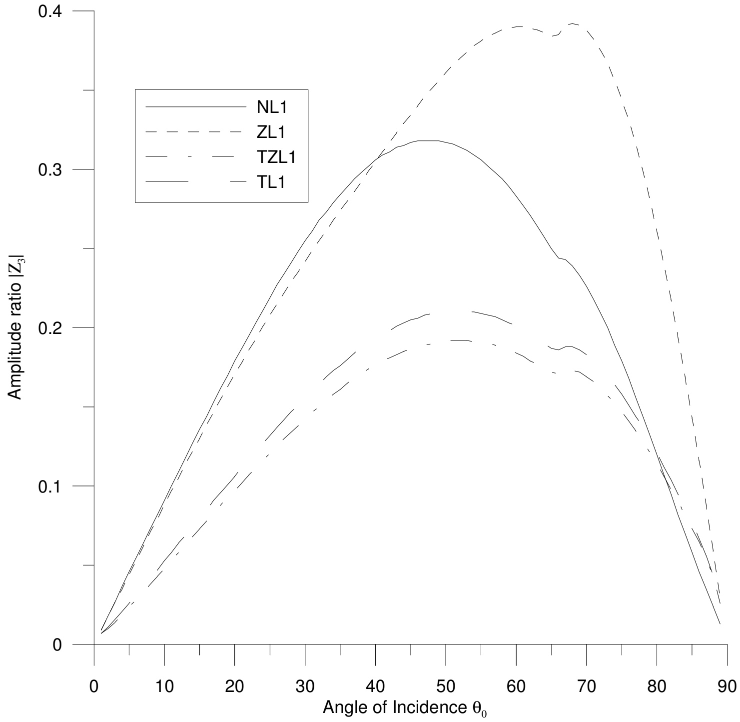

Figure 4. Variation of amplitude ratio with angle of incidence for LD wave.

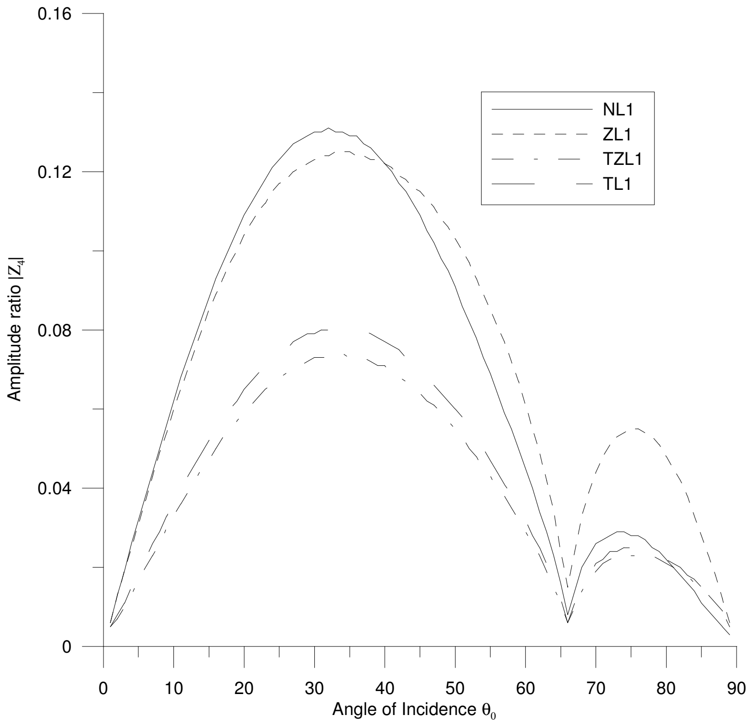

Figure 5. Variation of amplitude ratio with angle of incidence for LD wave.

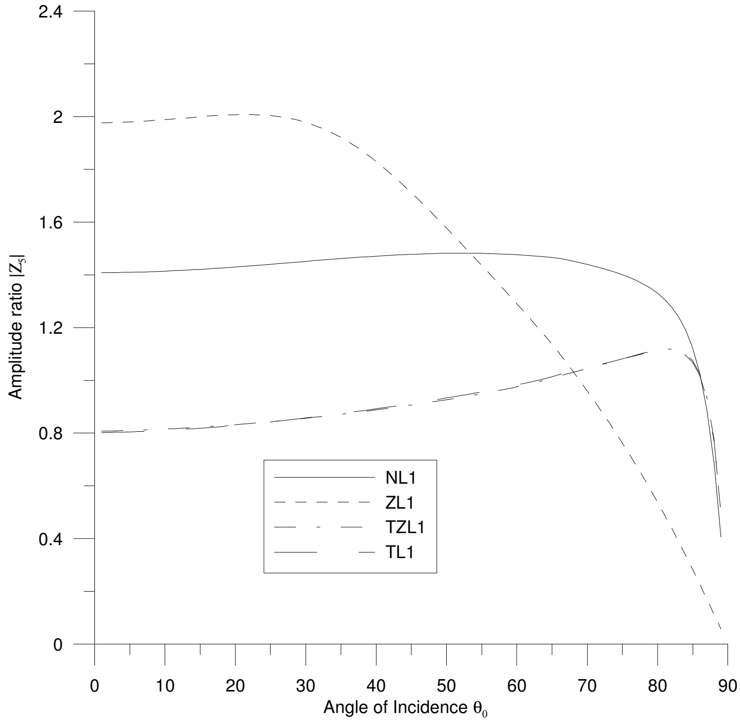

Figure 6. Variation of amplitude ratio with angle of incidence for LD wave.

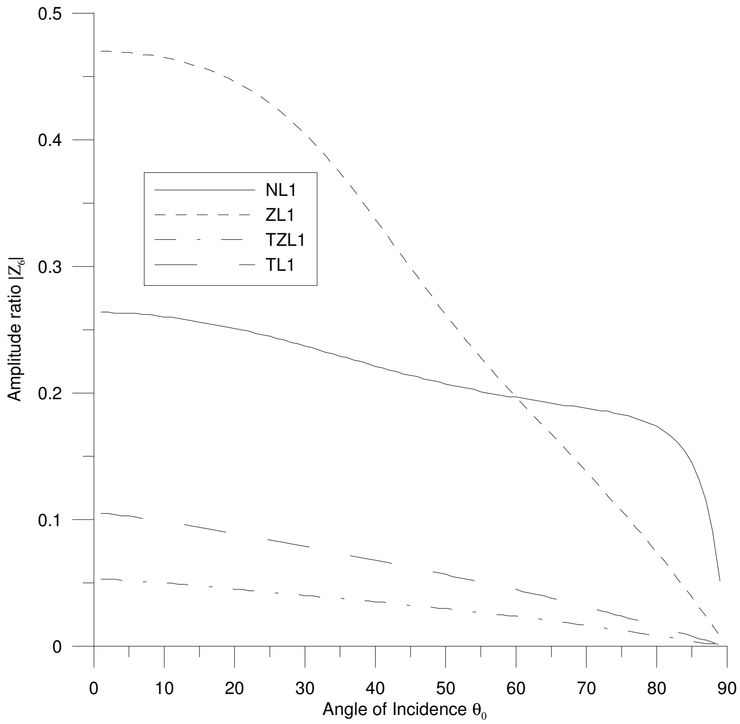

Figure 7. Variation of amplitude ratio with angle of incidence for LD wave.

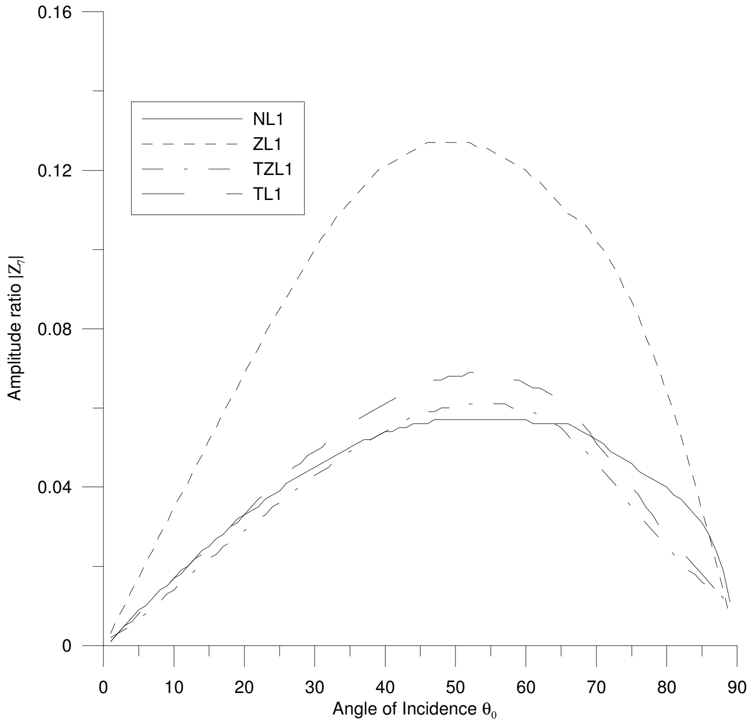

Figure 8. Variation of amplitude ratio with angle of incidence for LD wave.

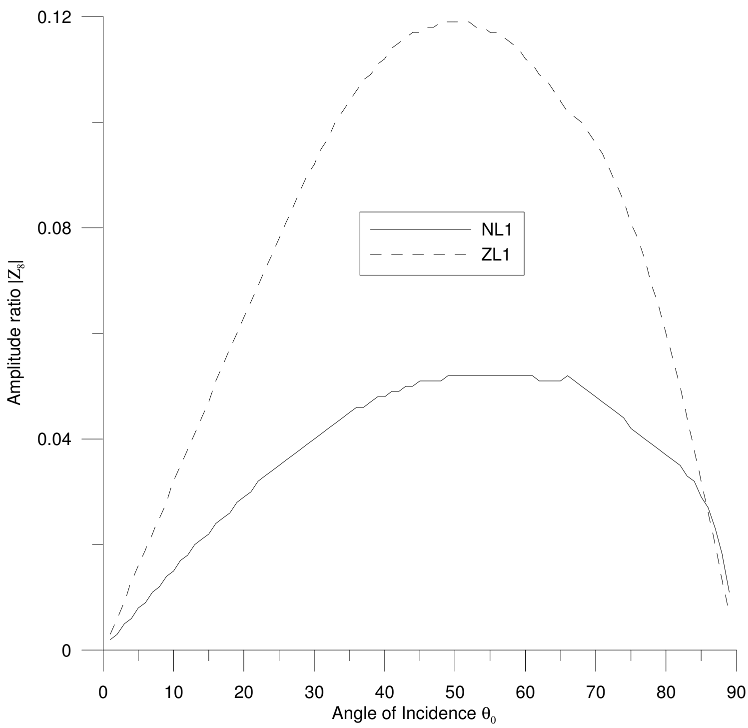

Figure 9. Variation of amplitude ratio with angle of incidence for LD wave.

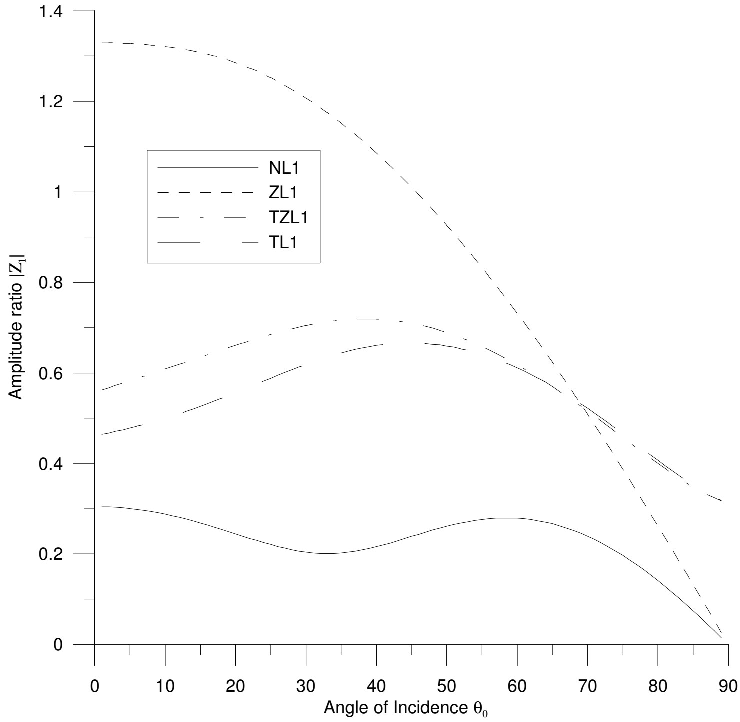

Figure 10. Variation of amplitude ratio with angle of incidence for T wave.

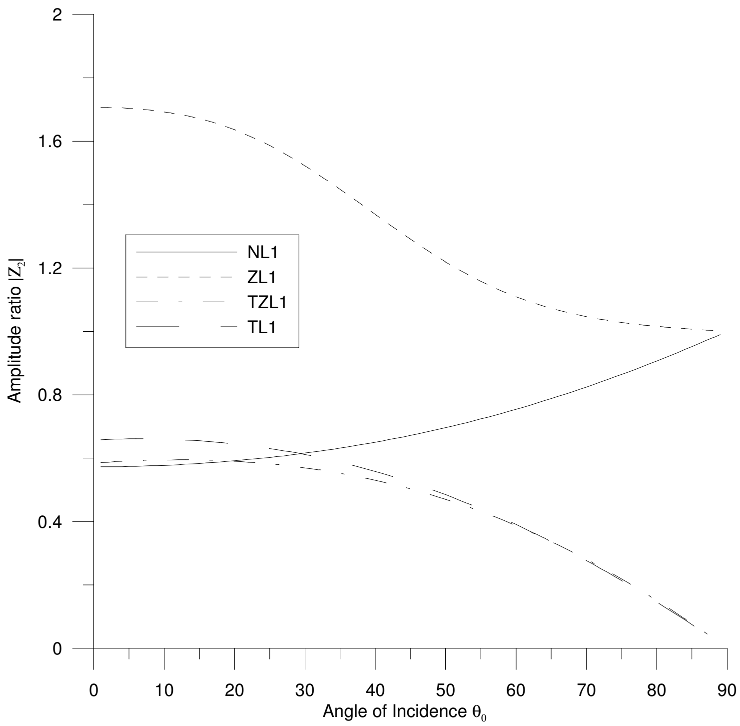

Figure 11. Variation of amplitude ratio with angle of incidence for T wave.

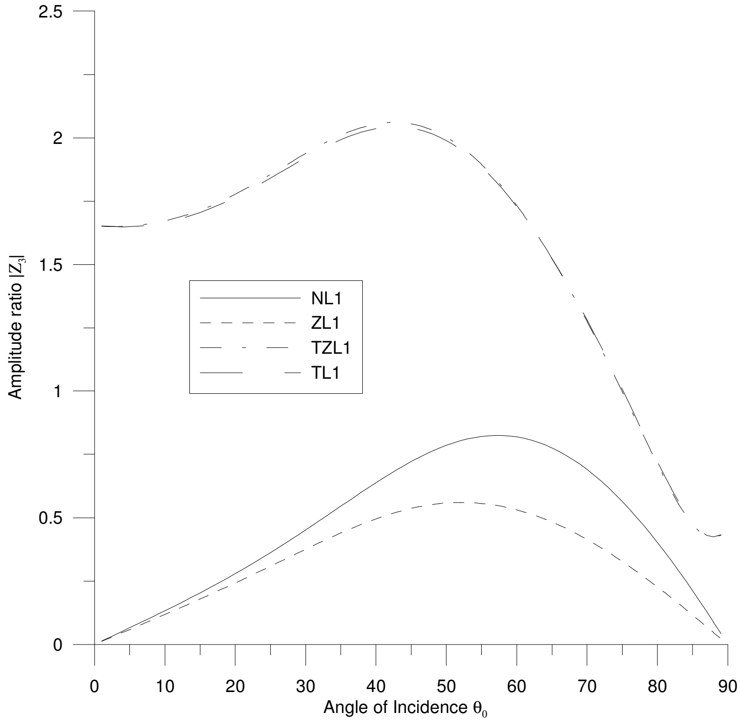

Figure 12. Variation of amplitude ratio with angle of incidence for T wave.

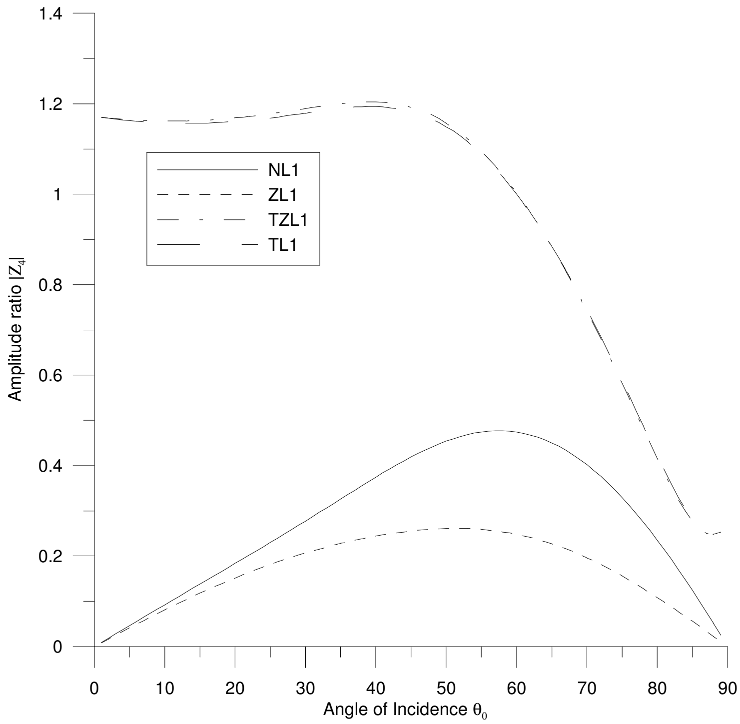

Figure 13. Variation of amplitude ratio with angle of incidence for T wave.

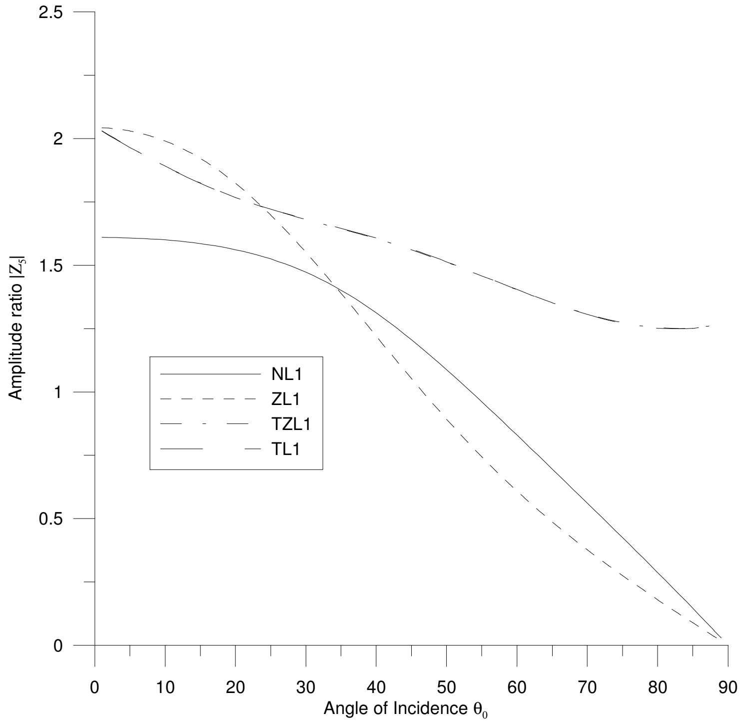

Figure 14. Variation of amplitude ratio with angle of incidence for T wave.

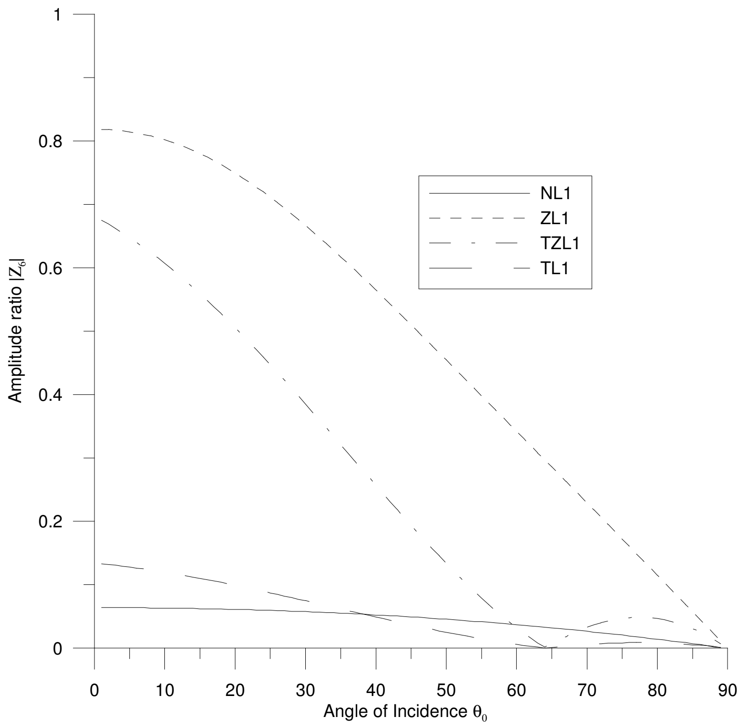

Figure 15. Variation of amplitude ratio with angle of incidence for T wave.

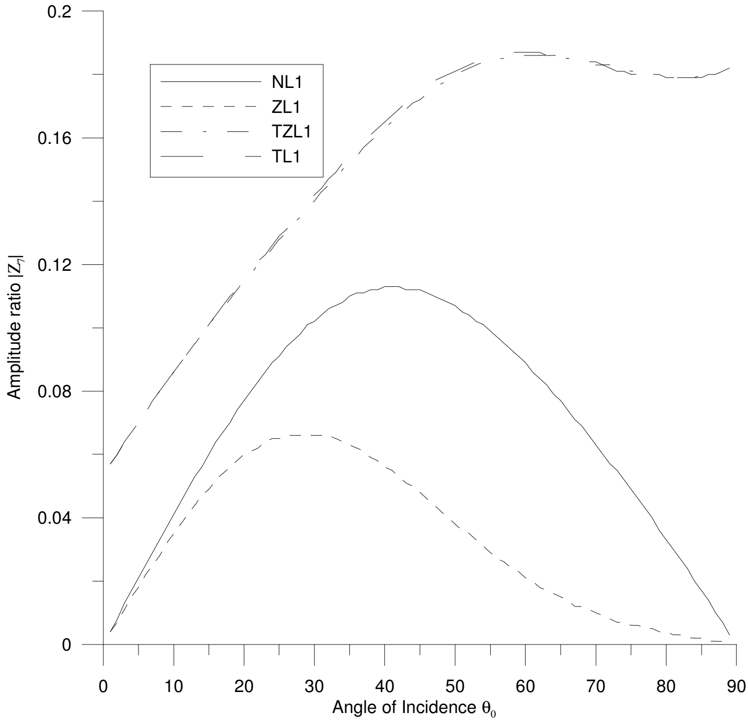

Figure 16. Variation of amplitude ratio with angle of incidence for T wave.

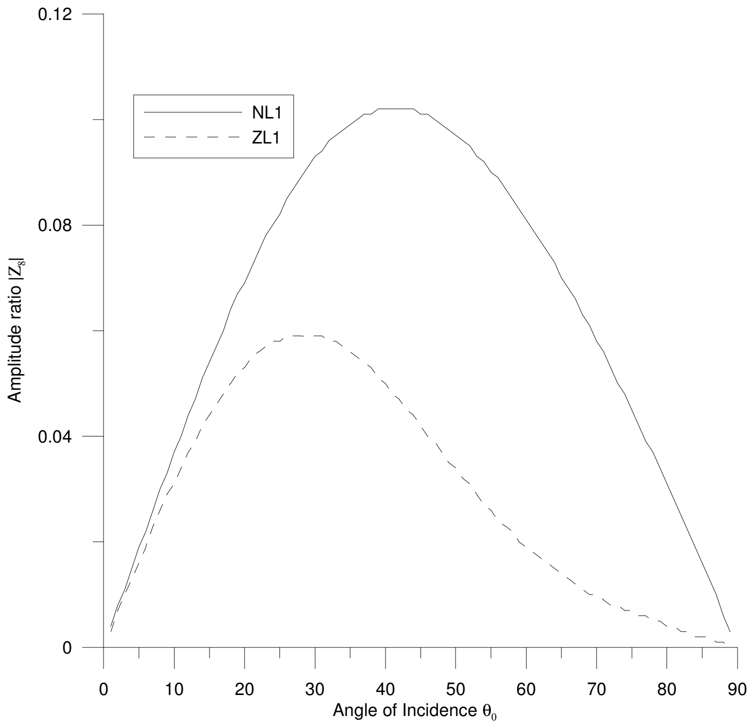

Figure 17. Variation of amplitude ratio with angle of incidence for T wave.

of $|Z_3|$ and $|Z_4|$ for NL1, ZL1, TZL1, and TL1 are oscillatory and the values for TL1 remain greater than the values for TZL1 in the whole range.

Figure 6 shows that the values of $|Z_5|$ for ZL1, TL1, and TZL1 increase in the whole range, except near the grazing incidence, where the values decrease sharply. Figure 7 shows that the values of $|Z_6|$ for NL1, ZL1, TL1, and TZL1 decrease in the whole range. The values of $|Z_6|$ for ZL1 and TL1 are magnified by a factor of 10 and for NL1 by 10^3.

Notice from Figures 8 and 9 that the values of $|Z_7|$ and $|Z_8|$ for ZL1 remain greater than the values for NL1 in the whole range, except near the grazing incidence, where the values for NL1 are greater. The values of $|Z_7|$ for TZL1 and TL1 are magnified by a factor of 10.

7.2. ***Incident T wave.*** Variations of the amplitude ratios $|Z_i|$, $1 \le i \le 8$, with angle of incidence θ_0, for incident T waves are shown in Figures 10–17.

Figure 10 shows that the values of $|Z_1|$ for ZL1 remain greater than the values for NL1 in the whole domain. The values of $|Z_1|$ for NL1, ZL1, TZL1, and TL1 are reduced by a factor of 10. It is evident from Figure 11 that the values of $|Z_2|$ for NL1 increase and those for ZL1, TZL1, and TL1 decrease in the whole range.

Figure 12 shows that the values of $|Z_3|$ for TL1 are greater than those for NL1, and the values for TZL1 are greater than those for ZL1, in the whole range. Figure 13 shows that the behavior of the variation of the amplitude ratio $|Z_4|$ is similar to that of $|Z_3|$ with a different magnitude. Figure 14 shows that the values of the amplitude ratio $|Z_5|$ for NL1 are greater than for ZL1, except in the range $0° < \theta_0 < 33°$, where the behavior is reversed.

Figure 15 shows that the values of $|Z_6|$ for NL1, ZL1, TZL1, and TL1 decrease in the whole range, while the values for TZL1 and TL1 follow an oscillatory pattern near the grazing incidence. The values of $|Z_6|$ for ZL1, TZL1, and TL1 and the values of $|Z_7|$ for TZL1 and TL1 are magnified by multiplying the original value by 10. It can be noticed from Figure 16 that values of $|Z_7|$ for NL1, ZL1, TZL1, and TL1 are oscillatory in the whole range. Figure 17 shows that the values of $|Z_8|$ for NL1 remain greater than the values for ZL1 in the whole range.

7.3. ***Incident CD-I wave.*** Variations of the amplitude ratios $|Z_i|$, $1 \le i \le 8$, with angle of incidence θ_0, for incident CD-I waves are shown in Figures 18–25.

Figures 18 and 19 show that the values of the amplitude ratios $|Z_1|$ and $|Z_2|$ for NL1 and ZL1 oscillate, while the values for TZL1 and TL1 increase with increase in θ_0. The values of $|Z_1|$ for TZL1 and NL1 remain greater than those for TL1 and ZL1, respectively, in the whole range. The values of $|Z_2|$ for NL1 are magnified by a factor of 10^2.

Figures 20 and 21 show that the values of $|Z_3|$ and $|Z_4|$ for NL1 are greater than those for TL1, and the values for ZL1 are greater than those for TZL1, in the whole domain, which reveals the effect of micropolarity. Figures 22 and 23 show that the values of $|Z_5|$ and $|Z_6|$ for ZL1 remain greater than the values for NL1 in the whole range. The values of $|Z_6|$ for NL1 are magnified by a factor of 10^4 and for TL1 and TZL1 by a factor of 10.

It is shown in Figures 24 and 25 that the values of $|Z_7|$ and $|Z_8|$ for NL1 and ZL1 decrease in the whole range, except near $\theta_0 = 90°$, where the values increase. The values of $|Z_7|$ for TL1 and TZL1 are magnified by a factor of 10^2.

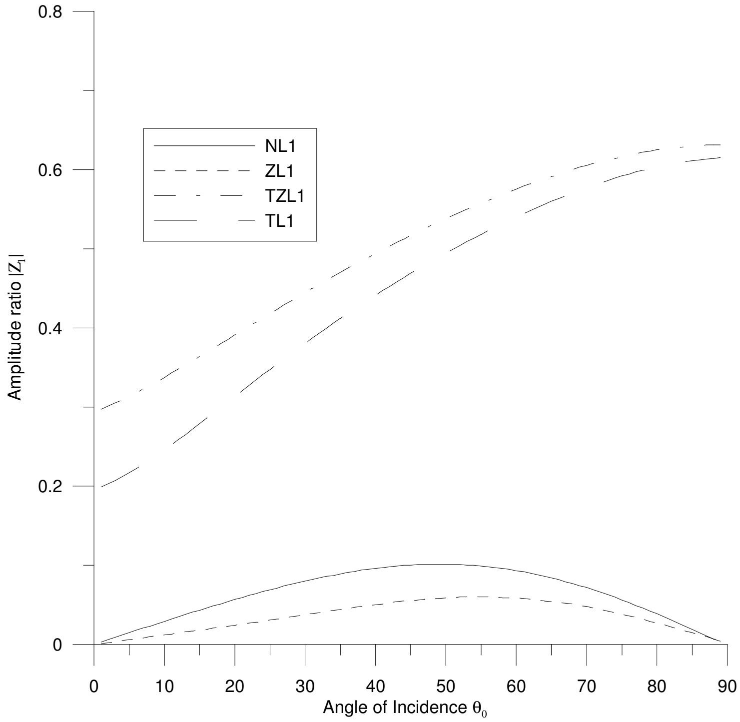

Figure 18. Variation of amplitude ratio with angle of incidence for CD-I wave.

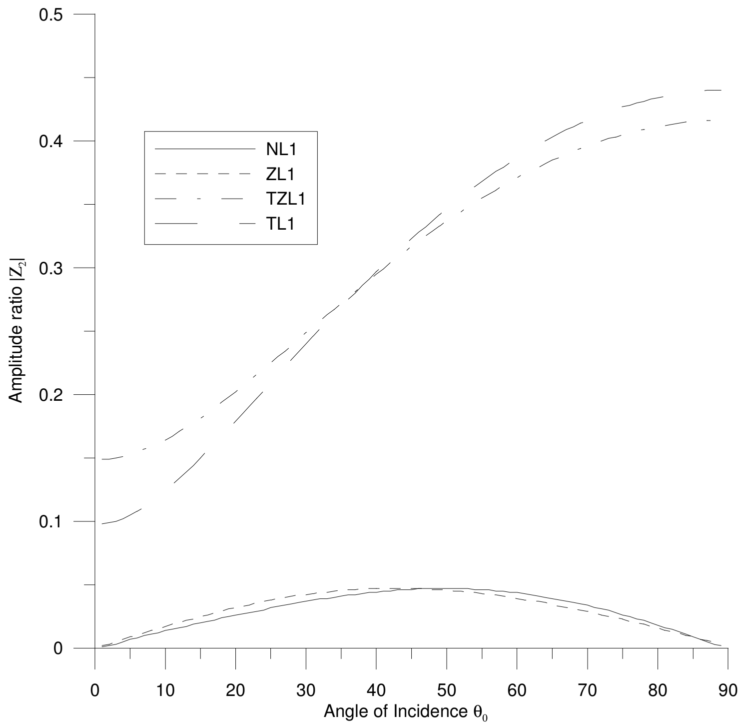

Figure 19. Variation of amplitude ratio with angle of incidence for CD-I wave.

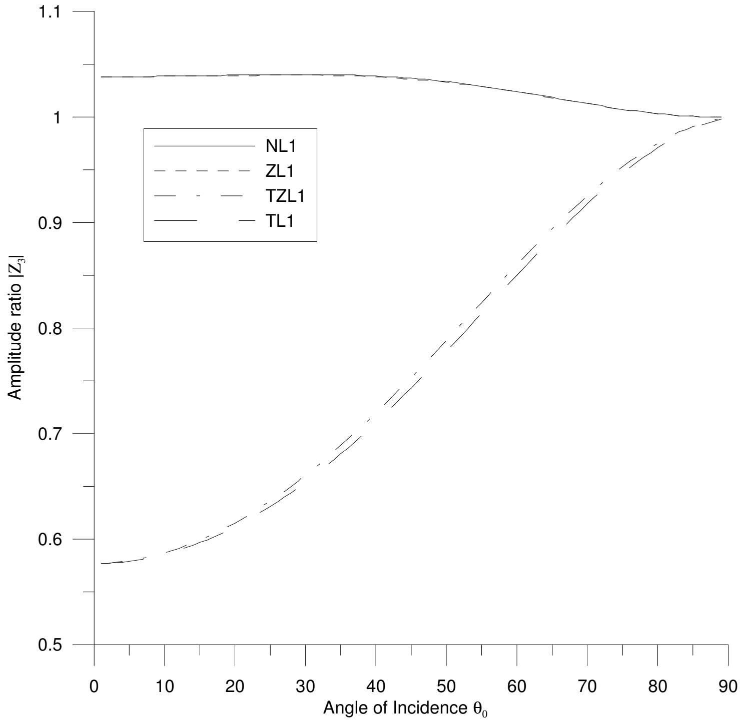

Figure 20. Variation of amplitude ratio with angle of incidence for CD-I wave.

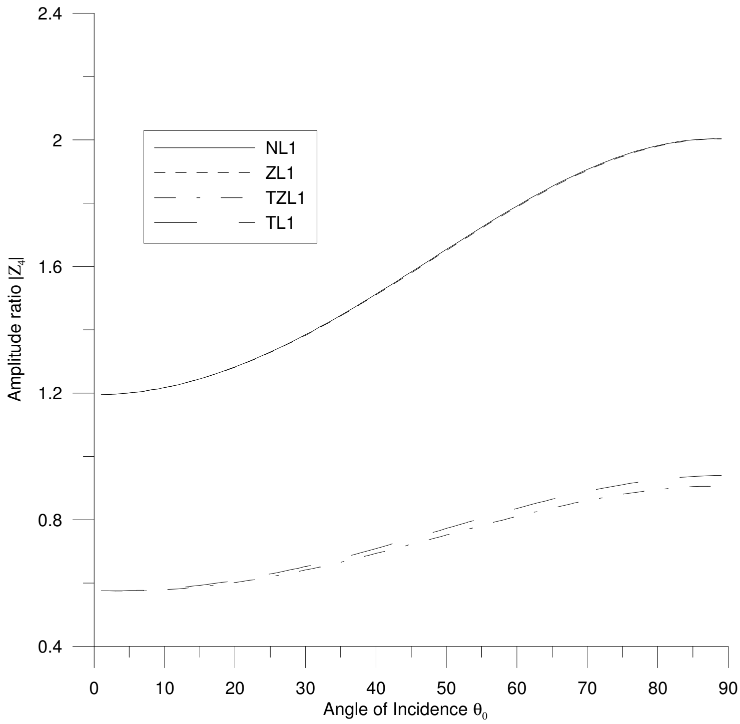

Figure 21. Variation of amplitude ratio with angle of incidence for CD-I wave.

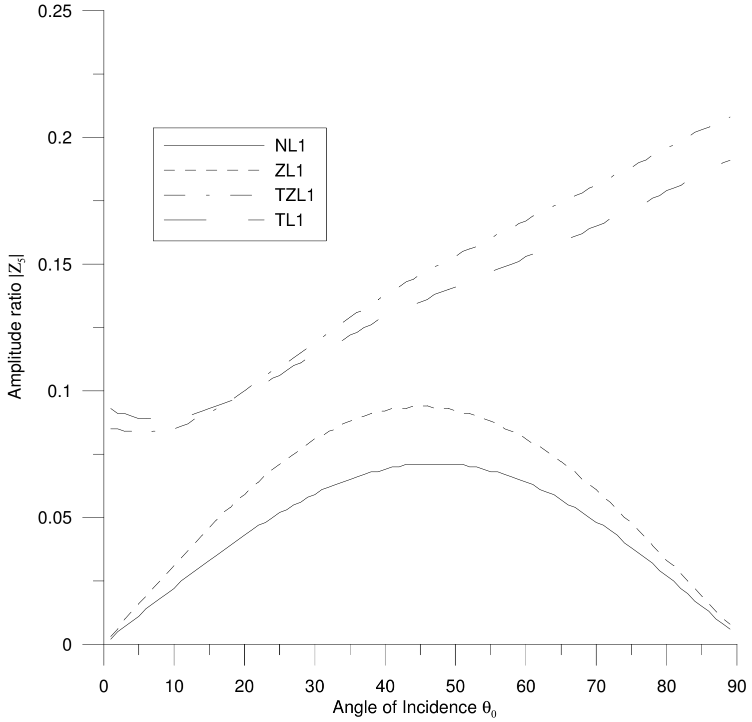

Figure 22. Variation of amplitude ratio with angle of incidence for CD-I wave.

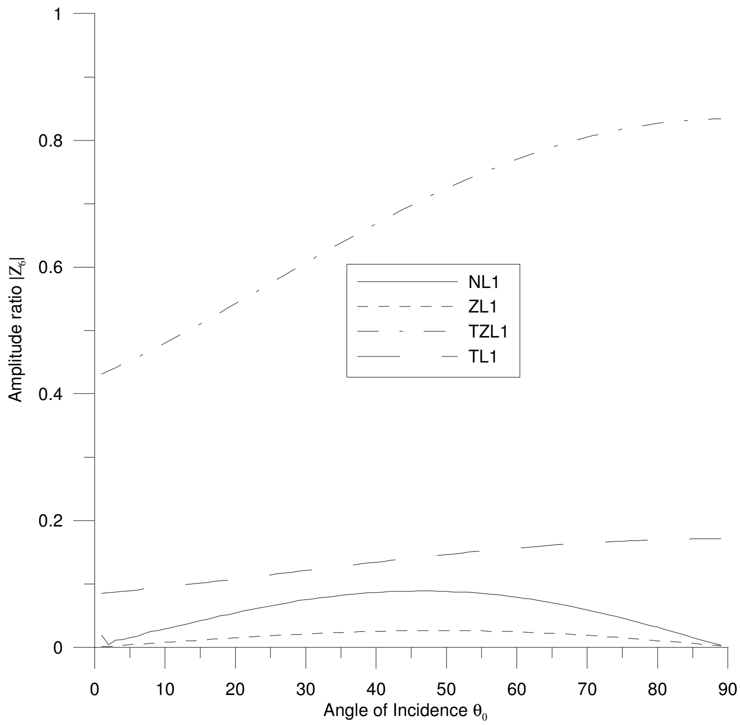

Figure 23. Variation of amplitude ratio with angle of incidence for CD-I wave.

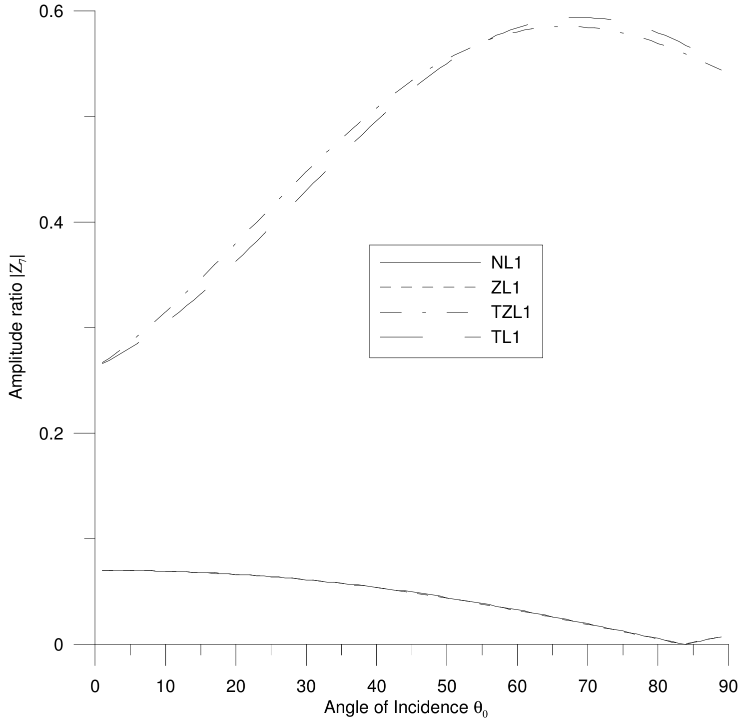

Figure 24. Variation of amplitude ratio with angle of incidence for CD-I wave.

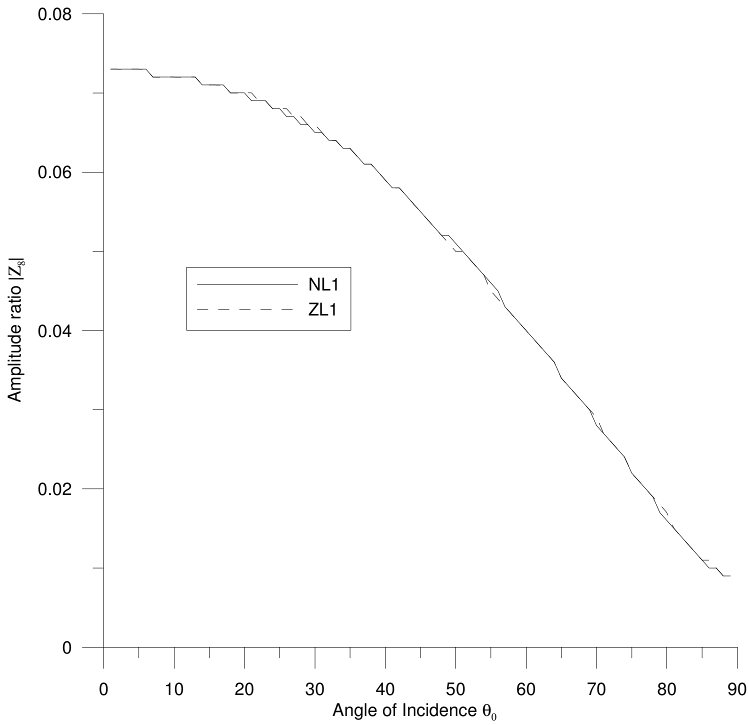

Figure 25. Variation of amplitude ratio with angle of incidence for CD-I wave.

8. Conclusion

The expressions for the reflection and transmission coefficients of various reflected and transmitted waves have been derived. When an LD wave is incident, the values of the amplitude ratios for NL1 and ZL1 follow an oscillatory pattern and the magnitudes of the amplitude ratios $|Z_i|$, $1 \le i \le 8$, for TL1 remain greater than the values for TZL1. When a T wave is incident, the values of $|Z_i|$; $i = 3, 4, 7, 8$ for NL1 remain more than the values for TL1 that reveals the effect of two temperatures. When a CD-I wave is incident the values of the amplitude ratios $|Z_1|$, $|Z_2|$, $|Z_5|$, $|Z_6|$, and $|Z_7|$ for TZL1 and TL1 remain greater than the values for ZL1 and NL1, respectively, due to the effect of micropolarity.

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