

Pacific Journal of Mathematics

**SIMPLIFIED PROOFS OF "SOME TAUBERIAN THEOREMS"
OF JAKIMOVSKI: ADDENDUM AND CORRIGENDUM**

C. T. RAJAGOPAL

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Dr. B. Kuttner has kindly drawn my attention to a paper by F. Hausdorff, *Die Äquivalenz der Hölderschen und Cesaròschen Grenzwerte negativer Ordnung*, Math. Z., 31 (1930), 186–196, which contains a generalization of Jakimovski's fundamental theorem discussed in § 2 of my paper (this volume, pp. 955–960) and Szász's product-theorem referred to in § 3 of my paper, under numbers VI and III respectively in the list of numbered results I–VIII. There is a close connection between Hausdorff's paper and mine, as shown, for instance, by a comparison of Lemmas 1, 3 in the latter with the interpretation of Γ_{-k} and the result numbered VII in the former (pp. 195–6). It is unfortunate that I should have been ignorant of Hausdorff's paper and that the paper should have escaped mention in the lists of references provided by such works as G. H. Hardy's *Divergent series* and O. Szász's *Introduction to the theory of divergent series*.

Dr. Kuttner has also been good enough to call my attention to the fact that my step numbered (6) in p. 958 is not a valid deduction from my Lemma 2. For the convenience of the reader, I add that my incorrect argument may be replaced by the following, after the deletion of the last two lines of p. 957 and the lines 1, 2, 6, 7, 8, 9 of p. 958.

Since, if $k=1$ we infer at once from Lemma 2 that $s_n=o(1)$, we suppose that $k \geq 2$ and reduce this case to the case $k=1$. When $k \geq 2$, (7) in p. 958 shows that

$$\sum_{r=0}^{\infty} \Delta^k s_{r-k} x^r = o(1), \quad x \rightarrow 1-0,$$

that is, that the series $\sum \Delta^k s_{r-k}$ is summable (A) to 0. In this series, the n th term $\Delta^k s_{n-k} = o(n^{-k})$, $n \rightarrow \infty$, by hypothesis, so that the series is convergent and necessarily to 0. Therefore

$$\Delta^{k-1} s_{n-k+1} = - \sum_{r=0}^n \Delta^k s_{r-k} = \sum_{r=n+1}^{\infty} \Delta^k s_{r-k} = \sum_{r=n+1}^{\infty} o(r^{-k}) = o(n^{-k+1}), \quad n \rightarrow \infty.$$

By repetitions of this argument (if necessary), we reduce $k \dots$

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Pacific Journal of Mathematics

Vol. 7, No. 4

April, 1957

Robert Geroge Buschman, <i>A substitution theorem for the Laplace transformation and its generalization to transformations with symmetric kernel</i>	1529
S. D. Conte, <i>Numerical solution of vibration problems in two space variables</i>	1535
Paul Dedecker, <i>A property of differential forms in the calculus of variations</i>	1545
H. Delange and Heini Halberstam, <i>A note on additive functions</i>	1551
Jerald L. Ericksen, <i>Characteristic direction for equations of motion of non-Newtonian fluids</i>	1557
Avner Friedman, <i>On two theorems of Phragmén-Lindelöf for linear elliptic and parabolic differential equations of the second order</i>	1563
Ronald Kay Gettoor, <i>Additive functionals of a Markov process</i>	1577
U. C. Guha, <i>(γ, k)-summability of series</i>	1593
Alvin Hausner, <i>The tauberian theorem for group algebras of vector-valued functions</i>	1603
Lester J. Heider, <i>T-sets and abstract (L)-spaces</i>	1611
Melvin Henriksen, <i>Some remarks on a paper of Aronszajn and Panitchpakdi</i>	1619
H. M. Lieberstein, <i>On the generalized radiation problem of A. Weinstein</i>	1623
Robert Osserman, <i>On the inequality $\Delta u \geq f(u)$</i>	1641
Calvin R. Putnam, <i>On semi-normal operators</i>	1649
Binyamin Schwarz, <i>Bounds for the principal frequency of the non-homogeneous membrane and for the generalized Dirichlet integral</i>	1653
Edward Silverman, <i>Morrey's representation theorem for surfaces in metric spaces</i>	1677
V. N. Singh, <i>Certain generalized hypergeometric identities of the Rogers-Ramanujan type. II</i>	1691
R. J. Smith, <i>A determinant in continuous rings</i>	1701
Drury William Wall, <i>Sub-quasigroups of finite quasigroups</i>	1711
Sadayuki Yamamuro, <i>Monotone completeness of normed semi-ordered linear spaces</i>	1715
C. T. Rajagopal, <i>Simplified proofs of "Some Tauberian theorems" of Jakimovski: Addendum and corrigendum</i>	1727
N. Aronszajn and Prom Panitchpakdi, <i>Correction to: "Extension of uniformly continuous transformations in hyperconvex metric spaces"</i>	1729
Alfred Huber, <i>Correction to: "The reflection principle for polyharmonic functions"</i>	1731