Pacific Journal of Mathematics

ISOMORPHISME D'HYPERGROUPOÏ DES ISOTOPES

A. Sade

Vol. 9, No. 2 June 1959

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Un *Hypergroupoïde* [2], est un ensemble, E, muni d'une loi de composition faisant correspondre à tout couple ordonné d'éléments $x, y \in E$ un sous-ensemble non vide de $E, x * y = (a, b, c, \cdots) \subseteqq E$. Un *scalaire*, [1, p. 707], est un élément s, tel que, $\forall x \in E$, $x * s$ et $s * x$ se réduisent à un seul élément. Un Hypergroupoïde est *associatif si* $\forall x, y, z \in E$, $(x * y) * z = x * (y * z)$. Une *unité scalaire* est un *élément*, u, tel que $\forall x \in E, x * u = u * x = x$. Deux hypergroupoïdes $H(*)$ et $H'(\times)$, définis sur le même ensemble E, sont *isotopes* si, ξ, η, ζ, étant trois applications biunivoques de E sur lui-même,

$$\forall x, y, z \in E, \ x * y = z \rightleftarrows x\xi \times y\eta = z\zeta \ , \tag{1}$$

ou $x * y = (x\xi \times y\eta)\zeta^{-1}$. Ils sont *isomorphes* si $\xi = \eta = \zeta$.

Théorème. *Si un hypergroupoïde $H(\times)$, avec l'unité scalaire u, est isotope (ξ, η, ζ) d'un hypergroupoïde associatif $G(*)$, défini sur le même ensemble, alors G et H coïncident par l'isomorphisme $x \to x\theta = x\xi\zeta^{-1}\eta$.*

Preuve. Puisque G est associatif,

$$\forall x, y, z \in E, \ x * (y * z) = (x * y) * z \ .$$

Donc, sur l'isotope H, (1)

$$(x\xi \times (y\xi \times z\eta)\zeta^{-1}\eta)\zeta^{-1} = ((x\xi \times y\eta)\zeta^{-1}\xi \times z\eta)\zeta^{-1} \ . \tag{2}$$

ξ, η étant des permutations de E,

$$(\exists) x, z \in E, x\xi = u, z\eta = u \ .$$

Puisque (2) est vérifiée $\forall x, y$, en faisant $x\xi = u$, on a

$$(y\xi \times z\eta)\zeta^{-1}\eta = y\eta\zeta^{-1}\xi \times z\eta \ ,$$

$$(x * y)\eta = x\eta\zeta^{-1}\xi \times y\eta \ . \tag{3}$$

En faisant au contraire $z\eta = u$, on a

$$x\xi \times y\xi\zeta^{-1}\eta = (x\xi \times y\eta)\zeta^{-1}\xi \ ,$$

$$\forall t \in E, t \times y\theta = (t \times y\eta)\zeta^{-1}\xi \ . \tag{4}$$

Received December 8, 1958.

Enfin, en faisant à la fois $x\xi = u$ et $z\eta = u$, on a

(5) $$\forall y,\ y\eta\zeta^{-1}\xi = y\xi\zeta^{-1}\eta = y\theta\ .$$

Donc, d'après (3),

$$(x * y)\eta\zeta^{-1}\xi = (x\eta\zeta^{-1}\xi \times y\eta)\zeta^{-1}\xi\ .$$

D'après (5)

$$(x * y)\theta = (x\theta \times y\eta)\zeta^{-1}\xi\ ;$$

d'après (4)

$$(x * y)\theta = x\theta \times y\theta\ .$$

La démonstration ne serait plus valable si u n'était pas scalaire bilatère et si l'associativité se réduisait à l'inclusion

$$(x * y) * z \supseteqq x * (y * z)\ .$$

RÉFÉRENCES

1. Melvin Dresher, and Oystein Ore, *Theory of multigroups,* Amer. J. Math. **60** (1938), 705–733.

2. Yuzo Utumi, *On Hypergroups of qroup right cosets*, Osaka Math. J. **1** (1949), 73–80.

MARSEILLE

PACIFIC JOURNAL OF MATHEMATICS

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be typewritten (double spaced), and the author should keep a complete copy. Manuscripts may be sent to any one of the four editors. All other communications to the editors should be addressed to the managing editor, L. J. Paige at the University of California, Los Angeles 24, California.

50 reprints per author of each article are furnished free of charge; additional copies may be obtained at cost in multiples of 50.

The *Pacific Journal of Mathematics* is published quarterly, in March, June, September, and December. The price per volume (4 numbers) is $12.00; single issues, $3.50. Back numbers are available. Special price to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: $4.00 per volume; single issues, $1.25.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 2120 Oxford Street, Berkeley 4, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), No. 6, 2-chome, Fujimi-cho, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

Pacific Journal of Mathematics
Vol. 9, No. 2 June, 1959

Lee William Anderson, *On the breadth and co-dimension of a topological lattice* . . . 327
Frank W. Anderson and Robert L. Blair, *Characterizations of certain lattices of functions* . . . 335
Donald Charles Benson, *Extensions of a theorem of Loewner on integral operators* . . . 365
Errett Albert Bishop, *A duality theorem for an arbitrary operator* . . . 379
Robert McCallum Blumenthal and Ronald Kay Getoor, *The asymptotic distribution of the eigenvalues for a class of Markov operators* . . . 399
Delmar L. Boyer and Elbert A. Walker, *Almost locally pure Abelian groups* . . . 409
Paul Civin and Bertram Yood, *Involutions on Banach algebras* . . . 415
Lincoln Kearney Durst, *Exceptional real Lehmer sequences* . . . 437
Eldon Dyer and Allen Lowell Shields, *Connectivity of topological lattices* . . . 443
Ronald Kay Getoor, *Markov operators and their associated semi-groups* . . . 449
Bernard Greenspan, *A bound for the orders of the components of a system of algebraic difference equations* . . . 473
Branko Grünbaum, *On some covering and intersection properties in Minkowski spaces* . . . 487
Bruno Harris, *Derivations of Jordan algebras* . . . 495
Henry Berge Helson, *Conjugate series in several variables* . . . 513
Isidore Isaac Hirschman, Jr., *A maximal problem in harmonic analysis. II* . . . 525
Alfred Horn and Robert Steinberg, *Eigenvalues of the unitary part of a matrix* . . . 541
Edith Hirsch Luchins, *On strictly semi-simple Banach algebras* . . . 551
William D. Munro, *Some iterative methods for determining zeros of functions of a complex variable* . . . 555
John Rainwater, *Spaces whose finest uniformity is metric* . . . 567
William T. Reid, *Variational aspects of generalized convex functions* . . . 571
A. Sade, *Isomorphisme d'hypergroupoï des isotopes* . . . 583
Isadore Manual Singer, *The geometric interpretation of a special connection* . . . 585
Charles Andrew Swanson, *Asymptotic perturbation series for characteristic value problems* . . . 591
Jack Phillip Tull, *Dirichlet multiplication in lattice point problems. II* . . . 609
Richard Steven Varga, *p-cyclic matrices: A generalization of the Young-Frankel successive overrelaxation scheme* . . . 617