

Pacific Journal of Mathematics

**CORRECTION TO “EQUIVALENCE AND
PERPENDICULARITY OF GAUSSIAN PROCESSES”**

JACOB FELDMAN

CORRECTON TO "EQUIVALENCE AND PERPENDICULARITY OF GAUSSIAN PROCESSES"

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It has been kindly pointed out to me by D. Lowdenslager that, as it stands, the argument in [1] only works when $L_2(\mu)$ and $L_2(\nu)$ are separable. In particular, the theorem of von Neumann from [2], which is used there, only holds in separable Hilbert spaces. Our theorem nevertheless holds in the non-separable case; an argument will be supplied here enabling one to go from the separable to the general case. We retain notation and terminology of [1].

For any countable subset C of L , let \mathcal{S}_C be the σ -subalgebra of \mathcal{S} generated by C , L_C the linear subspace of L spanned by C , and μ_C, ν_C the restrictions of μ, ν to \mathcal{S}_C . $\bigcup_C \mathcal{S}_C$ is a σ -algebra contained in \mathcal{S} , and, since each $x \in L$ is in some L_C , each x in L is measurable with respect to $\bigcup_C \mathcal{S}_C$. Therefore $\mathcal{S} = \bigcup_C \mathcal{S}_C$. Now, suppose, under the assumptions of the theorem of [1], that μ and ν are not equivalent. Then there is some set in \mathcal{S} with μ -measure 0 and ν -measure > 0 (or vice versa). This set is in some \mathcal{S}_C . So μ_C and ν_C are not equivalent. By the separable case of the theorem, they are mutually perpendicular, i.e., there is some set in \mathcal{S}_C with μ -measure 0 and ν -measure 1. Thus μ and ν are mutually perpendicular.

Next we show that $\mu \sim \nu$ implies that the correspondence $x^\nu \xrightarrow{T} x^\mu$ between equivalence classes of functions has the property that T extends to an equivalence operator between the linear subspaces \bar{L}_μ and \bar{L}_ν of $L_2(\mu), L_2(\nu)$ generated by L . Assume, then, that $\mu \sim \nu$. By using the separable case, we easily see that T and T^{-1} are bounded. An argument on p. 704 of [1] still works to show that the extension of T to an operator from \bar{L}_μ onto \bar{L}_ν still has the property that, given ξ in \bar{L}_μ , there is an \mathcal{S} -measurable x such that $x^\mu = \xi$ and $x^\nu = T\xi$. Write T^*T as $\int \lambda dF(\lambda)$. Let $E_n = F\left(1 + \frac{1}{n}\right) - F\left(1 - \frac{1}{n}\right)$, $n = 2, 3, 4, \dots$. Let $E = \bigcap_n E_n$. I now assert $(I - E)\bar{L}_\mu$ is separable. For otherwise $(I - E_n)\bar{L}_\mu$ would be inseparable for some n , and one could therefore find a countable orthonormal infinite set ξ_1, ξ_2, \dots of elements of \bar{L}_μ for which $\|(T^*T - I)\xi_i\| \geq \frac{1}{n} \|\xi_i\|$, all i . Let H be the Hilbert space spanned by the ξ_i . Let \tilde{L} be the set of μ -measurable functions x on S such that $x^\mu \in H$. Let $\tilde{\mathcal{S}}$ be the σ -algebra spanned by them. Let $\tilde{\mu}, \tilde{\nu}$ be the completions of μ and ν , restricted to $\tilde{\mathcal{S}}$. Then the Hilbert spaces $\tilde{\bar{L}}_\mu, \tilde{\bar{L}}_\nu$ are isometric to H and $T(H)$,

respectively, in a natural way. Therefore they are separable, and, since $\tilde{\mu} \sim \tilde{\nu}$, the operator \tilde{T} induced by the correspondence $\tilde{x}^\mu \longrightarrow \tilde{x}^\nu$ is an equivalence operator. But T is unitarily equivalent to $T|H$, and $T|H$ was constructed so as *not* to be an equivalence operator, giving a contradiction.

To show T is an equivalence operator, it suffices to show this for $T|(I - E) \bar{L}_\mu$. Since $(I - E) \bar{L}_\mu$ is separable, we can reduce to the separable case exactly as in the last five sentences of the previous paragraph, with $(I - E) \bar{L}_\mu$ playing the role played there by H to show that T is an equivalence operator.

Finally, suppose that, for $x \in L$, $x^\mu = 0 \iff x^\nu = 0$, and that the one-to-one operator T from L_μ to L_ν induced thereby extends to an equivalence operator from \bar{L}_μ to \bar{L}_ν . It must be shown that $\mu \sim \nu$. If μ is not equivalent to ν , then as shown in the first paragraph (and using the notation established there) there is some countable subset C of L such that μ_C and ν_C are not equivalent. But the operator T_C induced by sending x^μ to x^ν for $x \in L_C$ is precisely the restriction of T to those elements in L_μ which come from L_C . Now, the restriction of T to a subspace is again an equivalence operator, so T_C extends to an equivalence operator from $(\bar{L}_C)_\mu$ to $(\bar{L}_C)_\nu$, which contradicts the separable case of the theorem.

Also, in reviewing [1], E. Nelson noticed that Lemma 1 is misstated. It should read "positive" instead of "self-adjoint," and, in (b), " $A^2 - I$ " rather than " $(A - I)^2$."

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2. J. von Neumann, *Charakterisierung des spektrums eines integral-operatoren*, Actuales Sci. Ind. 229, Paris, 1935.

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