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ON THE RING-LOGIC CHARACTER OF CERTAIN RINGS

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Introduction. Boolean rings $(B, \times, +)$ and Boolean logics (= Boolean algebras) $(B, \cap, *)$ though historically and conceptionally different, are equationally interdefinable in a familiar way [6]. With this equational interdefinability as motivation, Foster introduced and studied the theory of ring-logics. In this theory, a ring (or an algebra) R is studied modulo K , where K is an arbitrary transformation group in R . The Boolean theory results from the special choice, for K , of the "Boolean group," generated by $x^* = 1 - x$ (order 2, $x^{**} = x$). More generally, let $(R, \times, +)$ be a commutative ring with identity 1, and let $K = \{\rho_1, \rho_2, \dots\}$ be a transformation group in R . The K -logic (or K -logical algebra) of the ring $(R, \times, +)$ is the (operationally closed) system $(R, \times, \rho_1, \rho_2, \dots)$ whose class R is identical with the class of ring elements, and whose operations are the ring product " \times " of the ring together with the unary operations ρ_1, ρ_2, \dots of K . The ring $(R, \times, +)$ is called a *ring-logic*, mod K if (1) the " $+$ " of the ring is *equationally* definable in terms of its K -logic $(R, \times; \rho_1, \rho_2, \dots)$, and (2) the " $+$ " of the ring is *fixed* by its K -logic. Of particular interest in the theory of ring-logics is the *normal group* D which was shown in [1] to be particularly adaptable to p^k -rings. Our present object is to extend further the class of ring-logics, modulo the normal group D itself. A by-product of this extension is the following result, namely, any finite commutative ring with zero radical is a ring-logic, mod D (see Corollary 8). Furthermore, in Corollary 10, we prove that, more generally, any (not necessarily finite) ring with unit which satisfies $x^n = x$ (n fixed, ≥ 2) is a ring-logic (mod D). Finally, we compare the normal group with the so-called *natural* group in regard to the ring-logic character of a certain important class of rings (see section 3).

1. The finite field case. Let $(F_{p^k}, \times, +)$ be a Galois (finite) field with exactly p^k elements (p prime). Then, as is well known, F_{p^k} contains a multiplicative generator, ξ ;

$$F_{p^k} = \{0, \xi, \xi^2, \dots, \xi^{p^k-1} (=1)\}.$$

We now have the following (compare with [1]).

THEOREM 1. *Let F_{p^k} be a Galois field, and let ξ be a generator of F_{p^k} . Then the mapping $x \rightarrow x^\frown$ defined by*

$$(1.1) \quad x^\frown = \xi x + (1 + \xi x + \xi^2 x^2 + \dots + \xi^{p^k-2} x^{p^k-2})$$

is a permutation of F_{p^k} , with inverse given by

$$(1.2) \quad x^\smile = \xi^{p^k-2}(1 + x + x^2 + \dots + x^{p^k-2}) + \xi^{p^k-2} x.$$

Furthermore, the permutation \frown is of period p^k ,

$$(1.3) \quad x^{\frown p^k} = (\dots (x^\frown)^\frown \dots)^\frown \quad (p^k\text{-iterations}) = x.$$

Proof. Since $a^{p^k-1} = 1$, $a \in F_{p^k}$, $a \neq 0$, therefore, by (1.1), $x^\frown = \xi x + \{[(1 - (\xi x)^{p^k-1})]/(1 - \xi x)\} = \xi x$, if $x \neq 0$ and $\xi x \neq 1$. Furthermore, by (1.1), $0^\frown = 1$ and $(1/\xi)^\frown = p^k \cdot 1 = 0$. Hence, $0^\frown = 1$, $1^\frown = \xi$, $\xi^\frown = \xi^2$, $(\xi^2)^\frown = \xi^3$, \dots , $(\xi^{p^k-2})^\frown = 0$. This proves (1.3). To prove (1.2), observe that the right-side of (1.2) is equal to

$$\frac{1}{\xi} x + \frac{1}{\xi} \left\{ \frac{1 - x^{p^k-1}}{1 - x} \right\} = \frac{1}{\xi} x, \quad \text{if } x \neq 1 \text{ and } x \neq 0.$$

Moreover, if $x \neq 0$ and $x \neq 1/\xi$, then $x^\frown = \xi x$ and hence $x^\smile = (1/\xi)x$. Since (1.2) clearly holds for $x = 0$, $x = 1/\xi$, and $x = 1$, therefore (1.2) is true for all elements of F_{p^k} , and the theorem is proved.

COROLLARY 2. Under the permutation \frown , F_{p^k} suffers the cyclic permutation

$$(1.4) \quad (0, 1, \xi, \xi^2, \xi^3, \dots, \xi^{p^k-2}).$$

Following [1], we call x^\frown the *normal negation* of x , and call the cyclic group D whose generator is x^\frown the *normal group*. By Theorem 1, it is now clear that

$$D = D(\xi) = \{\text{identity}, \frown, \frown^2, \frown^3, \dots, \frown^{p^k-1}\}.$$

As in [1], we define

$$(1.5) \quad a \times \frown b = (a^\frown \times b^\frown)^\smile.$$

It is readily verified that

$$(1.6) \quad a \times \frown 0 = a = 0 \times \frown a.$$

COROLLARY 3. The elements of F_{p^k} are equationally definable in terms of the D -logic.

Proof. By Corollary 2, it is easily seen that

$$\begin{aligned}
 0 &= xx \frown x \frown^2 \dots x \frown^{p^k-1} \\
 1 &= 0 \frown \\
 \xi &= 1 \frown \\
 \xi^2 &= \xi \frown \\
 &\dots \\
 \xi^{p^k-2} &= (\xi^{p^k-3}) \frown,
 \end{aligned}
 \tag{1.7}$$

and the corollary follows.

We recall from [3] the *characteristic function* $\delta_\mu(x)$, defined as follows: for a given $\mu \in F_{p^k}$,

$$\delta_\mu(x) = \begin{cases} 1 & \text{if } x = \mu \\ 0 & \text{if } x \neq \mu. \end{cases}
 \tag{1.8}$$

In view of Corollary 2, it is easily seen that, for any given $\mu \in F_{p^k}$, there exists an integer r such that $\mu \frown^r = 0$. Then, clearly,

$$\delta_\mu(x) = \delta_0(x \frown^r) \quad \text{where } \mu \frown^r = 0.
 \tag{1.9}$$

Now, let $\sum_{\alpha_i \in F} \alpha_i$ denote $\alpha_1 \times \frown \alpha_2 \times \frown \alpha_3 \dots$, where $\alpha_1, \alpha_2, \alpha_3, \dots$ are the elements of F . Then, by (1.6) and (1.8), we have the identity [3]

$$f(x, y, \dots) = \sum_{\alpha, \beta, \dots \in F_{p^k}} f(\alpha, \beta, \dots) (\delta_\alpha(x) \delta_\beta(y) \dots).
 \tag{1.10}$$

In (1.10), α, β, \dots range over all the elements of F_{p^k} while x, y, \dots are indeterminates over F_{p^k} . We shall use (1.9) and (1.10) presently.

LEMMA 4. *The characteristic functions $\delta_\mu(x)$, $\mu \in F_{p^k}$, are equationally definable in terms of the D-logic.*

Proof. Since $x^{p^k-1} = 1, x \neq 0, x \in F_{p^k}$, therefore, $\delta_0(x) = ((x^{p^k-1}) \frown)^{p^k-1}$. Hence $\delta_0(x)$ is *equationally* definable in terms of the D-logic. Therefore, by (1.9), $\delta_\mu(x)$ is also equationally definable in terms of the D-logic, and the lemma is proved.

We are now in a position to prove the following.

THEOREM 5. *The Galois field $(F_{p^k}, \times, +)$ is a ring-logic (mod D).*

Proof. By (1.10), we have,

$$x + y = \sum_{\alpha, \beta \in F_{p^k}} (\alpha + \beta) (\delta_\alpha(x) \delta_\beta(y)).$$

Now, by Corollary 3, $\alpha + \beta$ is equationally definable in terms of the

D-logic. Moreover, by Lemma 4, each of the characteristic functions $\delta_\alpha(x)$ and $\delta_\beta(y)$ is equationally definable in terms of the *D*-logic. Hence the “+” of F_{p^k} is *equationally* definable in terms of the *D*-logic $(F_{p^k}, \times, \frown, \smile)$. Next, we show that $(F_k, \times, +)$ is *fixed* by its *D*-logic. Suppose then that there exists another ring $(F_{p^k}, \times, +')$, with the same class of elements F_{p^k} and the same “ \times ” as $(F_{p^k}, \times, +)$ and which has the *same logic* as $(F_{p^k}, \times, +)$. To prove that $+ = +'$. Since both $(F_{p^k}, \times, +)$ and $(F_{p^k}, \times, +')$ have the *same* class of elements and the *same* “ \times ”, it readily follows that $(F_{p^k}, \times, +')$ is also a Galois field with exactly p^k elements. Since, up to isomorphism, there is *only one* Galois field with exactly p^k elements, therefore, $+ = +'$, and the theorem is proved.

2. The General Case. In order to extend Theorem 5 to *any* finite commutative ring with zero radical, the following concept of independence, introduced by Foster [2], is needed.

DEFINITION. Let $\bar{A} = \{A_1, A_2, \dots, A_n\}$ be a finite set of algebras of the same species S_p . We say that the algebras A_1, A_2, \dots, A_n are *independent* if, corresponding to each set $\{\varphi_i\}$ of expressions of species S_p ($i = 1, \dots, n$) there exists at least one expression ψ such that $\psi = \varphi_i \pmod{A_i}$ ($i = 1, \dots, n$). By an *expression* we mean some composition of one or more indeterminate-symbols ξ, \dots in terms of the primitive operations of A_1, A_2, \dots, A_n ; $\psi = \varphi \pmod{A}$ means that this is an identity of the algebra A .

We now examine the independence of the *D*-logics $(F_{p_i^{k_i}}, \times, \frown, \smile)$. Indeed, we have the following (compare with [2]).

THEOREM 6. *Let p_1, \dots, p_t be distinct primes. Then the *D*-logics $(F_{p_i^{k_i}}, \times, \frown, \smile)$ are independent.*

Proof. Let $n_i = p_i^{k_i}$, $F_i = F_{p_i}^{k_i} = \{0, 1, \lambda, \lambda^2, \dots, \lambda^{n_i-2}\}$, $n = \max_{1 \leq i \leq t} \{n_i\}$, $N = \prod_{j=1}^t n_j$, $n_i N_i = N$, $E = \xi \frown \xi \frown \xi \frown \dots \xi \frown_{n-1}$.

It is easily seen, since the n_i 's are *distinct prime* powers, that

$$|_i(\xi) = (E \frown_{N_i} n_i)^{n_i-1} = \begin{cases} 1 \pmod{F_i} \\ 0 \pmod{F_j} \end{cases} \quad (j \neq i).$$

Now, to prove the independence of the logics $(F_i, \times, \frown, \smile)$ ($i = 1, \dots, t$) let $\varphi_1, \dots, \varphi_t$ be any set of t expressions of species \times, \frown, \smile , i.e., primitive compositions of indeterminate-symbols in terms of the operations \times, \frown, \smile . Define an expression $K(\varphi_1, \dots, \varphi_t)$ as follows (compare with [2]):

$$K(\varphi_1, \dots, \varphi_t) = (\varphi_1 \cdot |_1(\xi)) \times \frown (\varphi_2 \cdot |_2(\xi)) \times \frown \dots \times \frown (\varphi_t \cdot |_t(\xi)).$$

Then it is easily seen that $K(\varphi_1, \dots, \varphi_t) = \varphi_i \pmod{F_i}$ ($i = 1, \dots, t$), since $a \times \frown 0 = 0 \times \frown a = a$, and the theorem is proved.

We shall now extend the concept of ring-logic to the direct sum of certain ring-logics. We denote the direct sum of A_1 and A_2 by $A_1 \oplus A_2$. The direct power A^m will denote $A \oplus A \oplus \dots \oplus A$ (m summands).

THEOREM 7. *Let A be any subdirect sum with identity of (not necessarily finite) subdirect powers of the Galois fields $F_{p_i^{k_i}}$ ($i = 1, \dots, t$). Then A is a ring-logic (mod D).*

Proof. Let q_1, \dots, q_r be the distinct primes in $\{p_1, \dots, p_t\}$. Since the Galois Fields $F_{p_i^{k_i}}$ and $F_{p_j^{k_j}}$ are both subfields of $F_{p_i^{k_i} p_j^{k_j}}$, it is easily seen that A is a subring of a direct sum of direct powers of $F_{q_i^{h_i}}$, ($i = 1, \dots, r$); i.e., A is a subring of $F_{q_1^{h_1}}^{m_1} \oplus \dots \oplus F_{q_r^{h_r}}^{m_r}$ for some positive integers h_1, \dots, h_r . Now, by Theorem 5, each $F_{q_i^{h_i}}$ is a ring-logic (mod D), and hence there exists a D -logical expression φ_i such that, for every $x_i, y_i \in F_{q_i^{h_i}}$ ($i = 1, \dots, r$),

$$x_i + y_i = \varphi_i(x_i, y_i; \times, \frown, \smile).$$

Since, by Theorem 6, the D -logics $(F_{q_i^{h_i}}, \times, \frown, \smile)$ ($i = 1, \dots, r$) are independent, there exists a D -logical expression K such that

$$K = \begin{cases} \varphi_1 \pmod{F_{q_1^{h_1}}} \\ \dots \\ \varphi_r \pmod{F_{q_r^{h_r}}} \end{cases}.$$

Therefore, for every $x_i, y_i \in F_{q_i^{h_i}}$ ($i = 1, \dots, r$),

$$x_i + y_i = \varphi_i = K(x_i, y_i; \times, \frown, \smile).$$

Hence, the D -logical expression K represents the “+” of each $F_{q_i^{h_i}}$. Since the operations are component-wise in the direct sum $F_{q_1^{h_1}}^{m_1} \oplus \dots \oplus F_{q_r^{h_r}}^{m_r}$, therefore, for all x, y in this direct sum, we have,

$$x + y = K(x, y; \times, \frown, \smile).$$

Hence, *a fortiori*, the “+” of the subring A is equationally definable in terms of the D -logic.

Next, we show that A is fixed by its D -logic. Suppose there exists a “+” such that $(A, \times, +')$ is a ring, with the same class of elements A and the same “ \times ” as the ring $(A, \times, +)$, and which has the same logic $(A, \times, \frown, \smile)$ as the ring $(A, \times, +)$. To prove that $+ = +'$. Now, since A is a subdirect sum of subdirect powers of $F_{p_i^{k_i}}$, therefore, a new “+” in A defines and is defined by a new

" $+_1$ " in $F_{p_1^{k_1}}$, " $+_2$ " in $F_{p_2^{k_2}}$, ..., " $+_t$ " in $F_{p_t^{k_t}}$, such that $(F_{p_i^{k_i}}, \times, +_i)$ is a ring ($i = 1, \dots, t$). Furthermore, the assumption that $(A, \times, +')$ has the same logic as $(A, \times, +)$ is equivalent to the assumption that each $(F_{p_i^{k_i}}, \times, +_i)$ has the same logic as $(F_{p_i^{k_i}}, \times, +)$ ($i = 1, \dots, t$). Since, by Theorem 5, $(F_{p_i^{k_i}}, \times, +)$ is a ring-logic, and hence with its " $+$ " fixed, it follows that $+'_i = +$ ($i = 1, \dots, t$). Hence $+ ' = +$, and the theorem is proved.

Now, it is well known (see [4]) that any finite commutative ring with zero radical and with more than one element is isomorphic to the complete direct sum of a finite number of finite fields. Hence, Theorem 7 has the following

COROLLARY 8. *Any finite commutative ring with zero radical is a ring-logic (mod D).*

It is also well known (see [1; 5]) that every p -ring (p prime) is isomorphic to a subdirect power of F_p , and every p^k -ring (p prime) is isomorphic to a subdirect power of F_{p^k} . Hence, by letting $t = 1$ in Theorem 7, we obtain the following (compare with [1; 7])

COROLLARY 9. *Any p -ring with identity, as well as any p^k -ring with identity, is a ring-logic (mod D).*

Now, let n be a fixed integer, $n \geq 2$. It is well known that a ring R which satisfies $x^n = x$ for all x in R is isomorphic to a subdirect sum of (not necessarily finite) subdirect powers of a finite set of Galois fields. Hence Theorem 7 has the following

COROLLARY 10. *Let R be a ring with unit such that $x^n = x$ for all x in R , where n is a fixed integer, $n \geq 2$. Then R is a ring-logic (mod D).*

3. The natural group and the normal group. Let $(R, \times, +)$ be a commutative ring with unit 1. We recall (see [1]) that the *natural group* N is the group generated by $x^\wedge = x + 1$ (with inverse $x^\vee = x - 1$). In [7], it was shown that $(F_{p^k}, \times, +)$ is a ring-logic (mod N), and hence the " $+$ " of F_{p^k} is equationally definable in terms of the N -logic $(F_{p^k}, \times, \wedge)$. Moreover, by Theorem 5, $(F_{p^k}, \times, +)$ is a ring-logic (mod D), and hence the " $+$ " of F_{p^k} is equationally definable in terms of the D -logic $(F_{p^k}, \times, \frown)$. Of the two rival logics, $(F_{p^k}, \times, \frown)$ requires only a knowledge of the multiplication table in F_{p^k} since, by Corollary 2, the effect of \frown on F_{p^k} is the cyclic permutation $(0, 1, \xi, \xi^2, \dots, \xi^{p^k-2})$. In this sense, the D -logical formula for the " $+$ " of F_{p^k} is a *strictly multiplicative formula*, and addition is thus

equationally definable in terms of multiplication whenever the generator ξ is chosen (compare with [1]). The situation is quite different in the case of the N -logical formula for the “+” of F_{p^k} , since the generator $x^\wedge = x + 1$ of the natural group N already involves a limited use of the addition table.

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