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A GENERALISATION OF W^* -ALGEBRAS

GEORGE A. REID

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Using the theory of double centralisers due to B. E. Johnson, we define a QW^* -algebra as being a B^* -algebra, A , such that the algebra of double centralisers of each closed $*$ -subalgebra B is contained in a suitable related closed $*$ -subalgebra B_{00} .

After obtaining explicit descriptions of the algebras of double centralisers of commutative and noncommutative B^* -algebras, we prove that in the general noncommutative case a W^* -algebra is a QW^* -algebra, and a QW^* -algebra is an AW^* -algebra, while in the commutative case the QW^* and AW^* conditions are equivalent.

We prove that if A is QW^* then so are its centre, any maximal commutative $*$ -subalgebra, and any subalgebra of the form eAe for e a projection in A .

We shall be concerned with centraliser theory, for the basic details of which reference may be made to Johnson [2], [3].

I should like to take this opportunity of expressing my sincere gratitude to Dr. J. H. Williamson, my research supervisor, for his advice and encouragement.

DEFINITION 1. A *left centraliser* \mathcal{T} of the algebra A is a linear map \mathcal{T} of A into itself such that $\mathcal{T}(xy) = (\mathcal{T}x)y$ for all $x, y \in A$.

A *right centraliser* \mathcal{S} is a linear operator on A such that $\mathcal{S}(xy) = x(\mathcal{S}y)$ for all $x, y \in A$.

A *double centraliser* (the concept is due to Johnson [2]) is a pair of linear operators $(\mathcal{T}, \mathcal{S})$ such that $x \cdot (\mathcal{T}y) = (\mathcal{S}x) \cdot y$ for all $x, y \in A$.

The set of all double centralisers on A is denoted by $Q(A)$.

We will assume throughout that $xA = 0$ or $Ax = 0$ only holds for $x = 0$. We note that this holds for B^* -algebras since $xA = 0 \Rightarrow xx^* = 0 \Rightarrow x = 0$, and $Ax = 0 \Rightarrow x^*x = 0 \Rightarrow x = 0$.

It is not difficult to see that defining $(\mathcal{T}_x, \mathcal{S}_x) \in Q(A)$ for $x \in A$ by $\mathcal{T}_x(y) = xy$, $\mathcal{S}_x(y) = yx$, and algebraic operations in $Q(A)$ by

$$\begin{aligned} \lambda_1(\mathcal{T}_1, \mathcal{S}_1) + \lambda_2(\mathcal{T}_2, \mathcal{S}_2) &= (\lambda_1\mathcal{T}_1 + \lambda_2\mathcal{T}_2, \lambda_1\mathcal{S}_1 + \lambda_2\mathcal{S}_2) \\ (\mathcal{T}_1, \mathcal{S}_1) \cdot (\mathcal{T}_2, \mathcal{S}_2) &= (\mathcal{T}_1\mathcal{T}_2, \mathcal{S}_2\mathcal{S}_1) \end{aligned}$$

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we have A embedded as a subalgebra of $Q(A)$, which is an algebra with identity. $A = Q(A)$ if and only if A has an identity. Also, for $(\mathcal{T}, \mathcal{S}) \in Q(A)$, \mathcal{T} is a left centraliser and \mathcal{S} is a right centraliser, and either of \mathcal{T}, \mathcal{S} determines the other uniquely.

If A is commutative, the notions of right, left and double centraliser coincide, and for $(\mathcal{T}, \mathcal{S}) \in Q(A)$ we have $\mathcal{T} = \mathcal{S}$.

PROPOSITION 1. If A is a Banach algebra then all double centralisers are continuous.

Proof. Suppose $(\mathcal{T}, \mathcal{S}) \in Q(A)$ and say $x_n \rightarrow x, \mathcal{T}x_n \rightarrow y$. Then

$$\begin{aligned} z \cdot (\mathcal{T}x_n) &= (\mathcal{S}z) \cdot x_n \\ \rightarrow z \cdot y &\quad \rightarrow (\mathcal{S}z) \cdot x = z \cdot (\mathcal{T}x) . \end{aligned}$$

So $z(y - \mathcal{T}x) = 0$ for all $z \in A$ i.e. $A(y - \mathcal{T}x) = 0$ and so $y = \mathcal{T}x$. Therefore \mathcal{T} is a closed operator on the Banach space A , hence by the Closed Graph Theorem, \mathcal{T} is continuous. Likewise so is \mathcal{S} .

We are particularly interested in C^* -algebras and in both the commutative and noncommutative cases explicit descriptions of their centraliser algebras may be given.

By the Gelfand Representation Theorem a commutative B^* -algebra is isometrically isomorphic to the space $C_0(Z)$ of all continuous functions vanishing at infinity on its carrier space, Z , a locally compact Hausdorff space.

PROPOSITION 2. For a locally compact Hausdorff space Z we have $QC_0(Z) = C(Z)$, the space of all bounded continuous functions on Z .

Proof. Certainly any $h \in C(Z)$ defines an element \mathcal{T}_h of $QC_0(Z)$ by $\mathcal{T}_h f = h \cdot f$ for $f \in C_0(Z)$, for

$$f \in C_0(Z), h \in C(Z) \Rightarrow hf \in C_0(Z)$$

and

$$h(fg) = (hf)g .$$

We clearly have $\|\mathcal{T}_h\| \leq \|h\|_\infty$. Suppose conversely we are given a centraliser \mathcal{T} on $C_0(Z)$. Then for $f, g \in C_0(Z)$ we have

$$(\mathcal{T}f)g = \mathcal{T}(fg) = \mathcal{T}(gf) = (\mathcal{T}g)f$$

so for $z \in Z$ taking any $f \in C_0(Z)$ such that $f(z) \neq 0$ and defining $h(z) = \mathcal{T}f(z)/f(z)$ we have $h(z)$ well defined independently of f .

Being a quotient of continuous functions, h is continuous at z , for each $z \in Z$. And for any $g \in C_0(Z)$,

$$\mathcal{T}g(z) = \frac{\mathcal{T}f(z)}{f(z)}g(z) = h(z)g(z)$$

so

$$\mathcal{T}g = hg = \mathcal{T}_h g .$$

Now by Proposition 1, \mathcal{T} is a bounded operator, so taking $f \in C_0(Z)$ such that $0 \leq f \leq 1$ and $f(z) = 1$ we have $h(z) = \mathcal{T}f(z)$ and $|\mathcal{T}f(z)| \leq \|\mathcal{T}f\|_\infty \leq \|\mathcal{T}\| \|f\|_\infty = \|\mathcal{T}\|$ so $\|h\|_\infty \leq \|\mathcal{T}\|$ and we see $h \in C(Z)$.

Hence all \mathcal{T} are of the form \mathcal{T}_h and $\|\mathcal{T}\| = \|h\|_\infty$. So $QC_0(Z) = C(Z)$.

PROPOSITION 3. If A is a C^* -algebra over H , principal identity E , then $Q(A)$ is isometrically isomorphic to

$$\{T \in \mathcal{B}(H) : T = ETE, TA \cup AT \subset A\} .$$

Proof. Recall that the principal identity of a C^* -algebra A is defined to be the orthogonal projection of H onto $M = H \ominus N$ where $N = \{\xi \in H : A\xi = 0\}$. Equivalently M is the closure of

$$M_1 = \{T\xi : T \in A, \xi \in H\} .$$

Suppose given $(\mathcal{T}, \mathcal{S}) \in Q(A)$, then \mathcal{T} is a bounded left centraliser.

Since A is a C^* -algebra it has an approximate identity (Segal [6]), $(Z_\lambda)_{\lambda \in I}$ say, so $\|Z_\lambda\| = 1$, and $SZ_\lambda \rightarrow S, Z_\lambda S \rightarrow S$ for each $S \in A$. So $\mathcal{T}(Z_\lambda S) \rightarrow \mathcal{T}(S)$. But $\mathcal{T}(Z_\lambda S) = \mathcal{T}(Z_\lambda)S = T_\lambda S$ where $T_\lambda = \mathcal{T}(Z_\lambda)$, so $\mathcal{T}(S) = \lim_\lambda T_\lambda S$ and $\|T_\lambda\| \leq \|\mathcal{T}\| \|Z_\lambda\| = \|\mathcal{T}\|$. For $\xi \in M_1$, $\xi = S\eta$ some $S \in A, \eta \in H$ so $\mathcal{T}(S)\eta = \lim_\lambda T_\lambda S\eta = \lim_\lambda T_\lambda \xi$. Define $T\xi = \lim_\lambda T_\lambda \xi = \mathcal{T}(S)\eta$, then T maps M_1 into M and $\|T\xi\| \leq \|\mathcal{T}\| \|\xi\|$ so $\|T\| \leq \|\mathcal{T}\|$.

So extend T to a map of M into M and define $T = 0$ on $H \ominus M$, so we have $T = ETE$ and $\mathcal{T}(S)\eta = \lim_\lambda T_\lambda S\eta = TS\eta$. Therefore $\mathcal{T}(S) = TS$ and $\|\mathcal{T}\| \leq \|T\|$. So $\|\mathcal{T}\| = \|T\|$.

We have

$$\begin{aligned} (\mathcal{S}S)Z_\lambda &= S(\mathcal{T}Z_\lambda) = STZ_\lambda \\ &\rightarrow \mathcal{S}S && \rightarrow ST . \end{aligned}$$

So $\mathcal{S}(S) = ST$ for all $S \in A$, and as for $\mathcal{T}, \|\mathcal{T}\| = \|T\|$. Since $TS, ST \in A$ for all $S \in A$ we have $TA \cup AT \subset A$. Conversely given any

T such that $T = ETE$ and $TA \cup AT \subset A$, the maps $S \rightarrow TS, S \rightarrow ST$ both map A into itself and define a double centraliser of A . Hence result.

Denote the set $\{T \in \mathcal{B}(H): T = ETE, TA \cup AT \subset A\}$ by $I(A)$, the idealiser of A in $E \cdot \mathcal{B}(H) \cdot E$.

Now let us suppose that B is a closed $*$ -subalgebra of the B^* -algebra A . We define $B_0 = \{x \in A: Bx = xB = 0\}$ and $B_{00} = (B_0)_0$. Then B_{00} is a closed $*$ -subalgebra of A containing B . Should it be necessary to make explicit mention of the algebra A we will write $B_0(A)$, etc.

Suppose two elements x_1, x_2 of B_{00} give the same double centraliser on B , so $x_1y = x_2y$ and $yx_1 = yx_2$ for all $y \in B$. Then $(x_1 - x_2)B = B(x_1 - x_2) = 0$ so $x_1 - x_2 \in B_0$. But $(x_1 - x_2)^* \in B_{00}$ so we have

$$(x_1 - x_2)^*(x_1 - x_2) = 0$$

and hence $x_1 - x_2 = 0$. So $x_1 = x_2$.

DEFINITION 2. A B^* -algebra A is said to be a QW^* -algebra if for each closed $*$ -subalgebra B of A all double centralisers of B are given by elements of B_{00} . We see that for each double centraliser the corresponding element of B_{00} is unique, and so we may briefly say that A is QW^* if and only if $Q(B) \subset B_{00}$ for all closed $*$ -subalgebras B .

We recall the definition of an AW^* -algebra (Kaplansky [4]).

DEFINITION 3. A B^* -algebra A is said to be an AW^* -algebra if

- (i) every set of orthogonal projections in A has a least upper bound in A .

- (ii) every maximal commutative $*$ -subalgebra B of A is generated by its projections.

We also recall that a W^* -algebra is a C^* -algebra, over H say, which is closed in the weak operator topology defined by seminorms $\|T\|_{\xi, \eta} = |\langle T\xi, \eta \rangle|$ for $\xi, \eta \in H$. Denote weak closure by $^-$.

PROPOSITION 4. For A a C^* -algebra, $I(A) \subset A^{-w}$.

Proof. By von Neumann's Double Commutant Theorem, $A^{-w} = \{T \in \mathcal{B}(H): T = ETE, T \in A''\}$ where as usual A'' denotes the double commutant of A .

Suppose $T \in I(A), S \in A', R \in A$, then certainly $T = ETE$ and $(ST - TS)R = S(TR) - T(SR) = TRS - TRS = 0$. So $(ST - TS)E = 0$

and therefore $ST = TSE$. Since $T^* \in I(A), S^* \in A'$ we have $S^*T^* = T^*S^*E$ so $TS = EST$. Thus $TS = EST = ETSE = TSE = ST$ and so $T \in A''$. Hence $I(A) \subset A''$.

THEOREM 1. *For a B^* -algebra $A, W^* \Rightarrow QW^* \Rightarrow AW^*$.*

If A is commutative, carrier space Z , then A is $QW^ \Leftrightarrow A$ is $AW^* \Leftrightarrow Z$ is extremally disconnected.*

Proof. If A is a W^* -algebra and B is a closed $*$ -subalgebra of A with principal identity E , then since A is W^* we note $E \in A$, and by Proposition 4, $I(B) \subset B^{-w} \subset A^{-w} = A$. Also we easily see that $B_0 = (I - E)A(I - E)$ so $B_0 = EAE$. Thus $Q(B) \subset B_0$ by Proposition 3 and hence A is QW^* .

Suppose now that A is a commutative B^* -algebra, carrier space Z , so by the Gelfand Representation Theorem A is isometrically isomorphic to $C_0(Z)$.

It is well known that A is AW^* if and only if Z is an extremally disconnected compact Hausdorff space.

Suppose A is QW^* , then taking $B = A$ we see that A has an identity, so Z is compact Hausdorff.

Let U be any open dense subset of Z .

Then taking $B = \{f \in C(Z) : f = 0 \text{ on } Z \setminus U\} = C_0(U)$, B is a closed $*$ -ideal in A so $Q(B) = C(U) \subset A$.

So any continuous function f on U is extendible to Z . Therefore Z is extremally disconnected (see Gillman and Jerison [1], p. 96).

Now suppose that Z is an extremally disconnected compact Hausdorff space, and suppose B is a closed $*$ -subalgebra of $A = C(Z)$.

Let $(Z_\lambda)_{\lambda \in \Lambda}$ be the sets of constancy of B (see Rickart [5], Ch. 3, § 2), then these form an upper semicontinuous decomposition of Z , so the space of these sets, Z' say, is a compact Hausdorff space and B may be considered as a space of continuous functions on Z' .

B is self-adjoint and separates points of Z' , so by the Stone-Weierstrass Theorem, either B consists of all continuous functions on Z' , in which case B has an identity so $Q(B) = B$, or B consists of all continuous functions on Z' vanishing at some point Z_0 of Z' . So $Q(B) =$ all continuous functions on $Z' \setminus \{Z_0\}$.

Given any function on $Z' \setminus \{Z_0\}$ it corresponds to a function f on $Z \setminus Z_0 = Y$ say.

Y is open, so \bar{Y} is a compact open subset of Z , and therefore \bar{Y} is extremally disconnected (Gillman and Jerison [1], p. 23). So there exists an extension of f to \bar{Y} , and defining $f = 0$ on $Z \setminus \bar{Y}$ we extend f to a continuous function on Z .

Now since

$$\begin{aligned} B_0 &= \{g \in C(Z): g = 0 \text{ on } Y\} \\ &= \{g \in C(Z): g = 0 \text{ on } \bar{Y}\} \end{aligned}$$

and

$$B_{00} = \{g \in C(Z): g = 0 \text{ on } Z \setminus \bar{Y}\}$$

we therefore have $Q(B) \subset B_{00}$.

So A is QW^* and we have proved our theorem for A commutative.

Now let us return to the general case and suppose A to be QW^* .

(i) Suppose (e_α) is a set of orthogonal projections in A (so $\alpha \neq \beta \Rightarrow e_\alpha e_\beta = 0$).

Let $B =$ closed $*$ -subalgebra of A generated by the e_α 's.
 $=$ closed linear hull of the e_α 's.

Now there exists a unique $e \in B_{00}$ such that $ex = xe = x$ for all $x \in B$ and $e^*, e^2 \in B_{00}$ with

$$\begin{aligned} e^*x &= xe^* = x \\ e^2x &= xe^2 = x \quad \text{for all } x \in B. \end{aligned}$$

So $e^2 = e^* = e$ and thus e is a projection.

Also $ee_\alpha = e_\alpha e = e_\alpha$ all α , so $e \geq e_\alpha$ all α .

Now suppose f is a projection in A such that $f \geq e_\alpha$ all α . Then $fe_\alpha = e_\alpha f = e_\alpha$ all α , so since all $x \in B$ are limits of linear combinations of the e_α 's, we have $fx = xf = x$ for all $x \in B$.

Now

$$\begin{aligned} y \in B_0 &\Rightarrow yfx = yx = 0 \\ xyf &= 0 \quad \text{all } x \in B \Rightarrow yf \in B_0 \end{aligned}$$

so for all $y \in B_0$,

$$\begin{aligned} fey &= f0 = 0 \\ yfe &= 0 \quad \text{thus } fe \in B_{00}. \end{aligned}$$

But

$$\begin{aligned} fex &= fx = x \\ xfe &= xe = x \end{aligned}$$

all $x \in B$, so since e is unique, $e = fe$.

So $ef = fe = e$ and $e \leq f$.

Hence e is a least upper bound in A for the e_α 's.

(ii) Suppose B is a maximal commutative $*$ -subalgebra of A . Then by Proposition 5 below, B is QW^* , thus since B is commutative it follows from the above result that B is AW^* , and is a maximal commutative $*$ -subalgebra of itself and therefore generated by its projections.

Thus we have both conditions for A to be AW^* .

The obvious question of interest arising from this theorem is whether or not the QW^* and the AW^* conditions are equivalent in the noncommutative case, but so far we have not been able to settle this problem.

We now prove some results for QW^* -algebras similar to those holding for W^* - and AW^* -algebras. We are indebted to the referee for pointing out case (iv) of Proposition 5 as generalising cases (i) and (ii).

PROPOSITION 5. If A is a QW^* -algebra then so also are the following closed $*$ -subalgebras of A :

- (i) the centre Z of A ,
- (ii) any maximal commutative $*$ -subalgebra of A ,
- (iii) the subalgebra eAe for any projection e in A ,
- (iv) S'' for any subset S of A such that $S^* = S$, where S'' is the double commutant of S in A .

Proof. We first prove (iv) from which (i) and (ii) follow immediately.

(iv) Suppose B is a closed $*$ -subalgebra of S'' .

Since A is QW^* any double centraliser on B is given by some $x \in B_{00}(A)$.

To prove $x \in B_{00}(S'')$, since $B_0(S'') \subset B_0(A)$, we need only show $x \in S''$.

Let $y \in S'$, $z \in B \subset S''$, then

$$(xy - yx)z = x(yz) - y(xz) = xzy - xzy = 0$$

$$z(xy - yx) = (zx)y - (zy)x = yzx - yzx = 0$$

so $xy - yx \in B_0(A)$.

Now

$$u \in B_0(A) \Rightarrow yuz = 0$$

$$zyu = yzu = 0 \quad \text{all } z \in B \Rightarrow yu \in B_0(A),$$

and likewise $u \in B_0(A) \Rightarrow uy \in B_0(A)$.

Therefore since $x \in B_{00}(A)$, $xyu = 0$ and $uxy = 0$ for all $u \in B_0(A)$, so $xy \in B_{00}(A)$, and likewise $yx \in B_{00}(A)$. So $(xy - yx)^* \in B_{00}(A)$ and hence $xy - yx = 0$ for all $y \in S'$. Thus $x \in S''$ and the result follows.

(i) We have $Z = A'$, $Z' = A$ so $Z = Z''$, and clearly $Z = Z^*$, so the result follows from (iv).

(ii) Suppose C is a maximal commutative $*$ -subalgebra of A , then by maximality C is closed and $C' = C$, so $C = C''$ and the result follows from (iv).

(iii) Let B be a closed $*$ -subalgebra of eAe , then since A is QW^*

any double centraliser on B is given by some $x \in B_{00}(A)$. Since $B \subset eAe$ we have $y \in B_0(A) \Rightarrow ey, ye \in B_0(A)$ and $x \in B_{00}(A) \Rightarrow exe \in B_{00}(A)$.

But for $z \in A$ we have

$$zexe = (zx)e = zx$$

$$exez = e(xz) = xz$$

so by the uniqueness of x in $B_{00}(A)$ we have $x = exe$.

Thus $x \in eAe$ and so $x \in B_{00}(eAe)$. Hence eAe is QW^* .

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