

# Pacific Journal of Mathematics

**SOME TOPOLOGICAL PROPERTIES OF CERTAIN SPACES OF  
DIFFERENTIABLE HOMEOMORPHISMS OF DISKS AND  
SPHERES**

JACK MAX ROBERTSON

# SOME TOPOLOGICAL PROPERTIES OF CERTAIN SPACES OF DIFFERENTIABLE HOMEOMORPHISMS OF DISKS AND SPHERES

JACK M. ROBERTSON

Let  $D_n = \{x \in E_n : |x| \leq 1\}$ , and  $S^n = \{x \in E_{n+1} : |x| = 1\}$ . We denote by  $H_n$  the space of  $C^\infty$  homeomorphisms of  $D_n$  onto itself leaving a neighborhood of the boundary fixed. Let  $K_n$  be the space of  $C^\infty$  orientation preserving homeomorphisms of  $S^n$  onto itself. It is not required that maps in the two spaces have differentiable inverses. In both space the  $C^k$  topology is used.

The purpose of this paper is to establish the following two theorems:

THEOREM 1.  $H_n$  is contractible to a point for any  $n$ .

THEOREM 2.  $K_n$  is arcwise connected for any  $n$ .

NOTATION.  $f(x) = (f_1(x_1, \dots, x_n), \dots, f_n(x_1, \dots, x_n))$  where  $x = (x_1, \dots, x_n)$ , or simply  $f(x)$  will denote mappings of  $E_n$  into  $E_n$ . The shorter form will be used where the meaning is clear.

The topological analog of Theorem 1 is established by a mapping described by Alexander (1923) [1]. Smale (1959) [4] proved the corresponding result for  $n = 2$  in the space of diffeomorphisms on  $D_n$  leaving a neighborhood of the boundary fixed. Kneser (1926) [3] proved that the space of all orientation preserving homeomorphisms of  $S^2$  onto  $S^2$  has the rotation group as a deformation retract, while Smale gave the corresponding result for the space of orientation preserving diffeomorphisms on  $S^2$  in the paper referred to above. Fisher's work (1960) [2] gives the analog of Theorem 2 in the topological case for  $n = 3$ .

II. Proof of Theorem 1. Let  $m(v)$  be a mapping on  $I$  (the unit interval  $[0, 1]$ ) with the following properties:

- (a)  $m(v) \in C^\infty$ ;
- (b)  $m'(v) > 0$  on  $(0, \frac{3}{4})$ ;
- (c)  $m(v) = 1$  on  $(\frac{3}{4}, 1]$ ;
- (d)  $m(v) = e^{-(1/v)}$  on  $(0, \frac{1}{4})$ ;

---

Received September 16, 1964. The work in this paper was partially supported by Summer Fellowships for Graduate Teaching Assistants in the summers of 1963 and 1964. This paper represents the major results of a thesis submitted to the University of Utah for the Ph.D. degree in August 1964.

$$(e) \quad m(0) = 0.$$

Now define  $k(v, t) = \begin{cases} 1 - (1 - e^{-(1/t)+1}) (1 - m(v)) & t \neq 0, \\ m(v) & t = 0, \end{cases}$

on  $I \times I$ . We see that:

$$(a') \quad k(v, t) \in C^\infty \text{ on } I \times I;$$

$$(b') \quad k(v, t) \text{ is monotonic in } v \text{ for each } t \in I;$$

$$(c') \quad k(v, t) = 1 \text{ for } v \geq \frac{3}{4} \text{ for all } t \in I;$$

$$(d') \quad k(v, 1) = 1 \text{ for all } v \in I;$$

$$(e') \quad k(v, 0) = m(v);$$

$$(f') \quad 0 \leq k(v, t) \leq 1 \text{ on } I \times I.$$

The mapping

$$(1) \quad x \rightarrow k(|x|^2, t)x$$

is in  $H_n$  for each  $t$ . At  $t = 1$  the mapping is the identity, while at  $t = 0$  the mapping has all partial derivatives of all orders zero at the origin.

The mapping given by Alexander was defined as follows:

$$f_t(x) = \begin{cases} tf\left(\frac{x}{t}\right), & t \neq 0 \text{ (} f \text{ extended to be the identity outside } D_n \text{)}, \\ x, & t = 0. \end{cases}$$

In the  $C^k$  topology the mapping of  $H_n \times I \rightarrow H_n$  defined by  $(f, t) \rightarrow f_t$  (the Alexander map) will not be continuous for  $k \geq 1$ . In general,  $\lim_{t \rightarrow 0} f_t \neq f_0$  because at the origin the derivatives of  $f_t$  do not converge to the derivatives of the identity mapping. However, by composing the Alexander mapping with (1), we obtain the mapping required in Theorem 1. Thus define

$$h : H_n \times I \rightarrow H_n$$

by

$$h(f, t) = kf_t$$

where

$$kf_t(x) = k(|f_t(x)|^2, t) f_t(x).$$

In particular  $h(f, 1) = f$  for all  $f \in H_n$ , while  $h(f, 0)$  is the mapping given by (1). Because of the form of map (1) at the origin, all derivatives of all orders of  $kf_t$  approach zero there and the problem mentioned above is removed. The argument that  $h$  is continuous is tedious but straightforward.

**III. Local straightening of mappings in  $E_n$ .** The proof of Theorem 2 requires some local straightening procedures for maps in  $E_n$  which we now give. For this purpose let  $L$  be the space of  $C^\infty$  orientation preserving homeomorphisms mapping  $U_r = \{x \in E_n : |x| \leq r\}$  into  $E_n$ , leaving the origin fixed and topologized by the  $C^k$  topology. We will use  $J(f)_p$  to represent the Jacobian matrix of  $f$  evaluated at  $p \in U_r$ , and  $|J(f)_p|$  the corresponding determinant.

**LEMMA 1.** *Suppose  $f \in L$  with  $J(f)_p = (a_{ij})$ ,  $p$  the origin and  $(a_{ij})$  nonsingular. Then there is a path  $f_t \in L$  from  $f$  to  $g$ , where  $g$  agrees with  $f$  in a neighborhood of the boundary of  $U_r$  and is the linear map with Jacobian  $(a_{ij})$  in a neighborhood of the origin. Also for all  $t$ ,  $f_t$  agrees with  $f$  in a neighborhood of the boundary of  $U_r$ .*

*Proof.* Let  $\sigma(v)$  be a mapping on  $[0, \infty)$  with the following properties:

- (a)  $\sigma(v) \in C^\infty$ ;
- (b)  $\sigma(v) = 1$  on  $[0, \alpha)$ ,  $\alpha > 0$ ;
- (c)  $\sigma(v) = 0$  for  $v \geq 1$ ;
- (d)  $\sigma'(v) \leq 0$  for  $v \in [0, \infty)$ .

We see that  $|\sigma'(v)| < M$  for some  $M$ . Let  $c < r$  be chosen so that for  $x \in U_c$ ,

$$(i) \quad \left| a_{ij} - \frac{\partial f_i(x_1, \dots, x_n)}{\partial x_j} \right| < \varepsilon, \varepsilon > 0; i = 1, \dots, n; j = 1, 2, \dots, n.$$

Then for  $x \in U_c$ ,

$$(ii) \quad |a_{i1}x_1 + \dots + a_{in}x_n - f_i(x_1, \dots, x_n)| < n\varepsilon c \text{ for } i = 1, 2, \dots, n.$$

Now define

$$\begin{aligned} f_t(x_1, \dots, x_n) = & \left( f_1(x_1, \dots, x_n) \right. \\ & + t\sigma\left(\frac{x_1^2 + \dots + x_n^2}{c^2}\right)(a_{11}x_1 + \dots + a_{1n}x_n \\ & - f_1(x_1, \dots, x_n)), \dots, f_n(x_1, \dots, x_n) \\ & + t\sigma\left(\frac{x_1^2 + \dots + x_n^2}{c^2}\right)(a_{n1}x_1 + \dots + a_{nn}x_n \\ & \left. - f_n(x_1, \dots, x_n)) \right). \end{aligned}$$

At  $t = 0$ ,  $f_t = f$ ; at  $t = 1$ ,  $f_t$  is linear with Jacobian  $(a_{ij})$  inside a neighborhood of the origin; for all  $t$ ,  $f_t$  agrees with  $f$  outside  $U_c$ . The element in the  $(i, j)$ th position of  $J(f_t)$  differs from  $a_{ij}$  by at most

$$\left| a_{ij} - \frac{\partial f_i(x_1, \dots, x_n)}{\partial x_j} \right|$$

$$\begin{aligned}
 &+ \left| \frac{2x_j t}{c^2} \sigma' \left( \frac{x_1^2 + \dots + x_n^2}{c^2} \right) (a_{i_1} x_1 + \dots + a_{i_n} x_n - f_i(x_1, \dots, x_n)) \right| \\
 &+ \left| t \sigma \left( \frac{x_1^2 + \dots + x_n^2}{c^2} \right) \left( a_{i_j} - \frac{\partial f_i(x_1, \dots, x_n)}{\partial x_j} \right) \right|.
 \end{aligned}$$

On  $U_c$ ,  $|x_j| \leq c$  so that the expression is bounded by  $\varepsilon + (2/c) Mn \varepsilon c + \varepsilon = (2 + 2Mn)\varepsilon$ . Hence by choosing  $\varepsilon$  sufficiently small,  $|J(f_t)|$  will remain positive on  $U_c$  for all  $t$  so that  $f_t$  will be a homeomorphism on  $U_r$ . Continuity of the path  $f_t$  in  $L$  is immediate from the definition of  $f_t$ .

**LEMMA 2.** *Let  $f(x_1, \dots, x_n) = (a_{11}x_1 + \dots + a_{1n}x_n, a_{22}x_2 + \dots + a_{2n}x_n, \dots, a_{nn}x_n) \in L$  with  $a_{11} \dots a_{nn} > 0$ . Then there is a path  $f_t \in L$  such that  $a \leq t \leq b$ ,  $f_t(x_1, \dots, x_n) = f(x_1, \dots, x_n)$  at  $t = a$ ,  $f_t(x_1, \dots, x_n) = (a_{11}x_1, \dots, a_{nn}x_n)$  in a neighborhood of the origin at  $t = b$ , and  $f_t(x_1, \dots, x_n) = f(x_1, \dots, x_n)$  in a neighborhood of the boundary of  $U_r$  for each  $t$ .*

*Proof.* We construct the path in  $n - 1$  arcs as follows. Choose a positive  $c_1$  less than  $r$ . Let  $k_1 > 1$  be sufficiently large so that whenever

$$x_1^2 + k_1^2 x_2^2 + \dots + k_1^2 x_n^2 \leq c_1^2,$$

we have  $|x_i| < \varepsilon$ ,  $i = 2, 3, \dots, n$

Now define

$$\begin{aligned}
 f_t(x_1, \dots, x_n) = &(a_{11}x_1 + \dots + a_{1n}x_n - t\sigma \left( \frac{x_1^2 + k_1^2 x_2^2 + \dots + k_1^2 x_n^2}{c_1^2} \right), \\
 &(a_{12}x_2 + \dots + a_{2n}x_n), a_{22}x_2 + \dots + a_{2n}x_n, \dots, a_{nn}x_n).
 \end{aligned}$$

Then  $f_t = f$  when  $t = 0$  and  $f_t$  at  $t = 1$  is the mapping

$$(a_{11}x_1, a_{22}x_2 + \dots + a_{2n}x_n, \dots, a_{nn}x_n)$$

in a neighborhood of the origin. For each  $t$ ,  $f_t = f$  outside

$$x_1^2 + k_1^2 x_2^2 + \dots + k_1^2 x_n^2 = c_1^2$$

so that  $f_t = f$  outside  $U_{c_1}$ . Also  $J(f_t)$  in the (1, 1) position differs from  $a_{11}$  by

$$\left| t \frac{2x_1}{c_1^2} \sigma' \left( \frac{x_1^2 + k_1^2 x_2^2 + \dots + k_1^2 x_n^2}{c_1^2} \right) (a_{12}x_2 + \dots + a_{1n}x_n) \right|.$$

This expression is zero outside the ellipsoid  $x_1^2 + k_1^2 x_2^2 + \dots + k_1^2 x_n^2 = c_1^2$ . Inside this,  $|x_1| < c_1$  so if  $|a_{ij}| < M_1$ ,  $j = 2, \dots, n$ , and  $M$  is a bound on the derivative of  $\sigma(v)$ , the expression is at most  $1 \cdot (2/c_1) \cdot M(n - 1) \cdot M_1 \cdot \varepsilon$ . This expression is small whenever  $\varepsilon$  is small (nothing that  $\varepsilon$

can be chosen independent of  $c_1$ ). Thus by choosing  $\varepsilon$  small,  $|J(f_t)|$  will remain positive inside the ellipsoid and  $f_t$  will be a homeomorphism for each  $t$ .

Thus we assume  $f \in L$  and for  $c_i > 0$  with  $|x| \leq c_i < r$  the mapping is given by

$$(a_{11}x_1, \dots, a_{i-1,i-1}x_{i-1}, a_{ii}x_i + a_{i,i+1}x_{i+1} + \dots + a_{in}x_n, \\ a_{i+1,i+1}x_{i+1} + \dots + a_{i+1,n}x_n, \dots, a_{nn}x_n).$$

Let  $k_i > 1$  be sufficiently large so that whenever  $x_1^2 + \dots + x_i^2 + k_i^2x_{i+1}^2 + \dots + k_i^2x_n^2 \leq c_i^2$ , it follows that  $|x_j| < \varepsilon$ ,  $j = i + 1, \dots, n$ . Define for  $x \in U_r$

$$f_t(x_1, \dots, x_n) \\ = (f_1(x_1, \dots, x_n), \dots, f_{i-1}(x_1, \dots, x_n), f_i(x_1, \dots, x_n) \\ - t\sigma\left(\frac{x_1^2 + \dots + x_i^2 + k_i^2x_{i+1}^2 + \dots + k_i^2x_n^2}{c_i^2}\right)(a_{i,i+1}x_{i+1} + \dots + a_{in}x_n), \\ f_{i+1}(x_1, \dots, x_n), \dots, f_n(x_1, \dots, x_n)).$$

For the proper choice of  $\varepsilon$ , we can repeat the argument given above.

**LEMMA 3.** *Suppose  $f(x_1, \dots, x_n) = (a_1x_1, \dots, a_nx_n) \in L$ ,  $a_i > 0$  for all  $i$ . There is a path  $f_t$  in  $L$  from  $f$  to a mapping which is the identity in a neighborhood of the origin, and  $f_t = f$  for all  $t$  in a neighborhood of the boundary of  $U_r$ .*

*Proof.* First, if  $a > 0$  let  $p(x)$  be a function on  $(-\infty, \infty)$  with:

- (a)  $p(x) \in C^\infty$ ;
- (b)  $p'(x) > 0$  on  $(-\infty, \infty)$ ;
- (c)  $p(x) = x$  in a neighborhood of the origin;
- (d)  $p(x) = ax$  outside  $(-s + \alpha, s - \alpha)$ ,  $\alpha > 0$ .

We again construct the arc in segments. Choose  $s_1 < r$  and define

$$f_t(x_1, \dots, x_n) \\ = (a_1x_1 + t\sigma\left(\frac{x_2^2 + \dots + x_n^2}{s_1^2}\right)(p_1(x_1) - a_1x_1), a_2x_2, \dots, a_nx_n),$$

where  $\sigma$  is defined in Lemma 1 and  $p_1(x_1)$  satisfies properties (a) – (d) above for  $s = s_1$ . At  $t = 0$ ,  $f_t = f$ ; at  $t = 1$  in a neighborhood of the origin  $f_t$  is the mapping  $(x_1, a_2x_2, \dots, a_nx_n)$ . Also for all  $t \in I$ ,  $f_t = f$  outside the cylinder  $x_2^2 + \dots + x_n^2 \leq s_1^2$ ,  $-s_1 \leq x_1 \leq s_1$ .  $J(f_t)$  in the  $(1, 1)$  position is

$$\left[ 1 - t\sigma\left(\frac{x_2^2 + \dots + x_n^2}{s_1^2}\right) \right] a_1 + t\sigma\left(\frac{x_2^2 + \dots + x_n^2}{s_1^2}\right) p_1'(x_1),$$

which is positive for all  $t$  on the cylinder given above. Hence  $f_t$  is a homeomorphism for each  $t$ .

Now there is an  $s_2$  with  $0 < s_2 < s_1$  so that on the cylinder  $x_1^2 + x_3^2 + \dots + x_n^2 \leq s_2^2, -s_2 \leq x_2 \leq s_2$  the mapping is given by  $(x_1 a_2 x_2, \dots, a_n x_n)$ . On this cylinder define

$$f_t(x_1, \dots, x_n) = \left( x_1 a_2 x_2 + t\sigma\left(\frac{x_1^2 + x_3^2 + \dots + x_n^2}{s_2^2}\right) \right. \\ \left. \times (p_2(x_2) - a_2 x_2), a_3 x_3, \dots, a_n x_n \right).$$

Here  $p_2(x_2)$  satisfies conditions (a)-(d) given above with  $s = s_2$ . Repeating the process we complete the desired path.

**IV. Proof of Theorem 2.** The proof now consists of fitting together properly the mappings already constructed.

Let  $f \in K_n$ . Then there is a point  $p$  on  $S^n$  so that  $f$  has non-singular Jacobian at that point. Let  $(0_1, P_1)$  be a coordinate neighborhood where  $0_1 = S^n - p_1(p_1$  antipodal to  $p)$  and  $P_1$  an associated stereographic projection. Now there is a path  $e_t, t \in I$ , in the rotation group on  $S^n$  so that  $e_0$  is the identity map,  $e_t f = g$  leaves  $p$  fixed and  $P_1(e_t f)P_1^{-1} = P_1 g P_1^{-1}$  has a triangular Jacobian with positive diagonal elements at the origin. Let  $C$  be a closed disk on  $S^n$  so that for some  $r > 0, U_r \subset P_1(C)$ . Applying Lemmas 1 – 3 there is a path  $(P_1 g P_1^{-1})_t, t \in I$ , in the space of mappings on  $U_r$  from  $P_1 g P_1^{-1}$  to a mapping which is the identity in a neighborhood of the origin. Furthermore, for all  $t, (P_1 g P_1^{-1})_t$  agrees with  $P_1 g P_1^{-1}$  for all  $x \in P_1(C)$  except on an interior set of  $U_r$ . Define  $g_t \in K_n$  by

$$g_t = \begin{cases} P_1^{-1}(P_1 g P_1^{-1})_t P_1 & \text{on } C \\ g & \text{outside } C. \end{cases}$$

Then  $g_0 = g$  and  $g_1$  is the identity in a neighborhood of  $p$ .

Next let  $C_1$  and  $C_2$  be two closed sets covering  $S^n$  where  $C_1$  is a circular disk on  $S^n$  with  $p$  the center of the disk, and so that  $C_1$  is in an open set left pointwise fixed by  $g_1$ . We further assume  $p \notin C_2$ . Let  $(0_2, P_2)$  be a coordinate neighborhood with  $C_2 \subset 0_2 = S^n - p$  and  $P_2$  an associated stereographic projection. Then except for a trivial dilation  $P_2 g_1 P_2^{-1}$  is an element of the space  $H_n$ . By Theorem 1 there is a path  $(P_2 g_1 P_2^{-1})_t$  from  $P_2 g_1 P_2^{-1}$  to the identity map on  $P_2(C_2)$ . We now define  $h_t \in K'_n$  by

$$h_t = \begin{cases} P_2^{-1}(P_2 g_1 P_2^{-1})_t P_2 & \text{on } C_2 \\ g_1 & \text{outside } C_2. \end{cases}$$

The path from  $f$  to the identity map is now complete and Theorem 2 is established.

The spaces  $H_n$  and  $K_n$  are intermediate spaces to the topological spaces of Alexander and Kneser, and the diffeomorphism spaces treated by Smale. It is interesting to note that methods used in this paper are related to methods used in the larger nondifferentiable spaces and the smaller diffeomorphism spaces. Alexander's mapping is altered to give Theorem 1, while Theorem 2 parallels Smale's work.

#### REFERENCES

1. J. W. Alexander, *On the deformation of an  $n$ -cell*, Proc. Nat. Acad. Sci. U.S.A. **9**(1923), 406-407.
2. G. M. Fisher, *On the group of all homeomorphisms of a manifold*, Trans. Amer. Math. Soc. **97**(1960), 193-212.
3. H. Kneser, *Die Dieformationssätze der einfach zusammenhängenden Flächen*, Math. Z. **25**(1926), 362-372.
4. S. Smale, *Diffeomorphisms on the 2-sphere*, Proc. Amer. Math. Soc. **10**(1959), 621-626.





# PACIFIC JOURNAL OF MATHEMATICS

## EDITORS

H. SAMELSON

Stanford University  
Stanford, California

R. M. BLUMENTHAL

University of Washington  
Seattle, Washington 98105

J. DUGUNDJI

University of Southern California  
Los Angeles, California 90007

\*RICHARD ARENS

University of California  
Los Angeles, California 90024

## ASSOCIATE EDITORS

E. F. BECKENBACH

B. H. NEUMANN

F. WOLF

K. YOSIDA

## SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA  
CALIFORNIA INSTITUTE OF TECHNOLOGY  
UNIVERSITY OF CALIFORNIA  
MONTANA STATE UNIVERSITY  
UNIVERSITY OF NEVADA  
NEW MEXICO STATE UNIVERSITY  
OREGON STATE UNIVERSITY  
UNIVERSITY OF OREGON  
OSAKA UNIVERSITY  
UNIVERSITY OF SOUTHERN CALIFORNIA

STANFORD UNIVERSITY  
UNIVERSITY OF TOKYO  
UNIVERSITY OF UTAH  
WASHINGTON STATE UNIVERSITY  
UNIVERSITY OF WASHINGTON  
\* \* \*  
AMERICAN MATHEMATICAL SOCIETY  
CALIFORNIA RESEARCH CORPORATION  
SPACE TECHNOLOGY LABORATORIES  
NAVAL ORDNANCE TEST STATION

---

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be typewritten (double spaced). The first paragraph or two must be capable of being used separately as a synopsis of the entire paper. It should not contain references to the bibliography. No separate author's resumé is required. Manuscripts may be sent to any one of the four editors. All other communications to the editors should be addressed to the managing editor, Richard Arens, at the University of California, Los Angeles, California 90024.

50 reprints per author of each article are furnished free of charge; additional copies may be obtained at cost in multiples of 50.

---

The *Pacific Journal of Mathematics* is published quarterly, in March, June, September, and December. Effective with Volume 13 the price per volume (4 numbers) is \$18.00; single issues, \$5.00. Special price for current issues to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: \$8.00 per volume; single issues \$2.50. Back numbers are available.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley 8, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), No. 6, 2-chome, Fujimi-cho, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

\* Basil Gordon, Acting Managing Editor until February 1, 1966.

# Pacific Journal of Mathematics

Vol. 15, No. 4

December, 1965

Robert James Blattner, <i>Group extension representations and the structure space</i> .....	1101
Glen Eugene Bredon, <i>On the continuous image of a singular chain complex</i> .....	1115
David Hilding Carlson, <i>On real eigenvalues of complex matrices</i> .....	1119
Hsin Chu, <i>Fixed points in a transformation group</i> .....	1131
Howard Benton Curtis, Jr., <i>The uniformizing function for certain simply connected Riemann surfaces</i> .....	1137
George Wesley Day, <i>Free complete extensions of Boolean algebras</i> .....	1145
Edward George Effros, <i>The Borel space of von Neumann algebras on a separable Hilbert space</i> .....	1153
Michel Mendès France, <i>A set of nonnormal numbers</i> .....	1165
Jack L. Goldberg, <i>Polynomials orthogonal over a denumerable set</i> .....	1171
Frederick Paul Greenleaf, <i>Norm decreasing homomorphisms of group algebras</i> .....	1187
Fletcher Gross, <i>The 2-length of a finite solvable group</i> .....	1221
Kenneth Myron Hoffman and Arlan Bruce Ramsay, <i>Algebras of bounded sequences</i> .....	1239
James Patrick Jans, <i>Some aspects of torsion</i> .....	1249
Laura Ketchum Kodama, <i>Boundary measures of analytic differentials and uniform approximation on a Riemann surface</i> .....	1261
Alan G. Konheim and Benjamin Weiss, <i>Functions which operate on characteristic functions</i> .....	1279
Ronald John Larsen, <i>Almost invariant measures</i> .....	1295
You-Feng Lin, <i>Generalized character semigroups: The Schwarz decomposition</i> .....	1307
Justin Thomas Lloyd, <i>Representations of lattice-ordered groups having a basis</i> .....	1313
Thomas Graham McLaughlin, <i>On relative coimmunity</i> .....	1319
Mitsuru Nakai, <i><math>\Phi</math>-bounded harmonic functions and classification of Riemann surfaces</i> .....	1329
L. G. Novoa, <i>On <math>n</math>-ordered sets and order completeness</i> .....	1337
Fredos Papangelou, <i>Some considerations on convergence in abelian lattice-groups</i> .....	1347
Frank Albert Raymond, <i>Some remarks on the coefficients used in the theory of homology manifolds</i> .....	1365
John R. Ringrose, <i>On sub-algebras of a <math>C^*</math>-algebra</i> .....	1377
Jack Max Robertson, <i>Some topological properties of certain spaces of differentiable homeomorphisms of disks and spheres</i> .....	1383
Zalman Rubinstein, <i>Some results in the location of zeros of polynomials</i> .....	1391
Arthur Argyle Sagle, <i>On simple algebras obtained from homogeneous general Lie triple systems</i> .....	1397
Hans Samelson, <i>On small maps of manifolds</i> .....	1401
Annette Sinclair, <i><math> \varepsilon(z) </math>-closeness of approximation</i> .....	1405
Edsel Ford Stiel, <i>Isometric immersions of manifolds of nonnegative constant sectional curvature</i> .....	1415
Earl J. Taft, <i>Invariant splitting in Jordan and alternative algebras</i> .....	1421
L. E. Ward, <i>On a conjecture of R. J. Koch</i> .....	1429
Neil Marchand Wigley, <i>Development of the mapping function at a corner</i> .....	1435
Horace C. Wiser, <i>Embedding a circle of trees in the plane</i> .....	1463
Adil Mohamed Yaqub, <i>Ring-logics and residue class rings</i> .....	1465
John W. Lamperti and Patrick Colonel Suppes, <i>Correction to: Chains of infinite order and their application to learning theory</i> .....	1471
Charles Vernon Coffman, <i>Correction to: Non-linear differential equations on cones in Banach spaces</i> .....	1472
P. H. Doyle, III, <i>Correction to: A sufficient condition that an arc in <math>S^n</math> be cellular</i> .....	1474
P. P. Saworotnow, <i>Correction to: On continuity of multiplication in a complemented algebra</i> .....	1474
Basil Gordon, <i>Correction to: A generalization of the coset decomposition of a finite group</i> .....	1474