Pacific Journal of Mathematics

CORRECTION TO: CHAINS OF INFINITE ORDER AND THEIR APPLICATION TO LEARNING THEORY

John W. Lamperti and Patrick Colonel Suppes

Vol. 15, No. 4 December 1965

ERRATA

Correction to

CHAINS OF INFINITE ORDER AND THEIR APPLICATION TO LEARNING THEORY

JOHN LAMPERTI AND PATRICK SUPPES

Volume 9 (1959), 739–754

Professor M. Iosifescu has pointed out to us an error in our paper [1]. The difficulty lies in the positivity condition

$$(2.3) \qquad p_{j_0}^{(n_0)}(x) \geqq \delta \geqq 0 \qquad \text{for every } x \ ,$$

which is not strong enough when $n_0 > 1$. Iosifescu has in fact given an example of a second order Markov chain satisfying (2.3) with $n_0 = 0$ for which $\lim_{n\to\infty} p_i^{(n)}(x)$ is not independent of x as asserted by Theorem 2.1.

The difficulty can be overcome by making the stronger assumption that for some state j_0, some positive integer n_0, and some sequence of positive numbers δ_m,

$$(2.3') \qquad p_{j_0^* m}^{(n_0)}(x) \geqq \delta_m \qquad \text{for every } x \text{ and } m \ .$$

Here $p_{x_m}^{(n)}(x)$ is the joint probability (defined formally by (2.11) and (2.12)) of executing the sequence x_m after n steps, given x, and $j_0^* m$ means a sequence of m repetitions of j_0. Thus we are asserting that the event, consisting of m consecutive visits to j_0 starting after a lapse of time n_0, has positive probability uniformly in x (not in m). If $n_0 = 1$, (2.3′) follows from (2.3) with $\delta_m = \delta^m$, and our error lay in the tacit use of (2.3′), rather than (2.3), in proving Lemma 2.2 in our paper. When (2.3′) is assumed the argument given is valid. Lemma 2.1 does in fact follow from (2.3) and (2.5) as asserted, and so with the new hypothesis the conclusions of § 2 are justified.

Let us consider the effect of this change on the application to linear learning models. Assumption (ii) (b) of Theorem 4.1, which is used to derive (2.3), is now seen to be inadequate for the conclusions of the theorem. However the special case (4.5), when $m_0 = 0$, yields (2.3) with $n_0 = 1$ and so the results are valid in this situation. Although (2.3′) could be adapted to yield greater generality, we take it that essentially all cases of interest are actually covered by (4.5), and

Received April 10, 1965.

shall leave the matter so. A similar remark applies to Theorem 4.2.

REFERENCE

1. John Lamperti and Patrick Suppes, '*Chains of infinite order and their application to learning theory*,' Pacific J. Math. **9** (1959), 739-754.

Correction to

NON-LINEAR DIFFERENTIAL EQUATIONS ON CONES IN BANACH SPACES

CHARLES V. COFFMAN

Volume 14 (1964), 9–15

In [1] the proof of a main lemma, Lemma 3.1, contains an error. The lemma itself is false without stronger hypotheses. The purpose of this note is to state and prove a lemma which can be used in place of Lemma 3.1 in the proofs of Theorem 4.1 and 5.1 in [1].

Let Y be a Banach space, let Γ be a closed linear manifolds in Y^* which is total for Y.[1] Assume that I is some real interval. The differential equation with which [1] is concerned is

$$dy/dt = f(t, y) \ , \tag{1}$$

where f is a function from $I \times C \rightarrow Y$ which is continuous with respect to the weak Γ-topology on Y; C is a subset of Y. The notation and terminology used here will be the same as that employed in [1]; the definition of a weak Γ-derivative, a weak Γ-solution of (1), etc., are to be found in [1].

Let $\mathscr{C}$ be the space of weakly Γ-continuous functions on I with values in C, furnished with the topology of uniform convergence (in the weak Γ-topology) on compact subintervals of I. If C is compact in the weak Γ-topology, then Ascoli's theorem implies that a set of equicontinuous functions in $\mathscr{C}$ is relatively compact in $\mathscr{C}$. However unless the topology on $\mathscr{C}$ satisfies the first axiom of countability one cannot conclude from Ascoli's theorem, as is done in [1], that an equicontinuous sequence of functions in $\mathscr{C}$ has a convergent subsequence. ($\mathscr{C}$ will satisfy the first axiom of countability, for example,

Received March 3, 1965.

[1] In [1] a total manifold is defined but is incorrectly called a determining manifold. The author wishes to thank the referee of this note for pointing out this mistake as well as for correcting an omission in the original proof of the lemma stated here.

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50 reprints per author of each article are furnished free of charge; additional copies may be obtained at cost in multiples of 50.

The *Pacific Journal of Mathematics* is published quarterly, in March, June, September, and December. Effective with Volume 13 the price per volume (4 numbers) is $18.00; single issues, $5.00. Special price for current issues to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: $8.00 per volume; single issues $2.50. Back numbers are available.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley 8, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), No. 6, 2-chome, Fujimi-cho, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

* Basil Gordon, Acting Managing Editor until February 1, 1966.

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