Pacific Journal of Mathematics

AUTOMORPHISMS OF POSTLIMINAL C*-ALGEBRAS

E. CHRISTOPHER LANCE

Vol. 23, No. 3

AUTOMORPHISMS OF POSTLIMINAL C*-ALGEBRAS

E. C. LANCE

Let $\alpha(\mathfrak{A})$ denote the group of automorphisms of a C^* algebra \mathfrak{A} . The object of this paper is to give an intrinsic algebraic characterization of those elements α of $\alpha(\mathfrak{A})$ which are induced by a unitary operator in the weak closure of \mathfrak{A} in every faithful representation, and it is attained for the class of C^* -algebras known as GCR, or more recently postliminal. The relevant condition is that α should map closed two-sided ideals of \mathfrak{A} into themselves, and the main theorem (Theorem 2) may be thought of as an analogue for C^* -algebras of Kaplansky's theorem for von Neumann algebras, namely that an automorphism of a Type I von Neumann algebra is inner if and only if it leaves the centre elementwise fixed. The proof of Theorem 2 requires the—probably unnecessary—assumption that \mathfrak{A} is separable.

By a C^* -algebra we mean a Banach algebra over the complex numbers, with a conjugate-linear anti-automorphic involution $A \rightarrow A^*$ satisfying $||A^*A|| = ||A^*|| \cdot ||A||$. The mappings of C^* -algebras which we consider (automorphisms, representations, etc.) will always be assumed to preserve the adjoint operation, and by a homomorphic image of a C^* -algebra \mathfrak{A} , we mean the image of a homomorphism from \mathfrak{A} into another C^{*}-algebra \mathfrak{B} (this is automatically a C^{*}-subalgebra of \mathfrak{B} [2; 1.8.3]). We shall refer to Dixmier's book [2] for all standard results that we need to quote concerning C^* -algebras. By the theorem of Gelfand-Naimark (see, e.g. [2; 2.6.1]), a C^{*}-algebra has an isometric representation as an algebra of operators on a Hilbert space, and we shall usually think of a given C^* -algebra as being "concretely" represented on some Hilbert space. A state of a C^* -algebra \mathfrak{A} is a positive linear functional of norm one. The set \mathfrak{S} of states of \mathfrak{A} is a convex subset of the (Banach) dual space of \mathfrak{A} . If \mathfrak{A} has an identity element then \mathfrak{S} is w^* -compact, but in any case \mathfrak{S} contains an abundance of extreme points, which are called *pure states*. The set of pure states of \mathfrak{A} will be denoted by \mathfrak{P} .

Given a state ρ of \mathfrak{A} , there is a representation ϕ_{ρ} of \mathfrak{A} on a Hilbert space H_{ρ} , and a unit vector x_{ρ} in H_{ρ} such that $\{\phi_{\rho}(A)x_{\rho}: A \in \mathfrak{A}\}$ is dense in H_{ρ} (i.e. the representation ϕ_{ρ} is cyclic) and

$$\rho(A) = \left\langle \phi_{\rho}(A) x_{\rho}, x_{\rho} \right\rangle$$

for each $A \in \mathfrak{A}$. ϕ_{ρ} is irreducible if and only if ρ is pure. Given a state ρ of \mathfrak{A} , and a representation ϕ of \mathfrak{A} on H, we say that ρ is a vector state (in the representation ϕ) if $\rho(A) = \langle \phi(A)x, x \rangle$ for some

unit vector x in H; and if ϕ is faithful, we say that ρ is normal if the map $A \to \rho(A)$ is continuous with respect to the topology induced on $\phi(\mathfrak{A})$ by the ultra-weak topology on the algebra $\mathfrak{L}(H)$ of all bounded operators on H. It is clear that a vector state is normal. Let \emptyset denote the universal representation of \mathfrak{A} , formed by choosing one element from each unitary equivalence class of cyclic representations of \mathfrak{A} and taking their direct sum; and let Ψ denote the reduced atomic representation of \mathfrak{A} , formed by choosing one element from each unitary equivalence class of irreducible representations of \mathfrak{A} and taking their direct sum. Both \emptyset and Ψ are faithful representations, and every state [resp. every pure state] of \mathfrak{A} is a vector state in the representation \emptyset [resp. Ψ].

Let $\hat{\mathfrak{A}}$ denote the structure space of \mathfrak{A} , i.e. the set of unitary equivalence classes of irreducible representations of \mathfrak{A} , with the Jacobson topology [2; § 3.1]. Following Dixmier, we shall call a C^* algebra *liminal* if every irreducible representation consists of compact operators, *postliminal* if every nonzero homomorphic image has a nonzero closed two-sided liminal ideal, and *antiliminal* if it possesses no nonzero closed two-sided liminal ideals. If \mathfrak{A} is postliminal then $\hat{\mathfrak{A}}$ is a T_0 -space [2; 4.3.7 (ii)], and every representation of \mathfrak{A} has a Type I von Neumann algebra as weak closure [2; 5.5.2]. Also, \mathfrak{A} has a composition series $(I_{\rho})_{0 \leq \rho \leq \delta}$ (i.e. an increasing nest of closed two-sided ideals of \mathfrak{A} indexed by the ordinals less than or equal to some ordinal $\hat{\delta}$, such that $I_0 = (0), I_{\delta} = \mathfrak{A}$ and I_{ρ} is the closure of $\bigcup_{\rho' < \rho} I_{\rho'}$ for every limit ordinal $\rho \leq \delta$) such that each difference algebra $I_{\rho+1} - I_{\rho}$ has Hausdorff structure space [2; 4.5.5 and 4.5.3].

Given a C*-algebra \mathfrak{A} , we denote by $\alpha(\mathfrak{A})$ the group of automorphisms of \mathfrak{A} . Each element of $\alpha(\mathfrak{A})$ is an isometric isomorphism of \mathfrak{A} onto itself [2; 1.3.7 and 1.8.1]. If ϕ is a faithful representation of \mathfrak{A} on H, an automorphism α of \mathfrak{A} is said to be *extendable* (in the representation ϕ) if there is an automorphism of the weak closure of $\phi(\mathfrak{A})$ which agrees with $\phi \circ \alpha \circ \phi^{-1}$ on $\phi(\mathfrak{A})$; and weakly-inner if $\phi(\alpha(A)) = U^* \phi(A) U$ for each A in \mathfrak{A} , where U is a unitary operator in the weak closure of $\phi(\mathfrak{A})$. If $\alpha(A) = U^* A U$ for a unitary operator U in \mathfrak{A} , then we say that α is *inner*. Following [6], we denote by $\varepsilon_{\phi}(\mathfrak{A})$ [resp. $\iota_{\phi}(\mathfrak{A})$] the set of elements of $\alpha(\mathfrak{A})$ which are extendable [resp. weakly-inner] in the representation ϕ , and by $\pi(\mathfrak{A})$ the intersection of all the sets $\iota_{\phi}(\mathfrak{A})$ as ϕ ranges through the faithful representations of \mathfrak{A} (the elements of $\pi(\mathfrak{A})$ are called *permanently weakly-inner*, or π -inner automorphisms). The sets $\varepsilon_{\phi}(\mathfrak{A}), \iota_{\phi}(\mathfrak{A})$ and $\pi(\mathfrak{A})$ are all subgroups of $\alpha(\mathfrak{A})$. According to [6; Lemma 3], $\alpha \in \varepsilon_{\phi}(\mathfrak{A})$ if $\phi \circ \alpha \circ \phi^{-1}$ is ultra-weakly bicontinuous, equivalently if $\rho \circ \alpha$ is a normal state in the representation ϕ if and only if ρ is. It follows that $\varepsilon_{\phi}(\mathfrak{A}) = \alpha(\mathfrak{A})$

since every state is normal in the universal representation.

If $\alpha \in \alpha(\mathfrak{A})$, we shall say that α preserves ideals if $\alpha(I) \subseteq I$ for every closed two-sided ideal I of \mathfrak{A} , and that α preserves ideals carefully if $\alpha(I) = I$ for each such ideal I. We shall denote by $\tau(\mathfrak{A})$ [resp. $\tau_0(\mathfrak{A})$] the set of elements of $\alpha(\mathfrak{A})$ which preserve ideals [resp. preserve ideals carefully]. It is clear that $\tau_0(\mathfrak{A})$ is a subgroup of $\alpha(\mathfrak{A})$, and that $\tau(\mathfrak{A})$ is a subsemigroup of $\alpha(\mathfrak{A})$, but it is not clear whether $\tau(\mathfrak{A})$ can contain elements not in $\tau_0(\mathfrak{A})$ (cf. Corollary 1 of Theorem 1). Since an automorphism preserves the property of being a maximal ideal, an element of $\tau(\mathfrak{A})$ must preserve maximal two-sided ideals carefully, so that $\tau_0(\mathfrak{A}) = \tau(\mathfrak{A})$ if every closed two-sided ideal of \mathfrak{A} is an intersection of maximal ones.

LEMMA 1. For any C*-algebra $\mathfrak{A}, \varepsilon_{\Psi}(\mathfrak{A}) = \alpha(\mathfrak{A}).$

Proof. To save writing Ψ constantly, we shall suppose that \mathfrak{A} is given in its reduced atomic representation. Let \mathfrak{R} denote the closure in the norm topology on \mathfrak{S} of the convex hull of \mathfrak{P} . Let $\alpha \in \alpha(\mathfrak{A})$, then it is easy to see that α preserves pure states, i.e. $\rho \in \mathfrak{P} \Leftrightarrow \rho \circ \alpha \in \mathfrak{P}$. Also, for any bounded linear functional f on \mathfrak{A} , $||f \circ \alpha|| = ||f||$. It follows that $\sigma \in \mathfrak{R} \Leftrightarrow \sigma \circ \alpha \in \mathfrak{R}$.

Let \mathfrak{N}_0 denote the set of normal states of \mathfrak{A} . We shall show that $\mathfrak{N}_0 = \mathfrak{N}$ from which it follows that α and α^{-1} preserve normal states and by [6; Lemma 3] the lemma will be proved. Now \mathfrak{N}_0 is norm-closed and convex, and contains \mathfrak{P} since every pure state is a vector state in the given representation, hence $\mathfrak{N}_0 \supseteq \mathfrak{N}$. Conversely, if $\rho \in \mathfrak{N}_0$, then ρ is a norm limit of convex combinations of vector states [1; Chap. I § 4 Théorème 1] so it will suffice to show that each vector state is in \mathfrak{N} .

Denote by ω_x the state $A \to \langle Ax, x \rangle$ where x is a unit vector in the space H on which \mathfrak{A} acts. Since \mathfrak{A} is given in the reduced atomic representation we can write $H = \bigoplus_{\tau \in \Gamma} H_{\tau}$ where each H_{τ} is a subspace of H invariant under \mathfrak{A} , and the restriction $\mathfrak{A} \mid_{H_{\gamma}}$ is irreducible. Write $x = \sum_{\tau \in \Gamma} x_{\tau}$, with $x_{\tau} \in H_{\tau}$. Then

$$egin{aligned} A \in \mathfrak{A} & \Longrightarrow & Ax_{ au} \in H_{ au} & ext{for each } \gamma \in arGamma \ & \longrightarrow & ig< Ax, \ x ig> & = \sum_{ au \in arGamma} ig< Ax_{ au}, \ x_{ au} ig> \,, \end{aligned}$$

so that

(1)
$$\omega_x = \sum_{\gamma \in \Gamma} \omega_{x_{\gamma}}$$
, where $\sum_{\gamma \in \Gamma} ||x_{\gamma}||^2 = 1$.

But $\omega_{x_{\gamma}}$ is either zero (if $x_{\gamma} = 0$) or a multiple $||x_{\gamma}||^{-2}$ of a vector state of an irreducible representation, which is pure. It follows from

(1) that $\omega_x \in \mathfrak{N}$, showing that $\mathfrak{N}_0 \subseteq \mathfrak{N}$.

LEMMA 2. For any C*-algebra $\mathfrak{A}, \iota_{\Psi}(\mathfrak{A}) \subseteq \tau_{\mathfrak{g}}(\mathfrak{A})$.

Proof. We shall again suppose that \mathfrak{A} is given in its reduced atomic representation with weak closure \mathfrak{A}^- . Writing $H = \bigoplus_{\tau \in \Gamma} H_{\tau}$ as in Lemma 1, we have ([3]) $\mathfrak{A}^- = \bigoplus_{\tau \in \Gamma} \mathfrak{L}(H_{\tau})$. If $\alpha \in \iota_{\Psi}(\mathfrak{A})$, let $U = \sum U_{\tau}$ be a unitary in \mathfrak{A}^- which induces α , where U_{τ} is a unitary operator on $H_{\tau}(\gamma \in \Gamma)$. Let π_{τ} be the irreducible representation of \mathfrak{A} on H_{τ} defined by $A \to A|_{H_{\gamma}}$ (for some $\gamma \in \Gamma$), and suppose $\pi_{\tau}(A) = 0$. Then

$$egin{array}{ll} \pi_r(lpha(A)) &= \ U^* \ A \ U \,|_{{}^{_{H_{oldsymbol{\gamma}}}}} \ &= \ U^*_r \ A \ U_r \ &= \ 0 \ . \end{array}$$

Thus a preserves the primitive ideal $\pi_r^{-1}(0)$. But every primitive ideal is of this form, and every closed two-sided ideal in \mathfrak{A} is an intersection of primitive ideals, hence α preserves ideals.

Since $c_{\Psi}(\mathfrak{A})$ is a group, α^{-1} also preserves ideals, and so α preserves ideals carefully.

As an immediate corollary to the above lemma, we have $\pi(\mathfrak{A}) \subseteq \tau_0(\mathfrak{A})$ for any C*-algebra \mathfrak{A} , a fact which has previously been noted by **R.** V. Kadison (private communication).

THEOREM 1. If \mathfrak{A} is a postliminal C*-algebra, then $\tau(\mathfrak{A}) = \iota_{\Psi}(\mathfrak{A})$.

Proof. We continue to assume that \mathfrak{A} is given in the reduced atomic representation, and we shall use the notation established in Lemma 2. By that lemma, we have only to prove that $\tau(\mathfrak{A}) \subseteq \iota_{\mathfrak{P}}(\mathfrak{A})$.

For each closed two-sided ideal I of \mathfrak{A} , define subsets $\mathfrak{U}(I)$ and $\mathfrak{V}(I)$ of the structure space $\widehat{\mathfrak{A}}$ by

$$\mathfrak{U}(I) = \{\pi \in \widehat{\mathfrak{A}} \colon \pi(I) = (0)\} \;, \ \mathfrak{V}(I) = \{\pi \in \widehat{\mathfrak{A}} \colon \pi(I)
eq (0)\} \;.$$

These sets are, respectively, closed and open in $\hat{\mathfrak{A}}$ [2; 3.2.1].

Suppose that $\alpha \in \tau(\mathfrak{A})$. By Lemma 1, α has an extension to an automorphism $\overline{\alpha}$ of $\mathfrak{A}^- = \bigoplus_{\tau \in \Gamma} \mathfrak{A}(H_{\tau})$. Given $\pi \in \mathfrak{A}$ there is a unique subspace H_{τ} of H such that π is unitarily equivalent to π_{τ} . Let $E_{\pi} \in \mathfrak{A}^-$ denote the projection from H onto H_{τ} . The elements $\{E_{\pi} : \pi \in \mathfrak{A}\}$ are precisely the minimal central projections of \mathfrak{A}^- , and they generate the centre of \mathfrak{A}^- (as a von Neumann algebra). An automorphism

550

preserves the property of being a minimal central projection, so $\bar{\alpha}$ permutes the E_{π} .

Let $(I_{\rho})_{0 \leq \rho \leq \delta}$ be a composition series for \mathfrak{A} such that each difference algebra $I_{\rho+1} - I_{\rho}$ has Hausdorff structure space. Suppose that σ is an ordinal $(0 < \sigma \leq \delta)$ and that for $\rho < \sigma$ we have shown that

$$(2) \qquad \qquad \bar{\alpha}(E_{\pi}) = E_{\pi} \quad \text{for all} \quad \pi \in \mathfrak{V}(I_{\rho}) \; .$$

Clearly (2) is (vacuously) satisfied for $\sigma = 1$. If σ is a limit ordinal then $\mathfrak{B}(I_{\sigma}) = \bigcup_{\rho < \sigma} \mathfrak{B}(I_{\rho})$ so that (2) holds with $\rho = \sigma$. Suppose that σ is not a limit ordinal, and let $\theta \in \mathfrak{B}(I_{\sigma})$. Let $\overline{\alpha}(E_{\theta}) = E_{\phi}$. We shall suppose $\phi \neq \theta$ and obtain a contradiction.

Let $\{\phi\}^-$ denote the closure of $\{\phi\}$ in the Jacobson topology. We shall first show that $\theta \notin \{\phi\}^-$. To see this, note that

$$\mathfrak{A} = \mathfrak{B}(I_{\sigma-1}) \cup (\mathfrak{B}(I_{\sigma}) \cap \mathfrak{U}(I_{\sigma-1})) \cup \mathfrak{U}(I_{\sigma}) \; ,$$

so that ϕ must belong to one of these three sets.

(i) for $\pi \in \mathfrak{V}(I_{\sigma-1})$ we have by (2), $\overline{\alpha}(E_{\pi}) = E_{\pi}$, so that all the elements $E_{\pi}(\pi \in \mathfrak{V}(I_{\sigma-1}))$ are already bespoken as values for the (injective) mapping $\overline{\alpha}$, hence it is not possible that $\phi \in \mathfrak{V}(I_{\sigma-1})$ unless $\theta = \phi$. Thus $\phi \notin \mathfrak{V}(I_{\sigma-1})$ and also $\theta \notin \mathfrak{V}(I_{\sigma-1})$.

(ii) $\mathfrak{V}(I_{\sigma}) \cap \mathfrak{U}(I_{\sigma_{-1}})$ is homeomorphic with the structure space of $I_{\sigma} - I_{\sigma_{-1}}$ [2; 3.2.1], and this is Hausdorff (and hence a T_1 -space) so that if $\phi \in \mathfrak{V}(I_{\sigma}) \cap \mathfrak{U}(I_{\sigma_{-1}}), \theta \notin \{\phi\}^-$ since by (i) θ is also in $\mathfrak{V}(I_{\sigma}) \cap \mathfrak{U}(I_{\sigma_{-1}})$.

(iii) $\mathfrak{U}(I_{\sigma})$ is closed, and $\theta \notin \mathfrak{U}(I_{\sigma})$. Thus if $\phi \in \mathfrak{U}(I_{\sigma})$, it follows that $\{\phi\}^{-} \subseteq \mathfrak{U}(I_{\sigma})$ and $\theta \notin \{\phi\}^{-}$.

Thus in any case $\theta \notin \{\phi\}^-$, i.e. Ker $(\phi) \nsubseteq \text{Ker}(\theta)$. Choose $A \in \mathfrak{A}$ such that $\phi(A) = 0, \ \theta(A) \neq 0$. Then

$$egin{aligned} & heta(A)
eq 0 & \longrightarrow AE_ heta
eq 0 \ & \longrightarrow ar lpha(AE_ heta)
eq 0 \ & \longrightarrow ar lpha(A) \cdot ar lpha(E_ heta)
eq 0 \ & \longrightarrow lpha(A) \cdot E_ heta
eq 0 \,. \end{aligned}$$

On the other hand, $\alpha \in \tau(\mathfrak{A})$ so α preserves Ker (ϕ) , hence

$$\phi(A) = 0 \Longrightarrow A \in \operatorname{Ker}(\phi)$$

 $\Longrightarrow \alpha(A) \in \operatorname{Ker}(\phi)$
 $\Longrightarrow \alpha(A) \cdot E_{\phi} = 0$

We have arrived at a contradiction, thus showing that $\overline{\alpha}(E_{\theta}) = E_{\theta}$ for $\theta \in \mathfrak{V}(I_{\sigma})$, i.e. (2) holds for $\rho = \sigma$.

By transfinite induction, $\overline{\alpha}(E_{\pi}) = E_{\pi}$ for all $\pi \in \mathfrak{A}(=\mathfrak{V}(I_{\delta}))$. Since the centre of \mathfrak{A}^- is generated as a von Neumann algebra by the E_{π} and $\overline{\alpha}$ is ultra-weakly continuous (*cf.* Lemma 1), $\overline{\alpha}$ leaves the centre elementwise fixed. But \mathfrak{A}^- is Type I, so by Kaplansky's theorem [7] $\overline{\alpha}$ is inner, which proves the theorem.

COROLLARY 1. If \mathfrak{A} is postliminal, then $\tau_0(\mathfrak{A}) = \tau(\mathfrak{A})$.

Proof. By Lemma 2 and Theorem 1 we have

 $\tau_{0}(\mathfrak{A}) \subseteq \tau(\mathfrak{A}) = \iota_{\mathfrak{p}}(\mathfrak{A}) \subseteq \tau_{0}(\mathfrak{A}) .$

COROLLARY 2. If \mathfrak{A} is postliminal, $\alpha \in \tau(\mathfrak{A})$ and ϕ is an irreducible representation of \mathfrak{A} , then α induces a weakly-inner automorphism α_{δ} of $\phi(\mathfrak{A})$.

Proof. Suppose that \mathfrak{A} is given in its reduced atomic representation. ϕ is unitarily equivalent to the map $A \to AE_{\pi}$ (for some $\pi \in \widehat{\mathfrak{A}}$). By Theorem 1, $\alpha(A) = U^* AU$ (for all $A \in \mathfrak{A}$) for some $U \in \mathfrak{A}^-$. The map $AE_{\pi} \to (UE_{\pi})^* AE_{\pi}(UE_{\pi})$ is then unitarily equivalent to the required automorphism of $\phi(\mathfrak{A})$.

Our results so far have mirrored those of Miles [8] on derivations. In the case of derivations, it is now known ([5] and [9]) that every derivation of a C^* -algebra is permanently weakly-inner. We shall now show that the analogous result holds for ideal-preserving automorphisms of (separable) postliminal C^* -algebras, by making use of the decomposition of a representation of such an algebra as a direct integral of irreducible representations. For an account of this decomposition, see [1; Chap. II] and [2; §8].

LEMMA 3. If \mathfrak{A} is a C*-algebra, $\alpha \in \tau_0(\mathfrak{A})$ and \mathfrak{B} is any homomorphic image of \mathfrak{A} , then α induces an automorphism in $\tau_0(\mathfrak{B})$.

Proof. Let ψ be a homomorphism from \mathfrak{A} onto \mathfrak{B} , with kernel *I*. Define a map $\tilde{\alpha}$ on \mathfrak{B} by $\tilde{\alpha}(\psi(A)) = \psi(\alpha(A))$. $\tilde{\alpha}$ is well-defined since α preserves *I*. It is clearly a homomorphism, with range the whole of \mathfrak{B} , and since α preserves *I* carefully it is injective. Thus it is an automorphism.

If J is a closed two-sided ideal in \mathfrak{B} then $\psi^{-1}(J)$ is a closed twosided ideal in \mathfrak{A} containing I and is carefully preserved by α , from which it follows that $\tilde{\alpha}$ carefully preserves J. Thus $\tilde{\alpha} \in \tau_0(\mathfrak{B})$.

THEOREM 2. If \mathfrak{A} is a separable postliminal C*-algebra then $\pi(\mathfrak{A}) = \tau(\mathfrak{A})$.

Proof. We have already noted that $\pi(\mathfrak{A}) \subseteq \tau(\mathfrak{A})$. Suppose $\alpha \in \tau(\mathfrak{A})$,

and let ϕ be any faithful representation of \mathfrak{A} . We have to show that α is weakly-inner in the representation ϕ . Since \mathfrak{A} is postliminal, the weak closure $\overline{\phi(\mathfrak{A})}$ is a Type I von Neumann algebra, so is isomorphic to an algebra with abelian commutant, i.e. ϕ is quasi-equivalent to a multiplicity-free representation (*cf.* [2; 5.4.1]). Since the property of being weakly-inner is preserved by quasi-equivalence, we may suppose that ϕ is multiplicity-free and $\phi(\mathfrak{A})'$ is abelian (we use a prime to denote the commutant of a set of operators). Since we are assuming that \mathfrak{A} is separable, $\overline{\phi(\mathfrak{A})}$ is generated (as a von Neumann algebra) by a countable set of operators.

Let E be a cyclic projection in $\phi(\mathfrak{A})'$ (which is the centre of $\overline{\phi(\mathfrak{A})}$). The restriction of $\phi(\mathfrak{A})$ to E is a homomorphic image of \mathfrak{A} , so by Lemma 3 α induces an ideal-preserving automorphism on it. If the automorphism so induced on each cyclic portion of the centre of $\overline{\phi(\mathfrak{A})}$ is weakly-inner, then (taking a maximal orthogonal family of cyclic central projections) it follows that α is weakly-inner. We may thus restrict to a cyclic central projection and we can therefore assume that ϕ acts on a separable Hilbert space H.

There exist [2; 8.3.2] a standard Borel space Z, a bounded positive measure μ on Z, a measurable field $\zeta \to H_{\zeta}$ of Hilbert spaces on Z, a measurable field of representations $\zeta \to \pi_{\zeta}$ of \mathfrak{A} on the field (H_{ζ}) and an isometry from H onto $\int^{\oplus} H_{\zeta} d\mu(\zeta)$, which transforms $\phi(\mathfrak{A})'$ into the diagonal operators and ϕ into $\int^{\oplus} \pi_{\zeta} d\mu(\zeta)$. We shall equate $H, \phi(\mathfrak{A}), \& c.$ with their transforms under this equivalence. Since $\phi(\mathfrak{A})'$ consists of diagonal operators, almost every π_{ζ} is irreducible [2; 8.5.1]. For almost all $\zeta \in Z, \alpha$ induces an automorphism α_{ζ} of $\pi_{\zeta}(\mathfrak{A})$, which by Corollary 2 of Theorem 1 is weakly-inner, and so in particular extends to an automorphism (which we still call α_{ζ}) of $\mathfrak{L}(H_{\zeta})$. Define $\alpha_{\zeta} = 0$ on the exceptional null set. α_{ζ} is ultra-weakly continuous, hence strongly continuous on bounded sets. Thus we have a field (which we do not yet know to be measurable) of automorphisms α_{ζ} , such that for each $A \in \mathfrak{A}, \phi(\alpha(A)) = \int^{\oplus} \alpha_{\zeta}(\pi_{\zeta}(A)) d\mu(\zeta)$.

We now show that α is weakly continuous on the unit ball of \mathfrak{A} (in the representation ϕ). To do this it suffices, by [4; Remark 2.2.3], to show that α is weakly continuous at zero on the set of positive operators in the unit ball of \mathfrak{A} . Since H is separable, the unit ball is metrizable in the weak topology, and we need only deal with sequences. Suppose that $I \ge A_n \ge 0$ and $\phi(A_n) \to 0$ weakly. Then $\phi(A_n^{1/2}) \to 0$ strongly and by [1; Chap. II § 2 Prop. 4 (i)] there is a subsequence (n_k) such that, locally almost everywhere, $\pi_{\zeta}(A_{n_k}^{1/2}) \to 0$ strongly. Since α_{ζ} is strongly continuous on bounded sets, we have locally almost everywhere, $\pi_{\zeta}(\alpha(A_{n_k}^{1/2})) \to 0$ strongly. Since the sequence (A_{n_k}) is bounded, it follows from [1; Chap. II § 2 Prop. 4 (ii)] that $\alpha(A_{n_k}^{1/2}) \to 0$ strongly and so $\alpha(A_{n_k}) \to 0$ weakly. Thus α (and similarly α^{-1}) is weakly continuous on bounded sets in the representation ϕ , hence ultra-weakly continuous, and so α is extendable to an automorphism $\overline{\alpha}$ of $\overline{\phi(\mathfrak{A})}$.

We shall next show that the field of automorphisms $\zeta \to \alpha_{\zeta}$ induces $\overline{\alpha}$ on $\overline{\phi(\mathfrak{A})}$ (and so is measurable). Let A be a fixed element of $\overline{\phi(\mathfrak{A})}$, and let $\zeta \to A_{\zeta}$ be a measurable operator field representing $\overline{\alpha}(A)$. By metrizability of the strong topology [1; p. 33] and Kaplansky's Density Theorem [1; Chap. I § 3 Th. 3], we can choose a sequence (A_n) in \mathfrak{A} such that $||A_n|| \leq ||A||$ and $\phi(A_n) \to A$ strongly. By passing to a subsequence and using [1; Chap. II § 2 Prop. 4(i)] again, we can even suppose that $\pi_{\zeta}(A_n) \to A_{\zeta}$ strongly, locally almost everywhere. Since $\overline{\alpha}$ is strongly continuous on bounded sets, $\phi(\alpha(A_n)) \to \overline{\alpha}(A) = \int_{-\infty}^{\oplus} B_{\zeta} d\mu(\zeta)$ strongly, locally almost everywhere. But since α_{ζ} is strongly continuous on bounded sets, $\varphi(\alpha(A_{n_k})) \to \alpha_{\zeta}(\alpha(A_{n_k})) \to \alpha_{\zeta}(A_{\zeta})$ strongly, locally almost everywhere. But since α_{ζ} is strongly continuous on bounded sets, we have $\pi_{\zeta}(\alpha(A_{n_k})) \to \alpha_{\zeta}(A_{\zeta})$ strongly locally almost everywhere, we have $B_{\zeta} = \alpha_{\zeta}(A_{\zeta})$. Thus $\overline{\alpha}(A) = \int_{-\infty}^{\oplus} \alpha_{\zeta}(A_{\zeta}) d\mu(\zeta)$, as required.

Now since $\overline{\alpha}$ is induced by the field $\zeta \to \alpha_{\zeta}$, it is clear that $\overline{\alpha}$ leaves each diagonal operator fixed, i.e. $\overline{\alpha}$ leaves the centre of $\overline{\phi(\mathfrak{A})}$ elementwise fixed. Hence by Kaplansky's Theorem $\overline{\alpha}$ is inner (since $\overline{\phi(\mathfrak{A})}$ is Type I), and the proof is complete.

It is possible for an automorphism of a postliminal algebra to be weakly-inner in some representation without being π -inner, as the following example shows. Let ν denote Lebesgue measure on the interval [0, 1], and let $H = L_2([0, 1], \nu)$. Let \Re denote the set of compact operators on H. For $f \in C([0, 1])$ let T_f denote the operator defined by

$$T_f x(t) = f(t) x(t) ,$$

and let $\mathfrak{T} = \{T_f : f \in C([0, 1])\} \subseteq \mathfrak{L}(H)$. Then $\mathfrak{A} = \mathfrak{R} + \mathfrak{T}$ is a C^* -algebra [2; 1.8.4] and is postliminal since $\{(0), \mathfrak{R}, \mathfrak{A}\}$ is a composition series for which each difference algebra has Hausdorff structure space (because $\mathfrak{A} - \mathfrak{R} \cong \mathfrak{T}$). Let $U \in \mathfrak{L}(H)$ be the unitary operator defined by

$$Ux(t) = x(1-t)$$

then U induces an automorphism of \mathfrak{A} : for if $K \in \mathfrak{R}$, $T_f \in \mathfrak{T}$ then $U^*(K + T_f) U = U^* K U + T_g$ (where g(t) = f(1 - t)). Let

$$I_0 = \{T_f \in \mathfrak{T}: f(t) = 0 \quad \text{for} \quad 0 \leq t \leq \frac{1}{2}\}$$

and let $I_1 = \Re + I_0$, then it is easy to see that $U^* \cdot U$ does not preserve I_1 , so by Theorem 2, $U^* \cdot U$ is not π -inner. (In fact, it is not weakly-inner in the representation of \mathfrak{A} on $H \bigoplus H$ defined by $K + T \longrightarrow (K + T) \bigoplus T$.) But it is clearly weakly-inner in the given representation, since this is irreducible.

This example also shows that an automorphism of a postliminal C^* -algebra can leave the centre elementwise fixed and yet not be π -inner: for the centre of $\Re + \mathfrak{T}$ consists just of scalar multiples of the identity.

We conclude with a few remarks about the antiliminal case. Let \mathfrak{A} be a factor of Type II₁. Then \mathfrak{A} has no nonzero proper closed two-sided ideals, so that $\tau_0(\mathfrak{A}) = \tau(\mathfrak{A}) = \alpha(\mathfrak{A})$ in this case. On the other hand, there are many outer automorphisms of \mathfrak{A} . Thus the sets $\tau_0(\mathfrak{A})$ and $\tau(\mathfrak{A})$ are probably not of great interest when \mathfrak{A} is antiliminal.

Let \mathfrak{A} be an antiliminal algebra with a faithful irreducible representation. Then \mathfrak{A} has uncountably many such representations, all inequivalent [2; 4.7.2]. Intuitively, it seems unlikely that an automorphism would be weakly-inner in all these representations without actually being inner. In [6; Ex. a] an example is given of such an algebra (the Fermion algebra \mathfrak{F}) together with an automorphism of \mathfrak{F} which is weakly-inner in one representation, but not π -inner. It would be interesting to have an example of an automorphism of \mathfrak{F} which is π -inner but not inner.

References

1. J. Dixmier, Les algèbres d'opérateurs dans l'espace hilbertien, Gauthier-Villars, Paris, 1957.

- 2. ____, Les C*-algèbres et leurs représentations, Gauthier-Villars, Paris, 1964.
- 3. J. Glimm and R. V. Kadison, Unitary operators in C*-algebras, Pacific J. Math. 10 (1960), 547-556.
- 4. R. V. Kadison, Unitary invariants for representations of operator algebras, Ann. of Math. **66** (1957), 304-379.
- 5. ____, Derivations of operator algebras, Ann. of Math. 83 (1966), 280-293.
- 6. R. V. Kadison and J. R. Ringrose, Derivations and automorphisms of operator algebras, Commun. Math. Phys. 4 (1967), 32-63.
- 7. I. Kaplansky, Algebras of type I, Ann. of Math. 56 (1952), 460-472.
- 8. P. Miles, Derivations on B*-algebras, Pacific J. Math. 14 (1964), 1359-1366.
- 9. S. Sakai, Derivations of W*-algebras, Ann. of Math. 83 (1966), 273-279.

Received January 17, 1967.

UNIVERSITY OF NEWCASTLE UPON TYNE

PACIFIC JOURNAL OF MATHEMATICS

EDITORS

H. ROYDEN

Stanford University Stanford, California

J. P. JANS University of Washington Seattle, Washington 98105 J. DUGUNDJI

Department of Mathematics Rice University Houston, Texas 77001

RICHARD ARENS University of California Los Angeles, California 90024

ASSOCIATE EDITORS

E. F. BECKENBACH

B. H. NEUMANN F. WOLF

K. Yosida

SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA	STANFORD UNIVERSITY
CALIFORNIA INSTITUTE OF TECHNOLOGY	UNIVERSITY OF TOKYO
UNIVERSITY OF CALIFORNIA	UNIVERSITY OF UTAH
MONTANA STATE UNIVERSITY	WASHINGTON STATE UNIVERSITY
UNIVERSITY OF NEVADA	UNIVERSITY OF WASHINGTON
NEW MEXICO STATE UNIVERSITY	* * *
OREGON STATE UNIVERSITY	AMERICAN MATHEMATICAL SOCIETY
UNIVERSITY OF OREGON	CHEVRON RESEARCH CORPORATION
OSAKA UNIVERSITY	TRW SYSTEMS
UNIVERSITY OF SOUTHERN CALIFORNIA	NAVAL ORDNANCE TEST STATION

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be typewritten (double spaced). The first paragraph or two must be capable of being used separately as a synopsis of the entire paper. It should not contain references to the bibliography. Manuscripts may be sent to any one of the four editors. All other communications to the editors should be addressed to the managing editor, Richard Arens at the University of California, Los Angeles, California 90024.

50 reprints per author of each article are furnished free of charge; additional copies may be obtained at cost in multiples of 50.

The *Pacific Journal of Mathematics* is published monthly. Effective with Volume 16 the price per volume (3 numbers) is \$8.00; single issues, \$3.00. Special price for current issues to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: \$4.00 per volume; single issues \$1.50. Back numbers are available.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley 8, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 7-17, Fujimi 2-chome, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

Pacific Journal of Mathematics

Vol. 23, No. 3 May, 1967

Charles Ballantine, Products of positive definite matrices. I	A. A. Aucoin, <i>Diophantine systems</i>	419
David Wilmot Barnette, A necessary condition for d-polyhedrality 435 James Clark Beidleman and Tae Kun Seo, Generalized Frattini subgroups of finite groups 441 Carlos Jorge Do Rego Borges, A study of multivalued functions 451 William Edwin Clark, Algebras of global dimension one with a finite ideal lattice 463 Richard Brian Darst, On a theorem of Nikodym with applications to weak convergence and von Neumann algebras 473 George Wesley Day, Superatomic Boolean algebras 479 Lawrence Fearnley, Characterization of the continuous images of all pseudocricles 491 Neil Robert Gray, Unstable points in the hyperspace of connected subsets 515 Franklin Haimo, Polynomials in central endomorphisms 521 John Sollion Hsia, Integral equivalence of vectors over local modular lattices 527 Jim Humphreys, Existence of Levi factors in certain algebraic groups 543 E. Christopher Lance, Automorphisms of postliminal C*-algebras 557 Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics 569 Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces 591 J. H. Reed, Inverse limits of indecomposable continuous images of complete metric spaces 621 Abraham	Charles Ballantine, Products of positive definite matrices. I	427
James Clark Beidleman and Tae Kun Seo, Generalized Frattini subgroups of finite 441 Carlos Jorge Do Rego Borges, A study of multivalued functions 451 William Edwin Clark, Algebras of global dimension one with a finite ideal 463 Richard Brian Darst, On a theorem of Nikodym with applications to weak 463 convergence and von Neumann algebras 473 George Wesley Day, Superatomic Boolean algebras 479 Lawrence Fearnley, Characterization of the continuous images of all pseudocircles pseudocircles 491 Neil Robert Gray, Unstable points in the hyperspace of connected subsets 515 Franklin Haimo, Polynomials in central endomorphisms 521 John Sollion Hsia, Integral equivalence of vectors over local modular lattices 527 Jim Humphreys, Existence of Levi factors in certain algebraic groups 543 E. Christopher Lance, Automorphisms of postliminal C*-algebras 547 Sibe Mardesic, Images of ordered compacta are locally peripherally metric 557 Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, 0rder-preserving functions: Applications to majorization and order statistics 569 Wellington Ham Ow, An extremal length criterion for the parabolicity of 86 Riemannian space	David Wilmot Barnette, A necessary condition for d-polyhedrality	
groups 441 Carlos Jorge Do Rego Borges, A study of multivalued functions 451 William Edwin Clark, Algebras of global dimension one with a finite ideal lattice 463 Richard Brian Darst, On a theorem of Nikodym with applications to weak convergence and von Neumann algebras 473 George Wesley Day, Superatomic Boolean algebras 473 George Wesley Day, Superatomic Boolean algebras 479 Lawrence Fearnley, Characterization of the continuous images of all pseudocircles 491 Neil Robert Gray, Unstable points in the hyperspace of connected subsets 515 Franklin Haimo, Polynomials in central endomorphisms 521 John Sollion Hsia, Integral equivalence of vectors over local modular lattices 527 Jim Humphreys, Existence of Levi factors in certain algebraic groups 543 E. Christopher Lance, Automorphisms of postiliminal C*-algebras 547 Sibe Mardesic, Images of ordered compacta are locally peripherally metric 557 Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics 569 Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces 585 Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces 591 <	James Clark Beidleman and Tae Kun Seo, Generalized Frattini subgroups of finite	441
Cartos Jorge Do Rego Borges, A study of militivalized junctions 451 William Edwin Clark, Algebras of global dimension one with a finite ideal lattice 463 Richard Brian Darst, On a theorem of Nikodym with applications to weak convergence and von Neumann algebras 473 George Wesley Day, Superatomic Boolean algebras 473 George Wesley Day, Superatomic Boolean algebras 479 Lawrence Fearnley, Characterization of the continuous images of all pseudocircles 491 Neil Robert Gray, Unstable points in the hyperspace of connected subsets 515 Franklin Haimo, Polynomials in central endomorphisms 521 John Sollion Hsia, Integral equivalence of vectors over local modular lattices 527 Jim Humphreys, Existence of Levi factors in certain algebraic groups 543 E. Christopher Lance, Automorphisms of postliminal C*-algebras 547 Sibe Mardesic, Images of ordered compacta are locally peripherally metric 557 Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics 569 Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces 591 J. H. Reed, Inverse limits of indecomposable continuo. 591 J. H. Reed, Inverse limits of indecomposable continuo simages of complete me	groups	441
William Edwin Clark, Algebras of global almension one with a junite ideal lattice 463 Richard Brian Darst, On a theorem of Nikodym with applications to weak convergence and von Neumann algebras 473 George Wesley Day, Superatomic Boolean algebras 479 Lawrence Fearnley, Characterization of the continuous images of all pseudocircles 491 Neil Robert Gray, Unstable points in the hyperspace of connected subsets 515 Franklin Haimo, Polynomials in central endomorphisms 521 John Sollion Hsia, Integral equivalence of vectors over local modular lattices 527 Jim Humphreys, Existence of Levi factors in certain algebraic groups 543 E. Christopher Lance, Automorphisms of postliminal C*-algebras 547 Sibe Mardesic, Images of ordered compacta are locally peripherally metric 557 Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets. 0rder-preserving functions: Applications to majorization and order statistics 585 Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces 591 J. H. Reed, Inverse limits of indecomposable continua 597 Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra 601 Roy Westwick, Transformations on tensor spaces 621 Abraham Zaks, A note on semi-primary hereditary ri	Villion Educio Chela Ala la calabela include a functions	431
Richard Brian Darst, On a theorem of Nikodym with applications to weak 405 Richard Brian Darst, On a theorem of Nikodym with applications to weak 473 George Wesley Day, Superatomic Boolean algebras 473 George Wesley Day, Superatomic Boolean algebras 479 Lawrence Fearnley, Characterization of the continuous images of all pseudocircles 491 Neil Robert Gray, Unstable points in the hyperspace of connected subsets 515 Franklin Haimo, Polynomials in central endomorphisms 521 John Sollion Hsia, Integral equivalence of vectors over local modular lattices 527 Jim Humphreys, Existence of Levi factors in certain algebraic groups 543 E. Christopher Lance, Automorphisms of postliminal C*-algebras 547 Sibe Mardesic, Images of ordered compacta are locally peripherally metric 557 Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics 569 Wellington Ham Ow, An extremal length criterion for the parabolicity of 885 Weilington Ham Ow, Criteria for zero capacity of ideal boundary components of 597 Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra 601 Roward Henry Wicke, The regular open continuous images of complete metric spaces <td< td=""><td>William Edwin Clark, Algebras of global dimension one with a finite ideal</td><td>162</td></td<>	William Edwin Clark, Algebras of global dimension one with a finite ideal	162
Richard Briar Dats, On a mearem of Nikodym win apprications to weak convergence and von Neumann algebras473George Wesley Day, Superatomic Boolean algebras479Lawrence Fearnley, Characterization of the continuous images of all pseudocircles491Neil Robert Gray, Unstable points in the hyperspace of connected subsets515Franklin Haimo, Polynomials in central endomorphisms521John Sollion Hsia, Integral equivalence of vectors over local modular lattices527Jim Humphreys, Existence of Levi factors in certain algebraic groups543E. Christopher Lance, Automorphisms of postliminal C*-algebras547Sibe Mardesic, Images of ordered compacta are locally peripherally metric557Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets. Order-preserving functions: Applications to majorization and order statistics569Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces585Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces591J. H. Reed, Inverse limits of indecomposable continua597Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra601Roward Henry Wicke, The regular open continuous images of complete metric spaces621Abraham Zaks, A note on semi-primary hereditary rings622Uppuluri V. Ramamohana Rao, Correction to: "A description of Multi (A ¹ ,, A ⁿ) by generators and relations"629Takesi Isiwata, Correction: "Mappings and spaces"631Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld,	Dishard Brian Darst. On a theorem of Nikedow with ambiasticus to weak	405
George Wesley Day, Superatomic Boolean algebras475George Wesley Day, Superatomic Boolean algebras479Lawrence Fearnley, Characterization of the continuous images of all491Neil Robert Gray, Unstable points in the hyperspace of connected subsets515Franklin Haimo, Polynomials in central endomorphisms521John Sollion Hsia, Integral equivalence of vectors over local modular lattices527Jim Humphreys, Existence of Levi factors in certain algebraic groups543E. Christopher Lance, Automorphisms of postliminal C*-algebras547Sibe Mardesic, Images of ordered compacta are locally peripherally metric557Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics569Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces585Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces591J. H. Reed, Inverse limits of indecomposable continua597Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra601Roy Westwick, Transformations on tensor spaces621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "A description of Mult ₁ (A ¹ ,, A ⁿ) by generators and relations"629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula"631Hamages of differential forms in n real variables"631James Calvert, Correction to: "An integral inequality	Richard Brian Darst, On a theorem of Nikodym with applications to weak	172
George Westey Day, Superatomic Boolean algebras479Lawrence Fearnley, Characterization of the continuous images of all pseudocircles491Neil Robert Gray, Unstable points in the hyperspace of connected subsets515Franklin Haimo, Polynomials in central endomorphisms521John Sollion Hsia, Integral equivalence of vectors over local modular lattices527Jim Humphreys, Existence of Levi factors in certain algebraic groups543E. Christopher Lance, Automorphisms of postliminal C*-algebras547Sibe Mardesic, Images of ordered compacta are locally peripherally metric557Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics569Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces591J. H. Reed, Inverse limits of indecomposable continua597Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra601Roy Westwick, Transformations on tensor spaces621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations"629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula"631Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables"631James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"631	Convergence and von Neumann algebras	475
Lawrence rearniey, Characterization of the continuous images of all pseudocircles491Neil Robert Gray, Unstable points in the hyperspace of connected subsets515Franklin Haimo, Polynomials in central endomorphisms521John Sollion Hsia, Integral equivalence of vectors over local modular lattices527Jim Humphreys, Existence of Levi factors in certain algebraic groups543E. Christopher Lance, Automorphisms of postliminal C*-algebras547Sibe Mardesic, Images of ordered compacta are locally peripherally metric557Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics569Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces585Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces591J. H. Reed, Inverse limits of indecomposable continua591J. H. Reed, Inverse limits of indecomposable continua591Roy Westwick, Transformations on tensor spaces613Howard Henry Wicke, The regular open continuous images of complete metric spaces621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "On a stronger version of Wallis' formula"629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula"631James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"631K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"	George Wesley Day, Superatomic Boolean algebras	4/9
Neil Robert Gray, Unstable points in the hyperspace of connected subsets 515 Franklin Haimo, Polynomials in central endomorphisms 521 John Sollion Hsia, Integral equivalence of vectors over local modular lattices 527 Jim Humphreys, Existence of Levi factors in certain algebraic groups 543 E. Christopher Lance, Automorphisms of postliminal C*-algebras 547 Sibe Mardesic, Images of ordered compacta are locally peripherally metric 557 Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics 569 Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces 585 Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces 591 J. H. Reed, Inverse limits of indecomposable continua 597 Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra 601 Roy Westwick, Transformations on tensor spaces 621 Abraham Zaks, A note on semi-primary hereditary rings 627 Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations" 629 Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula" 629 Takesi Isiw	<i>pseudocircles</i>	491
Franklin Haimo, Polynomials in central endomorphisms. 521 John Sollion Hsia, Integral equivalence of vectors over local modular lattices 527 Jim Humphreys, Existence of Levi factors in certain algebraic groups 543 E. Christopher Lance, Automorphisms of postliminal C*-algebras 547 Sibe Mardesic, Images of ordered compacta are locally peripherally metric 557 Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics 569 Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces 585 Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces 591 J. H. Reed, Inverse limits of indecomposable continua 597 Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra 601 Roy Westwick, Transformations on tensor spaces 613 Howard Henry Wicke, The regular open continuous images of complete metric spaces 627 Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations" 629 Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula" 629 Takesi Isiwata, Correction: "Mappings and spaces" 631 H	Neil Robert Gray, Unstable points in the hyperspace of connected subsets	515
John Sollion Hsia, Integral equivalence of vectors over local modular lattices527Jim Humphreys, Existence of Levi factors in certain algebraic groups543E. Christopher Lance, Automorphisms of postliminal C*-algebras547Sibe Mardesic, Images of ordered compacta are locally peripherally metric557Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics569Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces585Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces591J. H. Reed, Inverse limits of indecomposable continua597Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra601Roy Westwick, Transformations on tensor spaces613Howard Henry Wicke, The regular open continuous images of complete metric spaces627Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations"629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula"630Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables"631K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"632	Franklin Haimo, Polynomials in central endomorphisms	521
Jim Humphreys, Existence of Levi factors in certain algebraic groups543E. Christopher Lance, Automorphisms of postliminal C*-algebras547Sibe Mardesic, Images of ordered compacta are locally peripherally metric557Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics569Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces585Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces591J. H. Reed, Inverse limits of indecomposable continua597Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra601Roy Westwick, Transformations on tensor spaces613Howard Henry Wicke, The regular open continuous images of complete metric spaces621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "On a stronger version of Wallis' formula"629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula"631James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"631K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"632	John Sollion Hsia, Integral equivalence of vectors over local modular lattices	527
E. Christopher Lance, Automorphisms of postliminal C*-algebras 547 Sibe Mardesic, Images of ordered compacta are locally peripherally metric 557 Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics 569 Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces 585 Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces 591 J. H. Reed, Inverse limits of indecomposable continua 597 Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra 601 Roy Westwick, Transformations on tensor spaces 613 Howard Henry Wicke, The regular open continuous images of complete metric spaces 621 Abraham Zaks, A note on semi-primary hereditary rings 627 Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations" 629 Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula" 630 Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables" 631 James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem" 631	Jim Humphreys, <i>Existence of Levi factors in certain algebraic groups</i>	543
Sibe Mardesic, Images of ordered compacta are locally peripherally metric 557 Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics 569 Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces 585 Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces 591 J. H. Reed, Inverse limits of indecomposable continua 597 Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra 601 Roy Westwick, Transformations on tensor spaces 613 Howard Henry Wicke, The regular open continuous images of complete metric spaces 621 Abraham Zaks, A note on semi-primary hereditary rings 627 Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations" 629 Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula" 630 Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables" 631 James Calvert, Correction to: "Topology of some Kähler manifolds" 631	E. Christopher Lance, Automorphisms of postliminal C*-algebras	547
Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, Order-preserving functions: Applications to majorization and order statistics 569 Wellington Ham Ow, An extremal length criterion for the parabolicity of 585 Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of 585 Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of 591 J. H. Reed, Inverse limits of indecomposable continua 597 Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra 601 Roy Westwick, Transformations on tensor spaces 613 Howard Henry Wicke, The regular open continuous images of complete metric spaces spaces 621 Abraham Zaks, A note on semi-primary hereditary rings 627 Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) 629 Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' 629 Takesi Isiwata, Correction: "Mappings and spaces" 631 Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables" 631 James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem" 631	Sibe Mardesic, Images of ordered compacta are locally peripherally metric	557
Order-preserving functions: Applications to majorization and order statistics569Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces585Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces591J. H. Reed, Inverse limits of indecomposable continua597Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra601Roy Westwick, Transformations on tensor spaces613Howard Henry Wicke, The regular open continuous images of complete metric spaces621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "A description of Mult _i (A^1, \dots, A^n) by generators and relations"629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula"630Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables"631James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"631	Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets,	
statistics569Wellington Ham Ow, An extremal length criterion for the parabolicity of Riemannian spaces585Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces591J. H. Reed, Inverse limits of indecomposable continua597Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra601Roy Westwick, Transformations on tensor spaces613Howard Henry Wicke, The regular open continuous images of complete metric spaces621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations"629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula"620Takesi Isiwata, Correction: "Mappings and spaces"630Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables"631James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"631K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"632	Order-preserving functions: Applications to majorization and order	
Wellington Ham Ow, An extremal length criterion for the parabolicity of 585 Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of 591 Riemannian spaces. 591 J. H. Reed, Inverse limits of indecomposable continua 597 Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra 601 Roy Westwick, Transformations on tensor spaces 613 Howard Henry Wicke, The regular open continuous images of complete metric 592 spaces 621 Abraham Zaks, A note on semi-primary hereditary rings 627 Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) 629 by generators and relations" 629 Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' 630 Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: 631 James Calvert, Correction to: "An integral inequality with applications to the 631 Dirichlet problem". 631	statistics	569
Riemannian spaces585Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces591J. H. Reed, Inverse limits of indecomposable continua597Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra601Roy Westwick, Transformations on tensor spaces613Howard Henry Wicke, The regular open continuous images of complete metric spaces621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations"629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula"620Takesi Isiwata, Correction: "Mappings and spaces"630Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables"631James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"631K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"632	Wellington Ham Ow, An extremal length criterion for the parabolicity of	
Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of Riemannian spaces 591 J. H. Reed, Inverse limits of indecomposable continua 597 Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra 601 Roy Westwick, Transformations on tensor spaces 613 Howard Henry Wicke, The regular open continuous images of complete metric spaces 621 Abraham Zaks, A note on semi-primary hereditary rings 627 Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations" 629 Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula" 620 Thenry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables" 631 James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem" 631 K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds" 632	Riemannian spaces	585
Riemannian spaces591J. H. Reed, Inverse limits of indecomposable continua597Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra601Roy Westwick, Transformations on tensor spaces613Howard Henry Wicke, The regular open continuous images of complete metric621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ)629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis'629Takesi Isiwata, Correction: "Mappings and spaces"630Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables"631James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"631K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"632	Wellington Ham Ow, Criteria for zero capacity of ideal boundary components of	
J. H. Reed, Inverse limits of indecomposable continua	Riemannian spaces	591
Joseph Gail Stampfli, Minimal range theorems for operators with thin spectra601Roy Westwick, Transformations on tensor spaces613Howard Henry Wicke, The regular open continuous images of complete metric621spaces621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ)629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis'629Takesi Isiwata, Correction: "Mappings and spaces"630Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables"631James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"631K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"632	J. H. Reed, Inverse limits of indecomposable continua	597
Roy Westwick, Transformations on tensor spaces613Howard Henry Wicke, The regular open continuous images of complete metric spaces621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations"629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula"629Takesi Isiwata, Correction: "Mappings and spaces"630Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables"631James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"631K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"632	Joseph Gail Stampfli, <i>Minimal range theorems for operators with thin spectra</i>	601
 Howard Henry Wicke, <i>The regular open continuous images of complete metric spaces</i>	Roy Westwick, Transformations on tensor spaces	613
spaces621Abraham Zaks, A note on semi-primary hereditary rings627Thomas William Hungerford, Correction to: "A description of Mult _i (A ¹ ,, A ⁿ) by generators and relations"629Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis" formula"629Takesi Isiwata, Correction: "Mappings and spaces"630Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables"631James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"631K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"632	Howard Henry Wicke, <i>The regular open continuous images of complete metric</i>	
 Abraham Zaks, A note on semi-primary hereditary rings	spaces	621
 Thomas William Hungerford, Correction to: "A description of Mult_i (A¹,, Aⁿ) by generators and relations"	Abraham Zaks, A note on semi-primary hereditary rings	627
Uppuluri V. Ramamohana Rao, Correction to: "On a stronger version of Wallis' formula"	Thomas William Hungerford, <i>Correction to:</i> "A description of $Mult_i(A^1, \dots, A^n)$ by generators and relations"	629
formula" 629 Takesi Isiwata, Correction: "Mappings and spaces" 630 Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: 631 James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem" 631 K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds" 632	Uppuluri V Ramamohana Rao, Correction to: "On a stronger version of Wallis"	022
Takesi Isiwata, Correction: "Mappings and spaces" 630 Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: 631 "Properties of differential forms in n real variables" 631 James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem" 631 K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds" 632	formula"	629
 Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to: "Properties of differential forms in n real variables"	Takesi Isiwata, Correction: "Mappings and spaces"	630
 <i>"Properties of differential forms in n real variables"</i> 631 James Calvert, <i>Correction to: "An integral inequality with applications to the Dirichlet problem"</i> 631 K. Srinivasacharyulu, <i>Correction to: "Topology of some Kähler manifolds"</i> 632 	Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, Correction to:	
James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"	"Properties of differential forms in n real variables"	631
K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"	James Calvert, Correction to: "An integral inequality with applications to the Dirichlet problem"	631
	K. Srinivasacharyulu, Correction to: "Topology of some Kähler manifolds"	632