# Pacific Journal of Mathematics

# EXTENDING BOUNDED HOLOMORPHIC FUNCTIONS FROM CERTAIN SUBVARIETIES OF A POLYDISC

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Vol. 29, No. 3

July 1969

## EXTENDING BOUNDED HOLOMORPHIC FUNCTIONS FROM CERTAIN SUBVARIETIES OF A POLYDISC

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Let E be a subvariety of the unit polydisc

 $U^{N} = \{(z_{1}, \dots, z_{N}) \in C^{N} : |z_{i}| < 1, 1 \leq i \leq N\}$ 

such that E is the zero set of a holomorphic function f on  $U^N$ , i.e., E = Z(f) where  $Z(f) = \{z \in U^N : f(z) = 0\}$ . This amounts to saying that E is a subvariety of pure dimension N-1. In [2] Walter Rudin proved that if E is bounded away from the torus  $T^N = \{(z_1, \dots, z_N) \in C^N : |z_i| = 1, 1 \leq i \leq N\}$ , then there is a bounded holomorphic function F on  $U^N$  such that E = Z(F). Call such a subvariety E, that is, a pure N-1 dimensional subvariety of  $U^N$  bounded from  $T^N$ , a Rudin variety. We are interested in the following question: When is it possible to extend every bounded holomorphic function on a Rudin variety E to one on  $U^N$ ? Examples show this is not always possible. We will say that a pure N-1 dimensional subvariety E of  $U^N$  is a special Rudin variety if there exists an annular domain  $Q^N = \{(z_1, \dots, z_N) \in C^N : r < |z_i| < 1, 1 \leq i \leq N\}$  for some r(0 < r < 1) and a  $\delta > 0$  such that

(i) 
$$E \cap Q^N = \emptyset$$
 and

(ii) if  $1 \leq k \leq N$  and  $(z', \alpha, z'') \in (Q^{k-1} \times U \times Q^{N-k}) \cap E$ and  $(z', \beta, z'') \in (Q^{k-1} \times U \times Q^{N-k}) \cap E$  and  $\alpha \neq \beta$ , then  $|\alpha - \beta| \geq \delta$ . Obviously (i) implies that a special Rudin variety is a Rudin variety. We have the

THEOREM. If E is a special Rudin variety in  $U^N$ , then there exists a bounded linear transformation  $T: H^{\infty}(E) \to H^{\infty}(U^N)$ (where  $H^{\infty}$  is the corresponding Banach space of bounded holomorphic functions under sup norm) which extends each bounded holomorphic function on E to one on  $U^N$ .

REMARK. The proof of the theorem is a modification of the proof in [2] of Rudin's theorem: the changes reflecting the fact that we are dealing with an additive problem while Rudin's was of a multiplicative nature. I am further indebted to Professor Rudin for some comments (on a preliminary version of this paper) which led to improvement in the hypothesis of the theorem.

The following lemma is well-known and easy to prove.

LEMMA 1. If 
$$0 < r < 1$$
 and  $Q = \{\lambda \in C \colon r < |\lambda| < 1\}$  and  
 $h(\lambda) = \sum_{-\infty}^{\infty} a_n \lambda^n, h_1(\lambda) = \sum_{-\infty}^{-1} a_n \lambda^n$ 

for  $\lambda \in Q$ , then

$$||h_1||_q \leq K ||h||_q$$

where  $K \ (> 1)$  is a constant depending only on r.

If h is holomorphic on  $Q^N = \{(z_1, \dots, z_N): r < |z_i| < 1, 1 \leq i \leq N\}$ then h has a Laurent expansion

$$(1) h(z_1, z_2, \cdots, z_N) = \sum a(n_1, n_2, \cdots, n_N) z_1^{n_1} z_2^{n_2} \cdots z_N^{n_N}$$

Following [2], we define  $\pi_j h, 1 \leq j \leq N$ , to be the holomorphic function on  $Q^N$  whose Laurent series is obtained by deleting in (1) all terms in which  $n_j \geq 0$ . Lemma 1 implies

LEMMA 2. 
$$||\pi_j h||_{Q^N} \leq K ||h||_{Q^N}$$
.

Proof of the theorem. Since E is a subvariety of  $U^N$  of pure dimension N-1, there exists by [1, p. 251] a function f holomorphic on  $U^N$  such that at each point of  $U^N$  the germ of f generates the ideal of germs of holomorphic functions which vanish on the germ of E at the given point. In particular, E = Z(f). We will show that  $\partial f/\partial z_k \neq 0$ on  $(Q^{k-1} \times U \times Q^{N-k}) \cap E$  for  $1 \leq k \leq N$ . We give the proof for k = 1, the other cases are identical. Let  $(\alpha, \alpha') \in (U \times Q^{N-1}) \cap E$ . Now f is regular in the first coordinate [1, p. 13] at  $(\alpha, \alpha')$  since otherwise  $f(\zeta, \alpha')$  vanishes in a neighborhood of  $\alpha$  and hence for  $|\zeta| < 1$  and so  $E = Z(f) \supseteq \{(\zeta, \alpha'): |\zeta| < 1\}$ , contradicting (i) in the definition of a special Rudin variety. Thus we can apply the Weierstrass preparation theorem and write in some neighborhood of  $(\alpha, \alpha'), f = \Omega p$  where  $\Omega$  is invertible and p is a Weierstrass polynomial. Factor p into primes:  $p = p_i^{e_1} \cdots p_i^{e_t}$  where p and the  $p_i$ 's are of the form

$$(\zeta - \alpha)^n + a_{n-1}(\zeta')(\zeta - \alpha)^{n-1} + \cdots + a_0(\zeta')$$

for  $(\zeta, \zeta')$  near  $(\alpha, \alpha')$  with  $a_j(\alpha') = 0$ . Now the degree of each  $p_i$ must be equal to 1 since otherwise there would exist  $\zeta'_n \to \alpha'$  with  $\zeta'_n$  off the discriminant locus of some  $p_i$  and so there would exist  $\alpha_n \neq \beta_n$  near  $\alpha$  with  $p_i(\alpha_n, \zeta'_n) = 0 = p_i(\beta_n, \zeta'_n)$  and thus  $(\alpha_n, \zeta'_n)$  and  $(\beta_n, \zeta'_n)$  are in  $(U \times Q^{N-1}) \cap E$ , but  $\zeta'_n \to \alpha'$  implies  $\alpha_n \to \alpha$  and  $\beta_n \to \alpha$ and so  $|\alpha_n - \beta_n| \to 0$ , contradicting (ii). A similar argument also using (ii) shows that there cannot be more than one  $p_i$  and so  $f = \Omega p_i^{\epsilon_1}$ near  $(\alpha, \alpha')$ . Finally, since the germ of f generates the ideal of E at  $(\alpha, \alpha'), e_1$  must be equal to 1. Thus  $f(\zeta, \zeta') = \Omega(\zeta, \zeta')(\zeta - \alpha + \alpha_0(\zeta'))$ and  $\partial f/\partial \zeta(\alpha, \alpha') = \Omega(\alpha, \alpha') \neq 0$  as required.

Now by Theorem 1 of [2] applied to E = Z(f) there is a bounded holomorphic function F on  $U^{N}$  such that E = Z(F). Examination of the construction in [2] shows that 1/F is bounded on  $Q^N$  since  $F = f_1 e^{g-g_1}$ on  $Q^N$  and  $1/f_1$  and  $|\operatorname{Re}(g - g_1)|$  are bounded on  $Q^N$ . We will show that there is an  $\varepsilon > 0$  such that  $|\partial F/\partial z_k| > \varepsilon$  on  $(Q^{k-1} \times U \times Q^{N-k}) \cap E$ for  $1 \leq k \leq N$ . We do this for k = 1, the finitely many other cases are identical. From [2],  $F = f e^g$  for some g and so  $\partial f/\partial z_1 \neq 0$  on  $(U \times Q^{N-1}) \cap E$  implies  $\partial F/\partial z_1 \neq 0$  there. Now for  $z' \in Q^{N-1}$ 

$$z' \longrightarrow rac{1}{2\pi i} \int_{|\zeta|=r} rac{\partial F/\partial z_{\mathrm{i}}(\zeta,z')}{F(\zeta,z')} d\zeta$$

is a continuous integer-valued function and so is a constant  $m_1$  giving the number of zeros for  $F(\cdot, z')$  in U. Since these zeros are the points of  $(U \times Q^{N-1}) \cap E$  and  $\partial F/\partial z_1 \neq 0$  there, it follows that the  $m_1$  zeros  $\alpha_1(z'), \dots, \alpha_{m_1}(z')$  are distinct simple zeros. By (ii) then,  $|\alpha_i(z') - \alpha_j(z')| \geq \delta$ for  $i \neq j$ . Write  $F(\cdot, z') = BH$ , where B is the Blaschke product with zeros at  $\alpha_1(z'), \dots, \alpha_{m_1}(z')$ . Now since 1/F is bounded on  $Q^N$  1/H is bounded on U. But on  $E, \partial F/\partial z_1 = \partial B/\partial z_1 \cdot H$  and since

$$|lpha_i(z') - lpha_j(z')| \geq \delta, \partial B/\partial z_1$$

is bounded from zero on E by some constant depending on  $\delta$ , and as H is also bounded from zero independently of z', it follows that  $\partial F/\partial z_1$  is bounded from zero on  $(U \times Q^{N-1}) \cap E$ .

Let  $d = \text{dist}(E, Q^N)$  which we may assume is positive by increasing r if need be. Let g be a bounded holomorphic function on E. We shall extend g to a bounded function on  $U^N$ . By the general Oka-Cartan theory [1], there is a holomorphic extension G of g to  $U^N$ ; G need not be bounded. Since  $F \neq 0$  on  $Q^N$ , we may define a function  $h_1$  on  $U \times Q^{N-1}$  as follows: Let  $(z_1, z') \in U \times Q^{N-1}$ . Choose a circle  $\Gamma$  about 0 lying in Q and enclosing  $z_1$  with positive orientation and set

$$h_{_1}(z_{_1},z') = rac{1}{2\pi i} \int_{arGamma} rac{G(\zeta,z')/F(\zeta,z')}{\zeta-z_{_1}} \, d\zeta \; .$$

 $h_1$  is clearly independent of the choice of  $\Gamma$  and holomorphic on  $U \times Q^{N-1}$ . We claim that  $G/F - h_1$  is bounded on  $Q^N$ . Let  $(z_1, z') \in Q^N$  where  $z_1 \in Q$ ,  $z' \in Q^{N-1}$ . Let  $\Gamma_1, \Gamma_2, \dots, \Gamma_{m_1}$  be small circles about  $\alpha_1(z'), \dots, \alpha_{m_1}(z')$ , the zeros of  $F(\cdot, z')$ . Then the Cauchy integral formula reads

$$(G/F)(z_1, z') = \frac{1}{2\pi i} \int_{\Gamma - \Gamma_1 - \cdots + \Gamma_{m_1}} \frac{G(\zeta, z')/F(\zeta, z')}{\zeta - z_1} d\zeta$$

Therefore

$$(G/F - h_1)(z_1, z') = -\sum_{1}^{m_1} \frac{1}{2\pi i} \int_{\Gamma_k} \frac{G(\zeta, z')/F(\zeta, z')}{\zeta - z_1} d\zeta$$

Clearly for  $r_k$  = radius of  $\Gamma_k$ ,

$$\begin{split} & \frac{1}{2\pi i} \int_{\Gamma_k} \frac{G(\zeta,z')/F(\zeta,z')}{\zeta-z_1} \, d\zeta \\ &= \frac{1}{2\pi i} \int_{|\zeta-\alpha_k(z')|=r_k} \frac{G(\zeta,z')}{\zeta-z_1} \frac{\zeta-\alpha_k(z')}{F(\zeta,z')-F(\alpha_k(z'),z')} \frac{d\zeta}{\zeta-\alpha_k(z')} \\ & \to \frac{g(\alpha_k(z'),z')}{(\alpha_k(z')-z_1) \frac{\partial F}{\partial \zeta_1}(\alpha_k(z'),z')} \qquad \text{as} \ r_k \to 0 \ . \end{split}$$

So letting the radii of the  $\Gamma_k$  go to zero we get

$$(G/F - h_1)(z_1, z') = -\sum_{k=1}^{m_1} rac{g(lpha_k(z'), z')}{(lpha_k(z') - z_1) rac{\partial F}{\partial \zeta_1}(lpha_k(z'), z')}$$

Since  $(lpha_k(z'),z')\in (U imes Q^{N-1})\cap E$ , recalling the significance of d and arepsilon we get

$$|| \operatorname{\mathit{G}}/\operatorname{\mathit{F}} - h_{\scriptscriptstyle 1} ||_{\operatorname{\mathit{Q}}^N} \leqq rac{m_{\scriptscriptstyle 1} || \operatorname{\mathit{g}} ||_{\scriptscriptstyle E}}{d arepsilon} \ .$$

In the same way for each  $i, 1 < i \leq N$  we have an integer  $m_i$  and a function  $h_i$  holomorphic on  $Q^{i-1} \times U \times Q^{N-i}$  such that

$$|| \ G/F - h_i \, ||_{Q^N} \leq rac{|m_i|| \ g \, ||_{_E}}{darepsilon}$$

Now let  $m = \max \{m_i: 1 \leq i \leq N\}$  and let  $A = m/d\varepsilon$ . Subtracting in the above, we get  $||h_1 - h_i||_{Q^N} \leq 2A ||g||_{\varepsilon}$ . Now following [2] closely, set  $h = (1 - \pi_1)(1 - \pi_2) \cdots (1 - \pi_N)h_1$ . Since  $\pi_i h = 0$ , h extends (uniquely) to a holomorphic function on  $U^N$ . Since  $h_j$  is holomorphic on

$$Q^{{}_{j}-1} imes U imes Q^{{}_{N-j}}, \pi_j h_j = 0$$

and so  $\pi_i h_1 = \pi_i (h_1 - h_i)$  and therefore by Lemma 2,

$$||\, \pi_j h_1 \,||_{Q^N} = ||\, \pi_j (h_1 - h_j \,||_{Q^N} \leqq K \,||\, h_1 - h_j \,||_{Q^N} \leqq 2KA \,||\, g \,||_E$$
 .

Now, since  $h - h_1 = -\sum \pi_i h_1 + \sum \pi_i \pi_j h_1 - + \cdots$  and since we get by induction and by use of Lemma 2 that  $|| \pi_{i_1} \pi_{i_2} \cdots \pi_{i_S} h_1 ||_{Q^N} \leq 2K^S A || g ||_E$ , *it* follows that  $|| h - h_1 ||_{Q^N} \leq BA || g ||_E$  where *B* depends only on *K*. Now consider  $\overline{G} = G - Fh$ .  $\overline{G}$  is holomorphic on  $U^N$  and extends *g* since *G* does. On  $Q^N, \overline{G} = F(G/F - h_1) + F(h_1 - h)$ . Therefore  $|| \overline{G} ||_{Q^N} \leq || F ||_{U^N} A || g ||_E + || F ||_{U^N} BA || g ||_E$ . Thus  $\overline{G}$  is bounded on  $U^N$  and  $|| \overline{G} ||_{U^N} \leq \gamma || g ||_E$  where  $\gamma = A(1 + B) || F ||_{U^N}$  is independent of *g*.

Next we show that  $\overline{G}$  does not depend on the choice of G made at the beginning of the construction. Suppose  $\widetilde{G}$  were another (not necessarily bounded) extension of g to  $U^{\mathbb{N}}$ . As above we get

$$\widetilde{h}_{\scriptscriptstyle 1} = rac{1}{2\pi i}\int_{\scriptscriptstyle F} rac{\widetilde{G}/F}{\zeta-z_{\scriptscriptstyle 1}}\,d\zeta$$
 .

But then on  $U imes Q^{\scriptscriptstyle N-1}$ 

(2) 
$$h_1 - \tilde{h}_1 = \frac{1}{2\pi i} \int \frac{(G - \tilde{G})/F}{\zeta - z_1} d\zeta .$$

Since for  $z' \in Q^{N-1}$ ,  $(G - \tilde{G})(\cdot, z')$  vanishes at  $\alpha_1(z'), \dots, \alpha_{m_1}(z')$  and since  $F(\cdot, z')$  has simple zeros and only at these points,  $(G - \tilde{G})/F(\cdot, z')$  is holomorphic on U and the right hand side of (2) equals  $(G - \tilde{G})/F$  and so on  $U \times Q^{N-1}$ 

(3) 
$$h_1 - \tilde{h}_1 = (G - \tilde{G})/F$$
.

Since the left hand side of (3) is holomorphic on  $U \times Q^{N-1}$ , so is the right and consequently  $(G - \tilde{G})/F = (1 - \pi_1)((G - \tilde{G})/F)$  on  $Q^N$ . In the same way we see that for each j,  $(G - \tilde{G})/F = (1 - \pi_j)((G - \tilde{G})/F)$  on  $Q^N$ . Therefore on  $Q^N$  we have

$$(G-\widetilde{G})/F=\prod\limits_{j=1}^{N}{(1-\pi_j)(G-\widetilde{G})/F}=\prod\limits_{j=1}^{N}{(1-\pi_j)(h_1-\widetilde{h}_1)}=h-\widetilde{h}\;.$$

Thus  $G - Fh = \tilde{G} - F\tilde{h}$  on  $Q^N$  and so on  $U^N$ . Since the extensions thus coincide, we have a well-defined map  $T: H^{\infty}(E) \to H^{\infty}(U^N)$  such that  $|| T(g) ||_{U^N} \leq \gamma || g ||_E$ .

To see that T is linear, let g and  $\tilde{g}$  be bounded holomorphic functions on E and let  $\lambda$  be a complex number. Let G and  $\tilde{G}$  respectively be arbitrary holomorphic extensions to  $U^N$ . Let  $\tilde{\tilde{h}}_1, h_1, \tilde{h}_1$  and  $\tilde{\tilde{h}}, h, \tilde{h}$ be the  $h_1$  and the h for  $G + \lambda \tilde{G}, G$  and  $\tilde{G}$  respectively. Then

$$egin{aligned} \widetilde{\widetilde{h}}_1 &= rac{1}{2\pi i}\int rac{(G+\lambda\widetilde{G})/F}{\zeta-z_1}\,d\zeta \ &= rac{1}{2\pi i}\int \!\!rac{G/F}{\zeta-z_1}\,d\zeta + \lambda\!\cdot\!rac{1}{2\pi i}\int \!rac{\widetilde{G}}{\zeta-z_1}\,d\zeta = h_1 + \lambda\widetilde{h}_1 \end{aligned}$$

and  $\widetilde{\widetilde{h}} = \Pi(1-\pi_j)\widetilde{h}_1 = [\Pi(1-\pi_j)](h_1 + \lambda \widetilde{h}_1) = h + \lambda \widetilde{h}$ . Therefore

$$egin{aligned} T(g + \lambda \widetilde{g}) &= (G + \lambda \widetilde{G}) - F(h + \lambda \widetilde{h}) \ &= (G - Fh) + \lambda (\widetilde{G} - F\widetilde{h}) = T(g) + \lambda T(\widetilde{g}) \;. \end{aligned}$$

EXAMPLE. Let E be the Rudin variety in  $U^2$  given by  $E = Z((z_2 - \frac{1}{2})(z_1z_2 - \frac{1}{2}))$ . Then E is the disjoint union of  $Z(z_2 - \frac{1}{2})$  and  $Z(z_1z_2 - \frac{1}{2})$ . Let  $g \in H^{\infty}(E)$  be given by

$$g\mid Z\Bigl((z_2-rac{1}{2}\Bigr)=0 \quad ext{and} \quad g\mid Z\Bigl(z_1z_2-rac{1}{2}\Bigr)=1 \;.$$

Then g admits no bounded holomorphic extension to  $U^2$ . For if G were a bounded extension of g to  $U^2$  we would have for  $z \in U, z$  near 1,

$$egin{aligned} 1 &= G\Bigl(z,rac{1}{2z}\Bigr) - G\Bigl(z,rac{1}{2}\Bigr) = rac{1}{2\pi i}\int_{|\zeta|=1}G(z,\zeta) igg(rac{1}{\zeta-rac{1}{2z}} - rac{1}{\zeta-rac{1}{2}}\Bigr) d\zeta \ &= \Bigl(rac{1}{2z} - rac{1}{2}\Bigr)rac{1}{2\pi i}\int_{|\zeta|=1}rac{G(z,\zeta)}{\Bigl(\zeta-rac{1}{2z}\Bigr)\Bigl(\zeta-rac{1}{2}\Bigr)}\,d\zeta \ . \end{aligned}$$

But as  $z \to 1$ , the integral is bounded and  $(1/2z) - (1/2) \to 0$ , a contradiction.

## References

1. Robert C. Gunning and Hugo Rossi, Analytic functions of several complex variables, Prentice-Hall, Englewood Cliffs, New Jersey, 1965.

2. Walter Rudin, Zero-sets in polydiscs, Bull. Amer. Math. Soc. 73 (1967), 580-583.

Received January 8, 1968. The research for this paper war partially supported by the following contracts: NONR 222 (85) and NONR 3656 (08).

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Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley, California, 94708.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 7-17, Fujimi 2-chome, Chiyoda-ku, Tokyo, Japan.

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