

# Pacific Journal of Mathematics

**ON A CLASS OF NODAL ALGEBRAS**

MICHAEL RICH

## ON A CLASS OF NODAL ALGEBRAS

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**In this paper it is shown that there do not exist nodal algebras  $A$  satisfying the conditions:**

(I)  $x(xy) + (yx)x = 2(xy)x$

(II)  $(xy)x - x(yx)$  is in  $N$ , the set of nilpotent elements of  $A$ , over any field  $F$  of characteristic zero. Also several results regarding algebras satisfying (I) alone are established.

A finite dimensional power-associative algebra  $A$  with identity 1 over a field  $F$  is called a nodal algebra [7] if every element  $x$  of  $A$  can be represented in the form  $x = \alpha 1 + n$  where  $\alpha$  is in  $F$  and  $n$  is nilpotent and if the set  $N$  of nilpotent elements of  $A$  is not a subalgebra of  $A$ . It is known [5] that there are no nodal flexible algebras over any field  $F$  of characteristic zero. (An algebra is said to be flexible if the identity  $(xy)x = x(yx)$  is satisfied). There do exist, however, nodal algebras over fields  $F$  of characteristic zero in which  $(xy)x - x(yx)$  is in  $N$  for all elements  $x, y$  of the algebra [3]. Algebras satisfying (I) were first studied by Kosier [6]. The concern, however, was for algebras of degree  $>1$ .

Throughout, we shall be using the result of Albert [2, p. 526] who proved that there are no commutative nodal algebras over any field  $F$  of characteristic zero by showing that  $N$  forms a subalgebra. In the noncommutative case we let  $A^+$  be the same vector space as  $A$  with multiplication in  $A^+$  given by  $x \cdot y = 1/2(xy + yx)$ ,  $xy$  the multiplication in  $A$ . Then  $N$  is a subalgebra of  $A^+$ . In particular,  $N$  is a vector space. We use the standard notation,  $[x, y]$ , for the commutator  $xy - yx$  and  $(x, y, z)$  for the associator  $(xy)z - x(yz)$ .

2. It is a well known fact that if an algebra  $A$  is power-associative then  $A^+$  is power-associative. For algebras satisfying (I) the converse is also true.

**THEOREM 1.** *If  $A$  is an algebra satisfying (I) over a field  $F$  of characteristic  $\neq 2$  and if  $A^+$  is power-associative then  $A$  is power-associative.*

*Proof.* The following lemma is due to Withoft [8].

**LEMMA 1.1.**  $xx^n = x^n x$  for all  $x$  in  $A$  and for all  $n$ .

The proof is by induction on  $n$ . Trivially the lemma holds if

$n = 1$ . Assume it holds for  $n = k - 1$ . Then  $xx^{k-1} = x^{k-1}x = x^k$ . By (I), however,  $x(xx^{k-1}) + (x^{k-1}x)x = 2(xx^{k-1})x$  which reduces to  $xx^k = x^kx$  and the lemma holds by mathematical induction.

Now linearize (I) to get:

$$(1) \quad x(z y) + z(x y) + (y x) z + (y z) x = 2(x y) z + 2(z y) x .$$

Assume inductively that  $x^a x^b = x^{a+b}$  for all positive integers  $a, b$  such that  $a + b < n$ . This is certainly true if  $n = 3$ . The induction hypothesis leads to the following.

**LEMMA 1.2.**  $x^{n-k} x^k = x^k x^{n-k}$  for all  $k < n$ .

*Proof of Lemma 1.2.* In (1) let  $x = x^{n-k}$ ,  $y = x^{k-1}$ , and  $z = x$ . We get:

$$\begin{aligned} x^{n-k}(xx^{k-1}) + x(x^{n-k}x^{k-1}) + (x^{k-1}x^{n-k})x + (x^{k-1}x)x^{n-k} \\ = 2(x^{n-k}x^{k-1})x + 2(xx^{k-1})x^{n-k} . \end{aligned}$$

However, by hypothesis  $x^{n-k}x^{k-1} = x^{k-1}x^{n-k} = x^{n-1}$  since the degree of each of these terms is  $n - 1 < n$ . Also, by Lemma 1.1  $xx^{k-1} = x^{k-1}x = x^k$  and  $xx^{n-1} = x^{n-1}x = x^n$ . Therefore, the identity is reduced to  $x^{n-k}x^k + x^n + x^n + x^kx^{n-k} = 2x^n + 2x^kx^{n-k}$  or  $x^{n-k}x^k = x^kx^{n-k}$  as desired.

Now since  $A^+$  is power-associative we have  $x^n = x^{n-k} \cdot x^k$  for any  $k < n$ . Since  $x^{n-k}x^k = x^kx^{n-k}$  we get  $x^n = 2/2x^{n-k}x^k = x^{n-k}x^k$ . Suppose now that  $a + b = n$ . Then  $a = n - k$ ,  $b = k$  for some  $k \leq n$ . Then  $x^{a+b} = x^n = x^{n-k}x^k = x^a x^b$  and the result holds for  $a + b = n$ . It follows by mathematical induction that  $x^a x^b = x^{a+b}$  for all positive integers  $a, b$  and  $A$  is power-associative.

Clearly, Theorem 1 would also hold for a ring  $A$  in which the equation  $2x = a$  is solvable for all  $a$  in  $A$ . It should be noted that (I) alone is not sufficient to guarantee power-associativity of  $A$  since Albert [1, p. 25] has shown that commutativity does not guarantee power-associativity.

3. In this section we shall be considering finite dimensional, power-associative algebras with 1 every element of which is of the form  $\alpha 1 + n$  with  $n$  nilpotent. We call a nilpotent element  $w$  of such an algebra a commutator nilpotent if there are elements  $u, v$  in the algebra such that  $[u, v] = \alpha 1 + w$  for some  $\alpha$  in the base field. We write  $\text{tr.}(T)$  for the trace of an operator  $T$ .

**THEOREM 2.** *Let  $A$  be a finite dimensional algebra satisfying (I) over a field  $F$  of characteristic zero in which every element  $z$  is*

of the form  $z = \alpha 1 + n$  where  $\alpha$  is in  $F$  and  $n$  is nilpotent. Then a necessary and sufficient condition for the set  $N$  of nilpotent elements to form an ideal of  $A$  is that  $\text{tr.}(R(w)) = 0$  for every commutator nilpotent  $w$ . ( $R(w)$  is the operator which takes any  $x$  into  $xw$ .)

*Proof.* Gerstenhaber [4, p. 29] has shown that in a commutative power-associative algebra over a field of characteristic zero, the assumption that an element  $n$  is nilpotent implies that  $R(n)$  is nilpotent. We apply this result to the algebra  $A^+$  so that if  $a$  is a nilpotent element of  $A$  then  $R(a)^+ = 1/2(R(a) + L(a))$  is nilpotent and thus  $\text{tr.}[R(a)] + \text{tr.}[L(a)] = 0$ . Writing (1) in terms of operators we get:

$$(2) \quad R(y)L(x) + R(xy) + L(yx) + L(y)R(x) = 2L(xy) + 2R(y)R(x).$$

If we interchange  $x$  and  $y$  in (2) and subtract the result from (2) we get  $[L(y), R(x)] + [R(y), L(x)] + R([x, y]) + L([y, x]) = 2L([x, y]) + 2[R(y), R(x)]$  which gives rise to:

$$(3) \quad \text{tr.} R([x, y]) + \text{tr.} L([y, x]) = 2\text{tr.} L([x, y]).$$

Assume that  $\text{tr.} R(w) = 0$  for all commutator nilpotents  $w$  of  $A$ . Then  $\text{tr.} L(w) = \text{tr.} R(w) = 0$  also. Let  $x$  and  $y$  be arbitrary elements of  $N$ . Then  $[x, y] = \alpha 1 + n$  for some  $\alpha$  in  $F$  and  $n$  in  $N$  and  $n$  is a commutator nilpotent. Therefore (3) reduces to  $\text{tr.}[R(\alpha 1)] - \text{tr.}[L(\alpha 1)] = 2\text{tr.}[L(\alpha 1)]$  or  $\text{tr.}[R(\alpha 1)] = 3\text{tr.}[L(\alpha 1)]$  a contradiction unless  $\alpha = 0$ . Therefore,  $[x, y]$  is in  $N$  and by [2],  $xy$  and  $yx$  are in  $N$ . Thus  $N$  is an ideal of  $A$ .

Conversely, let  $N$  be an ideal of  $A$ . Therefore  $[x, y]$  is in  $N$  for all  $x, y$  in  $N$  and consequently for all  $x, y$  in  $A$ . Thus if  $w$  is a commutator nilpotent of  $A$  there is an  $x, y$  such that  $w = [x, y]$ . From (3) we have that  $\text{tr.} R(w) - \text{tr.} L(w) = 2\text{tr.} L(w)$ . But  $\text{tr.} R(w) + \text{tr.} L(w) = 0$ . Therefore  $\text{tr.} R(w) = 0$  and the result holds.

**THEOREM 3.** *There are no nodal Lie-admissible algebras satisfying (I) over any field  $F$  of characteristic zero.*

*Proof.* For if  $A$  is such a Lie-admissible algebra then for all  $u, v$  in  $N$  and  $w$  in  $A$  we have  $[[u, v], w] + [[v, w], u] + [[w, u], v] = 0$ . In operator form this becomes:

$$\begin{aligned} L([u, v]) - R([u, v]) + [L(v), R(u)] + [R(u), R(v)] \\ + [L(u), L(v)] + [R(v), L(u)] = 0. \end{aligned}$$

Therefore,  $\text{tr.} L([u, v]) = \text{tr.} R([u, v])$ .

Suppose that  $[u, v] = \alpha 1 + z$  with  $\alpha$  in  $F$  and  $z$  in  $N$ . Then  $\text{tr.} L(\alpha 1) + \text{tr.} L(z) = \text{tr.} R(\alpha 1) + \text{tr.} R(z)$ . Therefore,  $\text{tr.} R(z) = \text{tr.} L(z)$

for all commutator nilpotents  $z$ . From [4] we conclude that  $\text{tr. } R(z) = 0$  and by Theorem 2,  $N$  is an ideal of  $A$ . Therefore  $A$  is not a nodal algebra.

We say that  $N$  has nilindex  $p$  if  $p$  is the smallest positive integer such that  $n^p = 0$  for all  $n$  in  $N$ .

**LEMMA 1.** *There are no nodal algebras satisfying (I) over a field  $F$  of characteristic zero for which the nilindex of  $N$  is two.*

*Proof.* For if  $N$  has nilindex two, then  $xy + yx = 0$  for all  $x, y$  in  $N$ . Applying (I) to  $x$  and  $y$  in  $N$  we have  $x(xy) - (xy)x = 2(xy)x$  or  $x(xy) = 3(xy)x$ . If  $xy = \alpha 1 + z$  with  $\alpha$  in  $F$  and  $z$  in  $N$  the preceding identity becomes  $\alpha x + xz = 3\alpha x + 3zx$ . But  $xz = -zx$ . Therefore it reduces to  $2\alpha x = 4xz$  and since characteristic  $F \neq 2$  to  $\alpha x = 2xz$ . Multiplying on the left by  $x$  we have  $0 = \alpha x^2 = 2x(xz)$  or  $x(xz) = 0$ . But  $x[x(xy)] = x[x(\alpha 1 + z)] = x[\alpha x + xz] = \alpha x^2 + x(xz) = 0$ . Therefore we have  $yL(x)^3 = 0$  for all  $x, y$  in  $N$ .

Let  $\alpha 1 + n$  be a typical element of the algebra  $A$ . Then  $(\alpha 1 + n)L(x)^3 = \alpha x^3 + nL(x)^3$  and  $nL(x)^3 = 0$  as above. Therefore  $L(x)^3 = 0$ ,  $L(x)$  is a nilpotent operator of  $A$  and  $\text{tr. } L(x) = 0$ . As before, this implies that  $\text{tr. } R(x) = 0$ . By Theorem 2,  $N$  is an ideal of  $A$  and  $A$  is not a nodal algebra.

Anderson [3] has shown the existence of simple nodal algebras over a field of characteristic zero for which the associators  $(x, y, z)$  are nilpotent for all  $x, y$ , and  $z$ . The following theorem shows that no such algebras exist which satisfy (I).

**THEOREM 4.** *There are no simple nodal algebras satisfying (I) and (II) over any field  $F$  of characteristic zero.*

*Proof.* We first prove the following lemmas.

**LEMMA 4.1.** *If  $x$  and  $y$  are in  $N$  then  $xy^2$  and  $y^2x$  are also in  $N$ .*

For if we let  $xy = \alpha 1 + n$  with  $\alpha$  in  $F$  and  $n$  in  $N$ , then  $yx = 2x \cdot y - \alpha 1 - n$  and  $(x, y, x) = 2\alpha x + nx + xn - 2x(x \cdot y)$ . But  $xn + nx = 2x \cdot n$  is in  $N$ ,  $2\alpha x$  is in  $N$ , and by hypothesis  $(x, y, x)$  is in  $N$ . Therefore,  $2x(x \cdot y)$  and consequently  $x(x \cdot y)$  is in  $N$ . Linearizing this we have:

$$(4) \quad x(z \cdot y) + z(x \cdot y) \text{ is in } N \text{ if } x, y, z \text{ in } N.$$

Let  $z = y$  in (4). Then  $xy^2 + y(x \cdot y)$  is in  $N$ . But  $y(y \cdot x)$  is in  $N$  from the previous remark and we conclude that  $xy^2$  is in  $N$ . Since  $x \cdot y^2$  is in  $N$   $y^2x$  is also in  $N$ .

It can be further shown by mathematical induction that  $x^jy^k$  is in  $N$  if  $j > 1$  or  $k > 1$ .

LEMMA 4.2. *For any  $x, y$  in  $N$  the following elements are in  $N$ :  $(xy)x, x(xy), (yx)x,$  and  $x(yx)$ .*

For, since  $A$  is power-associative we have

$$(x, x, y) + (y, x, x) + (x, y, x) = 0 .$$

But  $(x, y, x)$  is in  $N$ . So we have that  $(x, x, y) + (y, x, x)$  is in  $N$  for all  $x, y$  in  $A$  or:  $x^2y - x(xy) + (yx)x - yx^2$  is in  $N$  for all  $x, y$  in  $A$ . If  $x$  and  $y$  are in  $N$  then by Lemma 4.1,  $x^2y - yx^2$  is in  $N$ . Thus,

$$(5) \quad (yx)x - x(xy) \text{ is in } N \text{ for all } x, y \text{ in } N .$$

We write  $x(xy) - (yx)x = n$  for some  $n$  in  $N$ . Adding (I) to this we get that  $2x(xy) = 2(xy)x + n$ . But characteristic  $F \neq 2$ . Therefore,  $x(xy) - (xy)x$  is in  $N$ . But  $x \cdot (xy)$  is in  $N$ . Thus,  $x(xy)$  and  $(xy)x$  are in  $N$  if  $x$  and  $y$  are in  $N$ . Applying (I) again  $(yx)x = 2(xy)x - x(xy)$ . By the previous remark the right side is in  $N$ . We conclude, therefore, that  $(yx)x$  and hence  $x(yx)$  is in  $N$  completing the proof of the lemma.

Since  $x(xy)$  is in  $N$ , it follows that:

$$(6) \quad x(zy) + z(xy) \text{ is in } N \text{ if } x, y, z \text{ are in } N .$$

Also  $(yx)x$  in  $N$  implies that:

$$(7) \quad (yx)z + (yz)x \text{ is in } N \text{ if } x, y, z \text{ are in } N .$$

Now, let  $y$  be an element of  $N$ . Then  $y^2$  is in  $N$ . We shall analyze the ideal  $I$  generated by the element  $y^2$ .  $I$  is the set of all sums of terms, each term being a product of elements of  $A$  at least one element of which is the element  $y^2$ . Consider the number of multiplications on  $y^2$  in a typical summand. If we multiply  $y^2$  by a single element in  $N$ , say  $z$ , we have either  $y^2z$  or  $zy^2$  which are in  $N$  by Lemma 4.1.

We prove by mathematical induction that any number of multiplications on  $y^2$  by elements of  $N$  maintains nilpotency. The result has been shown for one multiplication. Assume that  $n$  multiplications on  $y^2$  maintains nilpotency and consider  $n + 1$  multiplications by elements  $q_1, q_2, \dots, q_n, q_{n+1}$  of  $N$ . There are only four cases to consider:

$$\begin{array}{ll} (1) \quad \{(((( \dots (y^2) \dots )))\}q_n\}q_{n+1} & (2) \quad q_{n+1}\{(((( \dots (y^2) \dots )))\}q_n\} \\ (3) \quad q_{+1}\{q_n[((( \dots (y^2) \dots )))]\} & (4) \quad \{q_n[((( \dots (y^2) \dots )))]\}q_{n+1} \end{array}$$

for all other arrangements would involve  $n$  or less multiplications. Let

$b = (((\dots (y^2) \dots)))$ . By hypothesis  $b$  is in  $N$ . We must show then, that

$$(1) \quad (bq_n)q_{n+1} \quad (2) \quad q_{n+1}(bq_n) \quad (3) \quad q_{n+1}(q_nb) \quad (4) \quad (q_nb)q_{n+1}$$

are all in  $N$ .

In (6) let  $x = q_{n+1}$ ,  $z = b$ , and  $y = q_n$ . Then we have that  $q_{n+1}(bq_n) + b(q_{n+1}q_n)$  is in  $N$ . But  $b(q_{n+1}q_n)$  involves only  $n$  multiplications on  $y^2$ . Therefore, by the induction hypothesis it is in  $N$  and we conclude that  $q_{n+1}(bq_n)$  and therefore by [2]  $(bq_n)q_{n+1}$  are in  $N$ . Similarly, in (7) let  $x = b$ ,  $y = q_n$ , and  $z = q_{n+1}$ . Then we have  $(q_nb)q_{n+1} + (q_nq_{n+1})b$  are in  $N$ . As before this implies that  $(q_nb)q_{n+1}$  and consequently  $q_{n+1}(q_nb)$  are in  $N$ . Therefore  $n + 1$  multiplications on  $y^2$  by elements of  $N$  maintains nilpotency and the result holds for any number of multiplications. It follows easily that any number of multiplications on  $y^2$  by elements of  $A$  preserve nilpotency.

Now every element of  $I$  is a sum of terms of the above type and consequently nilpotent. Thus  $I \subseteq N$ . Hence,  $I$  is an ideal of  $A$  which does not encompass all of  $A$  and by the simplicity of  $A$ ,  $I = 0$ . But  $y^2$  is in  $I$ . Therefore  $y^2 = 0$ . This holds for all  $y$  in  $N$  and so the nilindex of  $N$  is two. By Lemma 1,  $A$  is not nodal.

**THEOREM 5.** *There are no nodal algebras satisfying (I) and (II) over any field  $F$  of characteristic zero.*

*Proof.* For let  $A$  be such an algebra. By Theorem 4,  $A$  is not simple. Let  $B$  be a maximal ideal of  $A$ . Then  $A/B$  is a simple nodal algebra satisfying (I) and (II) contradicting Theorem 4.

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