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IRREDUCIBLE CHARACTERS AND SOLVABILITY OF FINITE GROUPS

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IRREDUCIBLE CHARACTERS AND SOLVABILITY OF FINITE GROUPS

F. R. DEMEYER

The relationship between the degree of an irreducible character ζ on a finite group G induced from a nilpotent normal subgroup and the structure of the group G are studied when the degree of ζ is large. In particular if the square of the degree of ζ is the index of the center of G in G then G is solvable.

Let ζ be an irreducible (complex) character on the finite group G. What conditions on ζ insure that G is solvable? Of course, if ζ is a faithful linear character then G is cyclic. We are interested in the other extreme when the degree of ζ is large, in part because of the relationship to the theory of projective representations and the Schur multiplier. Let H be a nilpotent normal subgroup of G, assume $\zeta =$ ϕ^{a} for some character ϕ on H, and assume for each Sylow p-subgroup S of G that $\zeta|_{s} = m\lambda$ for some irreducible character λ on S where (m, p) = 1, then G is solvable. If Z is the center of G the last condition always holds if the degree of ζ is $[G: Z]^{1/2}$, that is, if G is a "group of central type" [2]. It is easy to see that no irreducible character on G can have degree larger than $[G; Z]^{1/2}$. Another upper bound for the degree of an irreducible character on G is d[G: H] where $d = \max \{ \text{degree } \rho | \rho \text{ is an irreducible character on } H \}$ ([3] 17.9 p. 570). If [G, G] is the commutator subgroup of G and $Z \cap [G, G]$ contains an element of order d[G: H] then there is an irreducible character ζ of degree d[G: H] on G. Moreover, $\zeta = \phi^{G}$ for some character ϕ on *H*, and for each Sylow *p*-subgroup *S* of *G*, $\zeta|_{S} = \sum_{i=1}^{n} \lambda_{i}$ where the λ_i are irreducible characters on S with $\lambda_i(1)$ equal to the *p*-part of $\zeta(1)$ $(j = 1, \dots, n)$. If n = 1 for each prime p dividing [G:1] then G is solvable. An example showing the necessity of the hypothesis on n is given. The conditions on the character ζ with respect to the Sylow subgroups S of G restrict the action of G on S. To illustrate this we show G is nilpotent if and only if for every Sylow subgroup S of G and every irreducible character χ on G, $\chi|_{s} =$ $m\lambda$ for some irreducible character λ on S.

In what follows all groups are finite and all characters and representations are taken over the complex numbers. If n is an integer and p is a prime integer we let n_p denote the largest factor of n which is a power of the prime p. Our standard reference is [3] and all unexplained terminology and notation coincides with [3].

THEOREM 1. Let ζ be an irreducible character on the group G and let H be a nilpotent normal subgroup of G. Assume

1. $\zeta = \phi^{a}$ for some character ϕ on H.

2. For each Sylow p-subgroup S of G, $\zeta|_s = m\lambda$ for some irreducible character λ on S where (p, m) = 1. Then G is solvable.

Proof. A theorem of P. Hall ([3] 1.10 p. 662) asserts that a group is solvable if every Sylow subgroup has a complement, this theorem will be applied to G/H. Let p be a prime dividing [G: 1], let P be the Sylow p-subgroup of H and S a Sylow p-subgroup of G. Since P is a characteristic subgroup of H, P is a normal subgroup of G and $P \subseteq S$. By Clifford's Theorem ([3] 17.3 p. 565)

$$\zeta|_P = e(\rho_1 + \cdots + \rho_n)$$

where the ρ_i are inequivalent irreducible characters on P conjugate in G. We determine the number n. By hypothesis 2, $\zeta|_P = m\lambda|_P$ so $\lambda|_P = e/m(\rho_1 + \cdots + \rho_n)$ and the ρ_i are conjugate in S by Clifford's Theorem. Now $(\phi, \zeta|_H) \geq 1$ so by relabeling we can say $(\rho_1, \phi|_P) \geq 1$. We claim $\rho_1^S = \lambda$ so e/m = 1 and n = [S: P]. To verify the claim hypothesis 2 says the p-part of $\zeta(1)$ is $\lambda(1)$. Also, $\phi|_P = q\rho_1$ (since His nilpotent) where $\rho_1(1)$ is a power of p so $\rho_1^S(1)$ divides the p-part of $\phi^a(1) = \zeta(1)$. Since λ is contained in ρ_1^S this implies $\lambda = \rho_1^S$ verifying the claim.

Now G acts on $\rho_1 \cdots \rho_n$ by conjugation and the inertia group H^* of the action of G on ρ_1 has index $n = \lambda(1)/\rho_1(1) = [S: P]$. Also H^* contains H since H is nilpotent so H^*/H is a p-complement in G/H. The Theorem of P. Hall completes the proof.

We next give a sufficient condition that ζ satisfy condition 2 of Theorem 1. (See [2] Theorem 2).

THEOREM 2. Let ζ be an irreducible character on G and let Z be the center of G. If $\zeta(1)^2 = [G: Z]$ then for each Sylow p-subgroup S of G, $\zeta|_S = m\lambda$ for some irreducible character λ on S and (p, m) = 1.

Proof. By Schur's lemma $\zeta|_Z = \zeta(1)\psi$ where ψ is a linear character on Z. Then by reciprocity $(\zeta, \psi^G) = (\zeta|_Z, \psi) = \zeta(1)$ so by counting degrees, $\zeta(1)\zeta = \psi^G$. Let S be a Sylow p-subgroup of G and let R be the subgroup of G generated by Z and S. Let λ be an irreducible character of R contained in ψ^R . By Schur's lemma $\lambda|_S$ remains irreducible because the elements of Z are represented by Sclars. Since λ is contained in ψ^R , $\lambda^G = m\zeta$ for some integer m. By counting degrees

$$m = [G; R] \lambda(1) / \zeta(1)$$
.

Since λ is irreducible on S, $\lambda(1) = p^a$ for some a, [G: R] is prime to p since R contains S. The p-part of $\zeta(1)^2$ is $[S: S \cap Z]$. Thus $\lambda(1)^2 = [S: S \cap Z]$ and $(\zeta, \lambda^c) = (\zeta|_R, \lambda) = (\zeta|_S, \lambda|_S) = [G: Z]/\lambda(1)^2$. Thus $\zeta|_S = m\lambda$ where m is the largest divisor of $\zeta(1)$ prime to p. We combine the first two results to obtain.

COROLLARY 1. Let ζ be an irreducible character on the group G, and let H be a nilpotent normal subgroup of G. Assume $\zeta = \phi^G$ for some character ϕ on H and $\zeta(1)^2 = [G; Z]$ where Z is the center of G. Then G is solvable.

The principal theorem of [1] is now an easy consequence of Corollary 1.

COROLLARY 2. Let ζ be an irreducible character on the finite group G, and let A be an abelian normal subgroup of G. If $\zeta(1)^2 = [G: A]^2 = [G. Z]$ where Z is the center of G then G is solvable.

Proof. Let ϕ be a linear constitutent of $\zeta|_A$. Then by reciprocity, ζ is a constitutent of ϕ^G . But $\zeta(1) = \phi^G(1) = [G: A]$ so $\phi^G = \zeta$. By Corollary 1, G is solvable.

We now verify some of the hypothesis of Theorem 1 in another situation. We begin by summarizing basic results relating ordinary representations, projective representions, and the Schur Multiplier. Our nontrivial assertions are the contents of 23.3, p. 629 of [3]. Let G be a finite group with center Z, assume n is the exponent of $[G, G] \cap Z$ and let $\overline{G} = G/Z$. Write

$$G = \bigcup_{g \in \overline{G}} ZR(g)$$

where R(g) is an element in G corresponding to g. Then $R(g_1)R(g_2) = A(g_1, g_2)R(g_1g_2)$ where $A(g_1, g_2) \in Z$. Let $a \in [G, G] \cap Z$ order n and let θ be a linear character on Z which is faithful on the cyclic group generated by a. Define a 2-cycle α on \overline{G} by

$$lpha(g_{\scriptscriptstyle 1},\,g_{\scriptscriptstyle 2})= heta(A(g_{\scriptscriptstyle 1},\,g_{\scriptscriptstyle 2}))$$
 .

Let K^* be the multiplicative group of the complex numbers. The element α represents in the Schur multipler $H^2(\overline{G}, K^*)$ has order *n*.

Form the projective group algebra $K\overline{G}_{\alpha}$ and let M be a left $K\overline{G}_{\alpha}$ module. For each $g \in \overline{G}$, left multiplication by g on M induces a Klinear transformation T(g) of M and

$$T(g_1) T(g_2) = \alpha(g_1, g_2) T(g_1g_2)$$
 .

If $x \in G$ then $x = z_i R(g_i)$ where $z_i \in Z$ and $g_i \in \overline{G}$. Let left multiplica-

tion by x on M be the linear transformation $T^*(x) = \theta(z_1) T(g_1)$. If $y = z_2 R(g_2) \in G$ then

$$xy = z_1 z_2 A(g_1, g_2) R(g_1 g_2)$$

and

$$T^*(x) \, T^*(y) \, = \, heta(z_1) \, T(g_1) heta(z_2) \, T(g_2) \, = \, heta(z_1 z_2) heta(A(g_1, \ g_2)) \, T(g_1 g_2) \, = \, T^*(xy)$$
 .

Thus M can be viewed as a KG-module. Notice that M is irreducible over KG if and only if M is irreducible over $K\overline{G}_{\alpha}$. Also, note that $T^*|_{Z} = T^*(1)$. This process can be reversed when M is a KG-module giving the G representation T^* if $T^*|_{Z} = T^*(1)\theta$ for the given linear character θ on Z. Define a linear character ψ on G by the equation $\psi(x) = \det(T^*(x))$. Since $a \in [G, G], \psi(a) = 1$. But $\psi(a) = \theta(a)^m$ where $m = T^*(1)$ so n divides $T^*(1)$.

Let S be a Sylow p-subgroup of G and \overline{S} the natural image of S in \overline{G} . The element the restriction of α to \overline{S} represents in $H^2(\overline{S}, K^*)$ is realized by the equation $\alpha(y_1, y_2) = \theta(A(y_1, y_2))$ in the group SZ. By ([3] 16.21, p. 118) α represents an element whose order is n_p in $H^2(\overline{S}, K^*)$. In the correspondence of ([3] 23.3, p. 629) this implies θ is faithful on a cyclic group of order n_p in $[S, S] \cap Z$. Form the projective group algebra $K\overline{S}_{\alpha}$. Now M can be viewed as a $K\overline{S}_{\alpha}$ -module, let $M = M_1 \bigoplus \cdots \bigoplus M_k$ where the M_i are irreducible $K\overline{S}_{\alpha}$ modules. As above, each M_i affords an ordinary representation T_i^* on SZ which is irreducible. The restriction of T_i^* to S is also irreducible since each T_i^* restricted to Z is $T_i^*(1)\theta$. Also, θ is faithful on a cyclic group of order n_p in $[S, S] \cap Z$ so arguing as before n_p must divide the degree of T_i^* .

LEMMA 1. Let G be a finite group with center Z, let $a \in [G, G] \cap Z$ of order n, and let θ be a linear character on Z faithful on the cyclic group generated by a. Then

(1) $\theta^{g} = \sum_{i=1}^{s} \zeta_{i}(1)\zeta_{i}$ where $n | \zeta_{i}(1)$ and the ζ_{i} are inequivalent irreducible characters of G.

(2) If ζ is an irreducible character on G with $(\theta^c, \zeta) \geq 1$ and S is a Sylow p-subgroup of G then $\zeta|_S = \sum_{j=1}^l b_j \lambda_j$ where $n_p|\lambda_j(1)$, the λ_j are inequivalent irreducible characters on S, and the b_j are positive integers.

Proof. Let ζ be an irreducible character on G. By Schur's lemma $\zeta|_Z = \zeta(1)\psi$ for a linear character ψ on Z. Now $(\zeta, \psi^G) = (\zeta|_Z, \psi) = \zeta(1)$. This shows $\theta^G = \sum_{i=1}^s \zeta_i(1)\zeta_i$ where the ζ_i are inequivalent irreducible characters of G. If T_i is the representation affording ζ_i then det T_i is a linear character on G. Since $a \in [G, G], 1 = \zeta_i$

det $(T_i(a)) = \det \left[\theta(a) T_i(1)\right] = \theta(a)^{\zeta_i(1)}$. Therefore $n | \zeta_i(1)$.

To prove (2) we need the analysis which preceded the lemma. Let T^* be the ordinary representation on G which affords ζ and T the corresponding projective representation on \overline{G} . In this situation we showed $T^*|_s = T_1^* + \cdots + T_k^*$ where the T_i^* are irreducible and their degree are divisible by n_p . Let $\lambda_1, \dots, \lambda_l$ be a full set of inequivalent characters afforded by the T_1^*, \dots, T_k^* . Then $\zeta|_s = \sum_{i=1}^l b_j \lambda_j$ where b_j is the multiplicity of λ_j in $\zeta|_s$ and $\lambda_j(1)$ is the degree of some T_i^* and so is divisible by n_p . We can now prove

THEOREM 3. Let G be group with center Z. Let H be a normal nilpotent subgroup of G and let $d = \max \{\rho(1) | \rho \text{ is an irreducible character of } H\}$. If $[G, G] \cap Z$ contains an element of order d[G: H] then there is an irreducible character ζ on G so that $\zeta = \phi^G$ for some character ϕ on H, and for each Sylow p-subgroup S of G, $\zeta|_S = \sum_{i=1}^n b_i \lambda_i$ where $\lambda_i(1) = \zeta(1)_p$. If n=1 for each p then G is solvable.

Proof. Let n = d[G: H] and let $a \in [G, G] \cap Z$ of order n. Let θ be a linear character on Z which is faithful on the cyclic group generated by a. By the first part of LEMMA 1

$$heta^{\scriptscriptstyle G} = \sum\limits_{i=1}^s \zeta_i(1) \zeta_i$$

where $n|\zeta_i(1)$ and the ζ_i are inequivalent irreducible characters of G. We will show each of the ζ_i satisfy the conclusion of the Theorem. By 17.9 p. 570 of [3], n is the largest possible degree of an irreducible character on G so $n = \zeta_i(1)(i = 1, \dots, s)$ and H is a maximal nilpotent normal subgroup of G so $Z \subseteq H$. Now $\theta^G(1) = [G: Z]$ so $[G: Z] = sn^2$ where s is the number of inequivalent ζ_i in θ^G . By Clifford's Theorem (17.3 p. 565, [3])

$$\zeta_i|_H = e(\phi_1^i + \cdots + \phi_m^i)$$

where the $\phi_i^i(j = 1, \dots, m)$ are inequivalent irreducible characters on H conjugate in G. Now ζ_i is a constitutent of $(\phi_j^i)^G$ and $(\phi_j^i)^G(1) \leq d[G: H] = \zeta_i(1)$ so for each $j, \phi_j^i(1) = d$ and $(\phi_j^i)^G = \zeta_i$. This verifies the first conclusion of Theorem 3 for each $i(i = 1, 2, \dots, s)$.

Let S be a Sylow p-subgroup of G. By the second part of LEMMA 1,

$$\zeta_i|_{\scriptscriptstyle S} = \sum\limits_{j=1}^l b_j \lambda_j^i$$

where the λ_j^i are inequivalent irreducible characters on S and n_p divides $\lambda_j^i(1)$. Since H is nilpotent, $d_p = \max \{\gamma(1) | \gamma \text{ is an irreducible character on } P\}$. If λ is an irreducible constitutent of $\lambda_j^i|_P$ then

 $\gamma^{s}(1) \leq \lambda_{j}^{i}(1)$ so $\gamma^{s} = \lambda_{j}^{i}$ and $\lambda_{j}^{i}(1) = d_{p}[S: P] = n_{p}$. This verifies the second conclusion of Theorem 3. If n = 1 for each p then $\zeta|_{s} = b_{1}\lambda_{1}$ and $\zeta(1) = b_{1}\lambda_{1}(1)$. But $\zeta(1)_{p} = \lambda_{1}(1)_{p}$ so $(p, b_{1}) = 1$ and by Theorem 1, G is solvable. This completes the proof.

For an example to show the necessity of Condition 2 in Theorem 1 let H be any group of order n and $J_n(H)$ the group algebra of H over the ring J_n of integers modulo n. Let $A = J_n(H)$ viewed as an additive group and let H act as a group of automorphisms of A by

$$h(ax) = ahx$$
 (regular representation) $x, h \in H, a \in J_n$

Let G be the semi-direct product of A by H with respect to this action. Let ϕ be the linear character defined on A by $\phi(\sum_{h \in H} a_h h) = \hat{\xi}^a$ where $\hat{\xi}$ is a primitive n^{th} root of 1 and a is an integer representing the coefficient in J_n of the identity e of H. One checks that $[G, A] \cap Z = Z$ where Z, the center of G, is $\{\sum a_h h | a_h = a_k \text{ all } h, k \in H\}$ and has exponent n. Also ϕ is distinct from all its conjugates so $\phi^G = \zeta$ is irreducible. Yet G need not be solvable. The problem is that the restriction of ζ to a Sylow subgroup does not behave properly. For example, if $H = A_5$ (the simple group of order 60), and S is the Sylow 5-subgroup of G then $\zeta|_S = \sum_{i=1}^{12} \lambda_i$ where the λ_i are 12 distinct irreducible characters on S of degree 5.

If G is a finite group with center Z and ζ is a faithful irreducible character on G with $\zeta|_s = m\lambda$ for some Sylow subgroup S and irreducible character λ on S then the center of S is $Z \cap S$. The proof of this observation also proves

THEOREM 4. The group G is nilpotent if and only if for each irreducible character ζ on G and each Sylow subgroup S of G, $\zeta|_s = m\lambda$ for some irreducible character λ on S.

Proof. Assume G is nilpotent, let ζ be an irreducible character on G and S a Sylow subgroup. Then S is normal in G so by Clifford's Theorem

$$\zeta|_{s} = e(\phi_{1} + \cdots + \phi_{m})$$

with the ϕ_i distinct conjugate irreducible characters on S. If $g \in G$ then $g = g_1g_2$ where g_1 centralizes S and $g_2 \in S$. Then, $\phi_1^g = \phi_1^{g_1g_2} = \phi^{g_2} = \phi$. So m = 1.

Conversely, let S be a Sylow subgroup of G and let a be an element of the center of S. Let ζ be an irreducible character on G, then $\zeta|_S = m\lambda$ where λ is an irreducible character on S. Let Z(S) be the center of S. Then by Schur's lemma, $\lambda|_{Z(S)} = \lambda(1)\theta$ for some linear character on Z(S). Thus $\zeta(a) = \zeta(1)\theta(a)$ so a is an element of

the center of $G/\ker \zeta$. Since this is true for all irreducible characters on G, a is an element of the center of G. If $\langle a \rangle$ is the central subgroup of G generated by a then the irreducible characters of $G/\langle a \rangle$ correspond to the irreducible characters of G with kernel $\langle a \rangle$. Thus $G/\langle a \rangle$ satisfies the same hypothesis G does so by induction Gis nilpotent.

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