CORRECTIONS TO: "ISOMORPHIC GROUPS AND GROUP RINGS"

Donald Steven Passman

ERRATA

Corrections to

ISOMORPHIC GROUPS AND GROUP RINGS

D. S. Passman

Volume 15 (1965), 561–583

Since the above named paper is apparently still of interest we discuss and correct two errors which occur in it.

We use the notation of the original paper [2].

1. The first error was pointed out to me many years ago by D. B. Coleman. Namely the characterization of the Frattini subgroup $\Phi(\mathcal{N})$ given in the first paragraph of page 569 is not right. What one gets is the intersection of all maximal subgroups $\mathcal{M}$ of $\mathcal{N}$ which are normal in $\mathcal{G}$ and this is just not $\Phi(\mathcal{N})$. For example if $\mathcal{G}$ is a nonabelian group of order p^3 and period p (for $p > 2$) and if $[\mathcal{G}:\mathcal{N}] = p$ then $\Phi(\mathcal{N}) = \langle 1 \rangle$ but this intersection is clearly the center of $\mathcal{G}$ which is not $\langle 1 \rangle$.

We correct this problem by essentially ignoring it. We just delete $\Phi(\mathcal{N})$ from part 4 of Theorem D. Note that if $\mathcal{G}$ is nilpotent then $\Phi(\mathcal{N})$ is of course given by $\Phi(\mathcal{N}) = \mathcal{N}'C^n(\mathcal{N})$ where n is the product of the distinct prime factors of $|\mathcal{N}|$. Thus $\Phi(\mathcal{N})$ is determined in this case and it can remain in the statement of Theorem E.

2. A more serious error was pointed out in a recent paper of T. Obayashi [1]. Namely Lemma 3 is just not right. The mistake occurs in the last line of the proof where it is assumed that $(\mathcal{N})(\mathcal{L}) = (\mathcal{L})(\mathcal{N})$. This fact is not true. For example it fails when $\mathcal{N} = \mathcal{G}$ is the dihedral group of order 8 and when $\mathcal{L}$ is the cyclic subgroup of index 2. However it is clear that no such trouble arises when $\mathcal{N}$ is abelian. Thus we replace Lemma 3 by

LEMMA 3*. *Let $\mathcal{L}, \mathcal{M}, \mathcal{N}$ be three normal subgroups of $\mathcal{G}$ with $\mathcal{L} \subsetneqq \mathcal{M} \subsetneqq \mathcal{N}$ and $\mathcal{N}$ abelian. Then*

$$(\mathcal{L}) \cap (\mathcal{M})(\mathcal{N}) \subseteqq (\mathcal{L})(\mathcal{N}) \,.$$

Proof. We first assume that $\mathcal{G} = \mathcal{N}$ and proceed as in the original proof. Then we use the fact that $R[\mathcal{G}]$ is free over $R[\mathcal{N}]$ to obtain the general result.

One can give an alternate proof in case $R = Z$ is the ring of rational integers using results of Whitcomb. If $\mathcal{L}$ is a subgroup

of $\mathscr{N}$ let $I(\mathscr{L})$ denote the augmentation ideal in the integral group ring $Z[\mathscr{L}]$. Suppose $\mathscr{N}$ is abelian. By [3], the map $g \to (1 - g) + I(\mathscr{N})^2$ is an isomorphism of $\mathscr{N}$ onto the additive group $I(\mathscr{N})/I(\mathscr{N})^2$. Thus clearly

$$\mathscr{L} \simeq \frac{I(\mathscr{L}) + I(\mathscr{N})^2}{I(\mathscr{N})^2} = \frac{I(\mathscr{L})Z[\mathscr{N}] + I(\mathscr{N})^2}{I(\mathscr{N})^2}$$

$$\simeq \frac{I(\mathscr{L})Z[\mathscr{N}]}{I(\mathscr{L})Z[\mathscr{N}] \cap I(\mathscr{N})^2} .$$

On the other hand we know by [3] that

$$\mathscr{L} \simeq \frac{I(\mathscr{L})Z[\mathscr{N}]}{I(\mathscr{L})I(\mathscr{N})} .$$

Since $\mathscr{L}$ is finite and since clearly $I(\mathscr{L})Z[\mathscr{N}] \cap I(\mathscr{N})^2 \supseteq I(\mathscr{L})I(\mathscr{N})$ this latter inclusion is therefore an equality.

If $x \in \mathscr{G}$ we let $x^{\mathscr{G}}$ denote the normal closure of the cyclic group $\langle x \rangle$ in $\mathscr{G}$. The crucial Proposition 4 is not only correct as stated but it is true in a slightly more general context.

PROPOSITION 4*. *Let K_x and K_y be two class sums in $R[\mathscr{G}]$. Suppose that $(x^{\mathscr{G}}, \mathscr{G}, y^{\mathscr{G}}) = (y^{\mathscr{G}}, \mathscr{G}, x^{\mathscr{G}}) = 1$. Then we can find $(x, y) \in \mathscr{X}$ in $R[\mathscr{G}]$.*

Proof. We proceed as in the original proof. Note that the normal subgroup $\mathscr{N}$ is defined by $\mathscr{N} = (x^{\mathscr{G}}, \mathscr{G}) \cap (y^{\mathscr{G}}, \mathscr{G})$. Since $y^{\mathscr{G}}$ centralizes $(x^{\mathscr{G}}, \mathscr{G})$ we see that $(y^{\mathscr{G}}, \mathscr{G}) \subseteq y^{\mathscr{G}}$ also centralizes $(x^{\mathscr{G}}, \mathscr{G})$ and thus $\mathscr{N}$ is abelian. With this observation we can apply Lemma 3* instead of Lemma 3 where needed and the result follows.

Thus, the proposition is generalized by dropping the assumption $(x, \mathscr{G}) \cap (y, \mathscr{G}) \subseteq \mathscr{X}_\infty$ of the original. We remark that as indicated on page 572 the notation $(x, \mathscr{G})$ used in [2] is just a shorthand for $(x^{\mathscr{G}}, \mathscr{G})$. Finally we point out a misprint in equation (10) of page 573. The first part should read $m_x \gamma_x \widehat{\mathscr{N}} = K_x \widehat{\mathscr{N}}$.

REFERENCES

1. T. Obayashi, *Integral group rings of finite groups*, Osaka J. Math., **7** (1970), 253-266.
2. D. S. Passman, *Isomorphic groups and group rings*, Pacific J. Math., **15** (1965), 561-583.
3. A. R. Whitcomb, *The group ring problem*, Ph. D. thesis, Univ. of Chicago, 1968.

UNIVERSITY OF WISCONSIN

PACIFIC JOURNAL OF MATHEMATICS

EDITORS

SUPPORTING INSTITUTIONS

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

Pacific Journal of Mathematics
Vol. 43, No. 3   May, 1972