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ABSOLUTE EXTENSOR SPACES: A CORRECTION AND AN ANSWER

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This paper has a two-fold purpose: The first is to make a minor correction in the proof of a result of ours, which states that any hyperconnected space is an AE (stratifiable) and the second is to give an affirmative answer to a question of Vaughan: Does Dugundji's Extension Theorem remain valid for linearly stratifiable spaces?

1. A correction. As it stands, the proof of Theorem 4.1 of [1] is incorrect, because the function g is not well-defined. (Obviously, for each $x \in X - A$, there is some implicit order in the selection of $p_{V_1}, \cdots, p_{V_n}$ such that $V_1, \cdots, V_n$ are the only elements $V \in \mathscr{V}$ for which $p_V(x) \neq 0$. However, no explicit mention of it is made.) The proof is easily corrected however, by taking the following three steps:

1. Assign a total order " $\leqq$ " to $\mathscr{V}$.
2. Add to the function g the sentence "and $V_1 \leqq V_2 \leqq \cdots \leqq V_n$."
3. On page 615 of [1], replace

(a) "say $V_1, \cdots, V_m, \cdots, V_{m+k}$" by "say $W_1, \cdots, W_{m+k}$ such that $W_1 \leqq \cdots \leqq W_{m+k}$".

(b) "$(p_{V_1}(x), \cdots, p_{V_m}(x), 0, \cdots, 0) \in P_{m+k-1}$" by

$$\text{"}(p_{W_1}(x), \cdots, p_{W_{m+k}}(x)) \in P_{m+k-1}\text{"} ,$$

(c) "$t \to (h_{m+k}(f(a_{V_1}), \cdots, f(a_{V_{m+k}}), t)$" by

$$\text{"}t \to h_{m+k}(f(a_{W_1}), \cdots, f(a_{W_{m+k}}), t)\text{"} ,$$

(d) "$p(y) = (p_{V_1}(y), \cdots, p_{V_{m+k}}(y))$" by

$$\text{"}p(y) = (p_{W_1}(y), \cdots, p_{W_{m+k}}(y))\text{"} .$$

2. An answer. Recently, Vaughan [7] asked if Dugundji's Extension Theorem (Theorem 4.1 of [6]) remains valid for linearly stratifiable spaces.[1] It turns out that the answer is affirmative and it requires little effort. Indeed, all our generalizations of Dugundji's Extension Theorem remain valid for linearly stratifiable spaces.

THEOREM 2.1. [2; *Theorem* 4.1], [3; *Theorem* 3.1], [4; *Theorem*

[1] A T_1-space X is said to be linearly stratifiable provided there exists some infinite cardinal number α such that to each open $U \subset X$ one can assign a family $\{U_\beta\}_{\beta<\alpha}$ of open subsets of X such that (a) $\overline{U_\beta} \subset U$ for all $\beta < \alpha$, (b) $U\{\beta \mid \beta < \alpha\} = U$, (c) $U_\beta \subset U_\beta$ whenever $U \subset V$, (d) $U_\gamma \subset U_\beta$ whenever $\gamma < \beta < \alpha$.

5.2] *and* [5; *Theorems* 4.1 *and* 4.2] *remain valid for linearly stratifiable spaces.*

Proof. All we need do is the following two alterations in Definition 4.1 of [2] and the proof of Theorem 4.1 of [2]. (The same alterations apply to the proofs of the other results):

1. In Definition 4.1 of [7] replace the word "integer" by the word "ordinal".

2. Replace the sentence "Note that $m(x) < \infty$ and, in fact, $m(x) < n(W, x)$" by the sentence "Note that $m(x) < n(W, x)$" on the fourth line of the proof of Theorem 4.3 of [2]. The same applies to the other proofs.

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