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CALCULATIONS OF THE SCHUR GROUP

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Let the field K be an abelian extension of the rational field Q. The Schur group of K, S(K), consists of those classes in the Brauer group of K which contain an algebra isomorphic to a simple component of a rational group algebra QG for some finite group G.

Suppose that K has a cyclic extension of the form $Q(\zeta)$ where ζ is a primitive *n*th root of unity. In this paper we calculate the 2-part of S(K) where K contains the fourth roots of unity.

An interesting facet of these results is that in some cases certain local indices of classes in S(K) are tied together. That is, a class in S(K) must have a nontrivial local index at an even number of the primes in a certain set. The tying together of local indices in these fields is caused by quadratic reciprocity and is not found in the *q*-part of S(K) where *q* is an odd prime number.

Let [A] be the class in the Brauer group of K which contains the K-central simple algebra A. The Hasse invariant of [A] at a prime \mathfrak{G} of K is denoted $\operatorname{inv}_{\mathfrak{G}}[A]$. Benard and Schacher [2] showed that each class [A] in S(K) has uniformly distributed invariants. That is, if the index of [A] is I, and $\sigma(\varepsilon_I) = \varepsilon_I^2$ where ε_I is a primitive Ith root of unity and $\sigma \in \operatorname{Gal}(K/Q)$, then $\operatorname{inv}_{\mathfrak{G}}[A] = \lambda \operatorname{inv}_{\sigma(\mathfrak{G})}[A]$ for each prime \mathfrak{G} in K. A corollary of this result is that the local index of a class [A] in S(K) is the same at each of the primes of K which divide a single rational prime p. This common index is called the p-local index of [A].

Set $L = Q(\xi)$ where ξ is a primitive $2^s n$ th root of unity, (2, n) = 1. Let K be a field contained in L such that $\operatorname{Gal}(L/K) = \langle \phi \rangle$ is a cyclic group of order $2^t t'$, (2, t') = 1. Let ζ be a primitive 2^s th root of unity and suppose that $\phi(\zeta) = \zeta^h$ where $h = 5^{2^{r-2}}$. Thus the 2^r th roots of unity lie in K. A theorem of Benard and Schacher [2] implies that the exponent of the 2-part of S(K) is at most 2^r .

Observe that there can be at most one rational prime p with even ramification index in L/K. This follows from the fact that the inertia group of a divisor of p is contained in Gal $(L/K(\varepsilon))$ where ε is a root of unity in L having largest possible order not divisible by p. If p is such a prime, then let:

- 2^k exactly divide p-1,
- 2° exactly divide e(p, L/K),
- 2^{d} exactly divide f(p, K/Q),

where e(p, L/K) is the ramification index of p in L/K and f(p, K/Q) is the residue class degree of p in K/Q.

Now suppose that q is a prime which does not divide 2n. We shall use the following notation:

 $2^{l(q)}$ exactly divides q-1,

- $2^{b(q)}$ exactly divides f(q, K/Q),
- $2^{a(q)}$ exactly divides A(q) where $\phi^{A(q)} = [L/K, q]$ is the Frobenius automorphism of q in L/K,
- $2^{v(q)}$ exactly divides V(q) where $h^{A(q)} q^{f(q, K/Q)} = V(q)2^s$.

In addition, for any prime p we denote $p^{f(p,K/Q)} - 1$ by $\Gamma(p)$.

Finally let $\lambda = \max\{s - t, 0\}$.

THEOREM. The 2-part of S(K) consists of those classes [A] in the Brauer group of K which have uniformly distributed invariants which satisfy the following conditions.

(I) If q does not divide n, then the q-invariants of [A] are integral multiples of $1/2^{I(q)}$ where

$$I(q) = egin{cases} \max{\{r-b(q),\, arsigma(q)-v(q),\, 0\}} & if \ arsigma(q) \leq r-\lambda \ \max{\{r-b(q),\, r-\lambda-v(q),\, 0\}} & if \ arsigma(q) \geq r-\lambda \ . \end{cases}$$

(II) If p divides n, then the p-invariants of [A] are integral multiples of $1/2^{I(p)}$ where

$$I(p) = egin{cases} 0 & if \ p=2 & or \ if \ e(p,\ L/K) \ is \ odd \ \max\left\{c-d+r-k,\ c-d+s-t-\lambda,\ 0
ight\} \ otherwise. \end{cases}$$

(III) Suppose that p divides n and $I(p) \neq 0$. If k > s, $k \neq t$, and $2^{k+s-t-\lambda}$ is greater than the power of 2 which divides p' - 1 for all primes $p' \neq p$ which divide n, then the q-invariants of [A] are odd multiples of $1/2^{I(q)}$ for an even number of primes q in the set.

$$\{p\} \cup \{q: (q/p) = -1, (q, 2n) = 1, \text{ and } \varkappa(q) \ge r - \lambda\}$$

where (q/p) is the Legendre symbol.

Proof. Let $K' \supset K$ be the field such that $[L:K'] = 2^t$. Then Lemma 2 of [5] implies that the set of permissible invariants for elements in the 2-part of S(K) is exactly the set of permissible invariants for elements in the 2-part of S(K'). Thus we may assume that $[L:K] = 2^t$ without any loss of generality.

Now we must determine the invariants of the crossed product algebras of the form

$$[L(arepsilon_q)/K,\,lpha] = \sum L(arepsilon_q) u_{\sigma}\,,\,\,\,\,\sigma\in \mathrm{Gal}\,(L(arepsilon_q)/K)$$

where ε_q is a primitive qth root of unity, q is an odd prime which

does not divide *n*, and α is a factor set from Gal $(L(\varepsilon_q)/K) \times$ Gal $(L(\varepsilon_q)/K)$ into $\langle \zeta \rangle$. The multiplication in these algebras is given by

$$egin{aligned} & u_{\sigma}u_{ au} &= lpha(\sigma,\, au)u_{\sigma au} \ , \ & u_{\sigma}w &= \sigma(w)u_{\sigma} \ , \end{aligned}$$

for $\sigma, \tau \in \text{Gal}(L(\varepsilon_q)/K)$ and $w \in L(\varepsilon_q)$. We know from Theorem 1 of [5] that the classes in the Brauer group of K which contain these classes generate the 2-part of S(K).

Let $\Delta_q = \Delta_q(x, y, z)$ be the algebra $(L(\varepsilon_q)/K, \alpha)$ where the values of α are in $\langle \zeta \rangle$ and q is an odd prime not dividing n. Set $\operatorname{Gal}(L(\varepsilon_q)/L) = \langle \gamma \rangle$. The factor set α is determined by the integers x, y, and z where

$$egin{aligned} & u_{ au} u_{\phi} = \zeta^{x} u_{\phi} u_{ au} \ , \ & (u_{ au})^{q-1} = \zeta^{y} \ , \ & (u_{\phi})^{2^{t}} = \zeta^{z} \ . \end{aligned}$$

We must have

$$egin{aligned} & u_{\phi}(\zeta^z) = \phi(\zeta^z) u_{\phi} = \zeta^z u_{\phi} \;, \ & u_{7}(\zeta^y) = \gamma(\zeta^y) u_{7} = \zeta^y u_{7} \;, \ & (u_{7} u_{\phi} u_{7}^{-1})^{z^t} = (\zeta^x u_{\phi})^{z^t} \;, \ & (u_{\phi} u_{7} u_{\phi}^{-1})^{q-1} = (\zeta^{-x} u_{7})^{q-1} \;. \end{aligned}$$

Thus

(a) 2^{s-r} divides z,
(b) 2ⁱ divides x,

(1)

(c)
$$y(h-1) + x(q-1) = Y2^s$$
 for some integer Y.

The Frobenius automorphism of q in L/K is $\phi^{A(q)}$. Thus

$$\phi^{\scriptscriptstyle A(q)}(\zeta) = \zeta^{\scriptscriptstyle h^{\scriptscriptstyle A(q)}} = \zeta^{\scriptscriptstyle q^{f(q,K|Q)}}$$
 .

Hence

$$h^{A(q)} - q^{f(q, K/Q)} = V2^s$$
 for some integer V.

Now applying Theorem 3 of [6] we get that the q-local index of $[\Delta_q]$ is given by

$$\frac{q-1}{(\nu(q)(q-1),\,q-1)}$$

where

(2) (a)
$$\nu(q) = \frac{1}{2^{*}} \left[x \frac{h^{A(q)} - 1}{h - 1} + y \frac{\Gamma(q)}{q - 1} \right]$$

(b) $= \frac{1}{h - 1} \left[Y \frac{\Gamma(q)}{q - 1} - xV \right].$

Thus, since 2^r exactly divides h-1, the q-local index of $[\varDelta_q]$ is $\max\{2^{r-\mu}, 1\}$ where 2^{μ} exactly divides $Y\Gamma(q)/(q-1) - xV$.

We know that $2^{i(q)}$ exactly divides $\Gamma(q)/(q-1)$. Moreover, we may make Y either odd or even without changing the power of 2 which divides x. If $\ell(q) \leq r - \lambda$, then equation (1)(c) implies that $2^{r-l(q)}$ is the smallest power of 2 which can divide x. If $\ell \geq r - \lambda$, then 2^{λ} is the least power of 2 which can divide x. Thus, the maximum q-local index of $[\Delta^q]$ is $2^{I(q)}$ where

$$I(q) = egin{cases} \max{\{r-b(q),\, arepsilon(q)-v(q),\, 0\}} & ext{if} \quad arepsilon(q) \leqq r-\lambda \ \max{\{r-b(q),\, r-\lambda-v(q),\, 0\}} & ext{if} \quad arepsilon(q) \geqq r-\lambda \ . \end{cases}$$

Now observe that for any prime q' which does not divide 2nq, q is unramified in $L(\varepsilon_{q'})/K$. Thus the q-invariants of $[\varDelta_{q'}]$ must be zero. This means that the only classes amongst the generators of the 2-part of S(K) which have non-zero invariants at the primes of K dividing q are those classes of the form $[\varDelta_q(x, y, z)]$. Thus we have proved (I).

If there is no prime which ramifies in L/K, then $[\varDelta_q]$ can have nonzero invariants only at the primes of K which divide q. If 2 ramifies in L/K, it must be the only prime which ramifies in L/K. So, since the 2-invariants of any class in S(K) must be zero by the results of Yamada [7], the only nonzero invariants that $[\varDelta_q]$ can have are at the primes of K which divide q. In both of these cases we are done and the theorem is proved.

So for the remainder of the proof let p be an odd prime which is ramified in L/K. Set $\phi^{g'}\gamma^{g}$ equal to a Frobenius automorphism for p in $L(\varepsilon_q)/K$. Observe that $\phi^{z^{t-c}}$ generates the inertia group of p in L/K where $2^c = e(p, L/K)$.

Applying Theorem 3 of [6] we get that the *p*-local index of $[\Delta_q]$ is given by

$$\frac{2^{\circ}}{(2^{\circ}\nu(p),\,2^{\circ})}$$

where

$$u(p)=rac{1}{2^s}igg[-xg\,rac{h^{2^{t-c}}-1}{h-1}+zrac{\Gamma(p)}{2^c}igg]\,.$$

Thus the *p*-local index of $[\varDelta_q]$ is max $\{2^{s-\eta}, 1\}$ where 2^{η} exactly divides

$$-xgrac{h^{2^{t-c}}-1}{h-1}+zrac{arGamma(p)}{2^c}$$
 .

We know that 2^{t-c} exactly divides $(h^{2^{t-c}}-1)/(h-1)$, that 2^{k+d-c} exactly divides $\Gamma(p)/2^c$, and that 2^{s-r} is the least power of 2 which divides z. Hence we need to find the smallest power of 2 which

divides xg.

We know that g must be an f(p, K/Q)th power, so picking q such that (q/p) = -1 we get that min $\{2^d, 2^{\ell(q)}\}$ is the smallest power of 2 which can divide g. If $\ell(q) \geq r - \lambda$, then 2^i must divide x, and if $\ell(q) \leq r - \lambda$, then $2^{r-\ell(q)}$ must divide x. Hence we find that min $\{2^{\lambda+d}, 2^r\}$ is the smallest power of 2 which can exactly divide xg. Thus the maximum p-local index of a class in the 2-part of S(K) is $2^{I(p)}$ where

$$egin{aligned} I(p) &= \max \left\{ c - d + s - t - \lambda, \, c - t + s - r, \, c - d + r - k, \, 0
ight\} \ &= \max \left\{ c - d + s - t - \lambda, \, c - d + r - k, \, 0
ight\} \end{aligned}$$

since $c \leq t - (s - r)$. This proves (II).

If I(p) = 0, then we are finished. So assume for the rest of the proof that I(p) > 0.

Now assume that k > s, $k \neq t$, and $2^{k+s-t-\lambda}$ is greater than the power of 2 which divides p'-1 for all primes p' which are unequal to p and which divide n.

Suppose that the p-local index of $[\varDelta_q(x, y, z)]$ is $2^{I(p)}$. Now $s - t - \lambda > r - k$ so $I(p) = c - d + s - t - \lambda$. Thus $2^{\lambda+d}$ exactly divides xg, indeed 2^{λ} must exactly divide x and 2^d must exactly divide g. Thus $\mathcal{E}(q) \geq r - \lambda$ and (q/p) = -1. Further, since $p \equiv 1 \mod 4$, (p/q) = -1 by the law of quadratic reciprocity. This, together with the hypotheses, implies that b(q) = k - t + a(q) where $2^{b(q)}$ exactly divides f(q, K/Q) and a(q) exactly divides A(q). Hence $\mathcal{E}(q) + b(q) > r + a(q)$. So, since $2^{\mathcal{E}(q)+b(q)}$ exactly divides $q^{f(q)} - 1$ and $2^{r+a(q)}$ exactly divides $h^{A(q)} - 1$, we get that r + a(q) = s + v(q). Thus

$$r-b(q) = r-k+t-a(q) < r-\lambda-a(q) \leq r-\lambda-v(q)$$

Hence $I(q) = r - \lambda - v(q)$ and the q-local index of $[\varDelta_q(x, y, z)]$ is $2^{I(q)}$. Observe that I(q) > 0 since the hypotheses insure that $a(q) < s - \lambda$, so that $v(q) < r - \lambda$.

Now let q be a prime such that (q/p) = -1, (q, 2n) = 1, and $\mathscr{E}(q) \geq r - \lambda$. Suppose that the q-local index of $[\varDelta_q(x, y, z)]$ is $2^{I(q)}$. We have seen that I(q) is positive and is equal to $r - \lambda - v(q)$ in this instance. Hence 2^{λ} must exactly divide x by equation (2)(b). Thus the p-local index of $[\varDelta_q(x, y, z)]$ is greater than or equal to $2^{\circ - d + s - t - \lambda}$. However $I(p) = c - d + s - t - \lambda > 0$, so the p-local index of $[\varDelta_q(x, y, z)]$ must be $2^{I(p)}$.

We have now shown that under the hypotheses of (III), the *p*-local index of $[\mathcal{A}_q]$ is $2^{I(p)}$ if and only if (q/p) = -1, $c(q) \ge r - \lambda$, and the *q*-local index of $[\mathcal{A}_q]$ is $2^{I(q)}$. This proves (III).

We now need to show that the restrictions on the invariants of elements in the 2-part of S(K) given in the theorem are the only

restrictions on the invariants of elements in the 2-part of S(K).

First assume that the hypotheses of (III) hold. Let $F = Q(\varepsilon_p, \varepsilon_{2^{s+1}}, \sqrt[4]{p})$ and let σ be the element in Gal(F/Q) such that $\sigma(\varepsilon_p) = \varepsilon_p^{-1}, \sigma(\sqrt[4]{p}) = -\sqrt[4]{p}$, and $\sigma(\varepsilon_{2^{s+1}}) = (\varepsilon_{2^{s+1}})^{\beta}$ where $\beta = 5^{2^{s-2}}$. Such a σ exists since p does not have a fourth root in $Q(\varepsilon_p, \varepsilon_{2^{s+1}})$. Let q be a prime not dividing n whose Frobenius automorphism in F/K is σ . There are infinitely many such primes by the Tchebotarev density theorem. This means that 2^s exactly divides q - 1, 2 exactly divides $f(q, Q(\varepsilon_p)/Q)$, and 2^{s-1} exactly divides $f(p, Q(\varepsilon_q)/Q)$. Thus the Frobenius automorphism of q in L/K is an odd power of $\phi^{2^{t-1}}$ if f(q, K/Q) is odd, and it is 1 if f(q, K/Q) is even. So we have that a(q) = t - 1 if f(q, K/Q) is odd, and a(q) = 0 if f(q, K/Q) is even. Further $\prime(q) \ge r - \lambda$.

Now the algebra class $[\varDelta_q(2^{\lambda}, 0, 0)]$ has q-local index 1 if f(q, K/Q) is even and $[\varDelta_q(2^{\lambda}, 2^{s-1}, 0)]$ has q-local index 1 if f(q, K/Q) is odd. This follows from equation (2)(a). Now both of these algebra classes have p-local index $2^{I(p)-1}$ since $2^{\lambda+d+1}$ divides xg in both cases. Thus the algebra class which has local index $2^{I(p)-1}$ at p and local index 1 at all other primes is in S(K). This implies that there are no further restrictions on the 2-part of S(K) in the case where the hypotheses of (III) hold.

Now assume that either $k \leq s$ or k = t > s. Let ψ_p be a generator of Gal $(L/Q(\zeta, \varepsilon_n))$ where $(n^*, p) = 1$ and n/n^* is a power of p. Also set ψ equal to the automorphism in Gal $(L/Q(\varepsilon_n))$ which sends ζ to ζ^5 . Now let q' be a prime whose Frobenius automorphism in L/Q is $\psi_p \psi^{2^{r+t-k-2}}$. This implies that (q'/p) = (p/q') = -1, that 2^{r+t-k} exactly divides q'-1, and that 2^{a+k-t} exactly divides f(q', K/Q). Consider the algebra class $[\Delta_{q'}(x_0, y_0, 0]$ where

$$x_{\scriptscriptstyle 0}\equiv 2^{\lambda-b(q')}\Big[rac{\varGamma(q')}{q'-1}\Big] \, {
m mod} \, 2^{s}$$

and

$$y_{\scriptscriptstyle 0}\equiv -\ 2^{{\scriptscriptstyle \lambda}-b(q')}\Big[rac{h^{{\scriptscriptstyle A}(q')}-1}{h-1}\Big] \, {
m mod} \ 2^s \ .$$

Observe that $x_0(q'-1) + y_0(h-1) \equiv 0 \mod 2^s$ so that equation (1)(c) is satisfied. Now we have that

$$x_{\scriptscriptstyle 0} \Big[rac{h^{{\scriptscriptstyle A}(q')}-1}{h-1} \Big] + \, y_{\scriptscriptstyle 0} \Big[rac{arGamma(q')}{q'-1} \Big] \equiv 0 \ \mathrm{mod} 2^s \, .$$

Hence the q'-local index of $[\Delta_{q'}(x_0, y_0, 0)]$ is 1. Further, 2^{λ} exactly divides $x_0, 2^{r-\lambda}$ divides q'-1, and (q'/p) = -1. Thus the p-local index of $[\Delta_{q'}(x_0, y_0, 0)]$ is max $\{2^{e-d+s-t-\lambda}, 1\}$.

Now consider the algebra class $[\Delta_{q'}(0, 0, 2^{s-r})]$. Its q'-local index

is 1 and its *p*-local index is max $\{2^{c-d+r-k}, 1\}$.

Thus S(K) contains the algebra class with local index $2^{I(p)}$ at p and local index 1 at all other primes. This implies that there are no extra restrictions on the 2-part of S(K) when either $k \leq s$ or k = t.

Finally assume that k > s, $k \neq t$, and that there is a prime $p' \neq p$ which divides n such that $2^{k+s-t-\lambda}$ divides p'-1. Let $\psi_{p'}$ be a generator of Gal $(L/Q(\zeta, \varepsilon_{n'}))$ where (n', p') = 1 and n/n' is a power of p'. Let ψ_{p} be as above.

Let q'' be a prime whose Frobenius automorphism in L/Q is $\psi_p \psi_{p'}$. Thus (q''/p) = -1 and 2^s divides q'' - 1. Further observe that if β is the smallest integer such that $(\psi_p \psi_{p'})^{\beta} \in \text{Gal}(L/K)$, then $2^{k+s-t-\lambda}$ must divide β . Hence $a(q'') \geq s - \lambda$. Thus $[\mathcal{A}_{q''}(2^{\lambda}, 0, 0)]$ has q''-local index 1 and p-local index $2^{s-d+s-t-\lambda}$. Since $k > s \geq r$, we have that $I(p) = c - d + s - t - \lambda$. So S(K) contains an algebra with local index $2^{I(p)}$ at p and local index 1 at all other primes. This implies that there are no further restrictions on the 2-part of S(K) in this case.

This completes the proof to the theorem.

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