# Pacific Journal of Mathematics

LEVEL SETS OF DERIVATIVES

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Vol. 83, No. 1

March 1979

## LEVEL SETS OF DERIVATIVES

R.P. BOAS, JR., AND G.T. CARGO

We consider real-valued functions defined on intervals on the real line R, and we denote the extended real line by  $\overline{R}$ .

The theme of this paper is the idea that, when a function has a derivative that is equal to some  $A \in \overline{R}$  on a dense set, the derivative can take other (finite) values only on a rather thin set. Our most general result shows that, in particular, the hypothesis "the derivative is equal to A on a dense set" can be replaced by "at each point of a dense set, at least one Dini derivate equals A." As corollaries we obtain unified and rather simple proofs of some more special known results, which we now state.

A function can be discontinuous at each point of a dense set and yet be continuous at each point of a co-meager (residual) subset of its domain. However, the following theorem of Fort [4] shows that such a function cannot be differentiable at each point of a nonmeager set.

THEOREM F. If  $f: I \rightarrow R$  where I is an open interval and if f is discontinuous at each point of a dense subset of I, then the set of points where f has a (finite) derivative is meager in I.

(For a different proof, see [1], p. 131; two rediscoveries are in [3] and [10].)

Recently, Cargo [2] used harmonic analysis to prove

THEOREM C. If f is a real-valued function of finite variation defined on a compact interval I, and if, for some  $A \in R$ , f'(x) = Aon a dense subset of I, then the set of those points at which f has a (finite) derivative different from A is meager in I.

In 1903 W. H. Young [11] proved

THEOREM Y. If  $f: I \to R$  where I is an open interval, then the set of all points at which at least one of the Dini derivates of f is infinite is a  $G_s$  subset of I.

In this paper we use real-variable methods to establish a result (Theorem 2) that includes Theorems F and C (without the hypothesis of finite variation) as corollaries. We also give a short, elementary proof of Theorem Y, observe that Theorem F is an easy consequence of Theorem Y, and then prove a theorem (Theorem 3) that has Theorems 2, Y, F, and C as corollaries.

### 2. The main theorems.

THEOREM 1. Let  $f: I \to R$  where I is an interval, and let  $A \in \overline{R}$ . If f'(x) = A on a dense subset of I, then the set of those points at which f has a (finite) derivative different from A is meager in I.

Note that Theorem C is an immediate consequence of Theorem 1. Since each interval is a Baire space with respect to the inherited metric, we have

COROLLARY 1. If  $f: I \to R$  has a (finite) derivative at each point of the interval I, if  $A \in R$ , and if f'(x) = A on a dense subset of I, then the set of points at which f'(x) = A is nonmeager and co-meager in I; and, hence, each subinterval of I contains uncountably many points at which f'(x) = A.

Theorem 1 is a special case of, but easier to prove than, the following result.

THEOREM 2. Let  $f: I \to R$  where I is an interval, and let  $A \in \overline{R}$ . If at each point of a dense subset of I at least one of the Dini derivates of f has the value A, then the set of those points at which f has a (finite) derivative different from A is meager in I.

Clearly, Theorem C is a corollary of Theorem 2.

To prove that Theorem F is a corollary of Theorem 2, suppose that a function f is discontinuous at each point of a dense subset of an open interval I. Let F denote the set of points in I at which fhas a (finite) derivative. We want to prove that F is meager in I. Let  $D_{+\infty}(D_{-\infty})$  denote the set of points in I at which at least one Dini derivate of f is equal to  $+\infty(-\infty)$ . Then  $D_{+\infty} \cup D_{-\infty}$  is dense in I, since f is clearly continuous at any point at which all Dini derivates are finite. Hence, each open subinterval of I contains an open interval in which either  $D_{+\infty}$  or  $D_{-\infty}$  is dense. Call an open subinterval of I distinguished if either  $D_{+\infty}$  or  $D_{-\infty}$  is dense in the subinterval, and let G denote the union of all distinguished intervals. Our previous observation shows that  $I \setminus G$  is nowhere dense in I. Clearly, G is separable since R is separable. According to Lindelöf's covering theorem,  $G = \bigcup_n G_n$  where  $\{G_1, G_2, \dots\}$  is a countable set of (not necessarily disjoint) distinguished intervals. According to Theorem 2, each  $F \cap G_n$  is meager in  $G_n$  and, hence, in *I*. Finally,  $F = \{F \cap (I \setminus G)\} \cup \bigcup_n (F \cap G_n)$  is meager in *I*, as desired.

Proofs of Theorems 1 and 2. In each theorem, it is enough to consider the set S where f'(x) < A, since the set where f'(x) > A is the set where (-f)'(x) < -A. If A is finite, S is contained in  $\bigcup_{n=1}^{\infty} \bigcup_{m=1}^{\infty} E_{n,m}$  where  $E_{n,m}$  consists of all points x in I such that  $y \in I$  and 0 < |y - x| < 1/n imply that (f(y) - f(x))/(y - x) < A - 1/m; if  $A = +\infty$ , replace A - 1/m by m. To show that S is meager in I, we have only to show that each  $E_{n,m}$  is nowhere dense.

Suppose that some  $E_{N,M}$  is dense in some open interval J. In Theorem 1, there is a dense set of points x at which f'(x) = A; let  $x_0$  be such a point in J. Since  $E_{N,M}$  is also dense in J, for each positive k, there exists  $x_k \in E_{N,M} \setminus \{x_0\}$  such that  $x_k \to x_0$  as  $k \to \infty$ . Thus, if k is so large that  $|x_0 - x_k| < 1/N$ , then  $(f(x_k) - f(x_0))/(x_k - x_0) < A - 1/M$  (or < M if  $A = +\infty$ ). Letting  $k \to \infty$ , we get  $f'(x_0) \leq A - 1/M$  (or  $\leq M$ ), contradicting  $f'(x_0) = A$ . Therefore, each  $E_{n,m}$  is nowhere dense.

In Theorem 2, at each point of a dense set least one of the Dini derivates has the value A; let  $x_0$  be a point of the dense set that is also in J. Then there exists, for each positive integer k, a point  $z_k \in J \setminus \{x_0\}$  such that, as  $k \to \infty$ ,  $z_k \to x_0$  and  $(f(z_k) - f(x_0))/(z_k - x_0) \to A$ . As for Theorem 1, for each positive integer k, there exists a point  $x_k \in E_{N,M} \setminus \{x_0\}$  between  $x_0$  and  $z_k$ . For all sufficiently large k, we have  $0 < |x_0 - x_k| < 1/N$  and  $0 < |z_k - x_k| < 1/N$ . Hence, since  $x_k \in E_{N,M}$ , for all sufficiently large k, we have  $(f(x_0) - f(x_k))/(x_0 - x_k) < A - 1/M$ (or M) and  $(f(z_k) - f(x_k))/(z_k - x_k) < A - 1/M$  (or M). Clearly,

$$rac{f(m{z}_k)-f(m{x}_0)}{m{z}_k-m{x}_0} = rac{f(m{z}_k)-f(m{x}_k)}{m{z}_k-m{x}_k}rac{m{z}_k-m{x}_k}{m{z}_k-m{x}_0} + rac{f(m{x}_k)-f(m{x}_0)}{m{x}_k-m{x}_0}rac{m{x}_k-m{x}_0}{m{x}_k-m{x}_0}\,;$$

and the right-hand side of the last equation is a convex combination of the two difference quotients, each of which is less than A - 1/M(or M) for all sufficiently large k. Letting  $k \to \infty$ , we obtain  $A \leq A - 1/M$  (or M), which is a contradiction; and, again, each  $E_{n,m}$ is nowhere dense in I.

The original proof of Theorem Y is quite complicated (see [11] or [9], pp. 402-404). We now give a simple, elementary proof.

Proof of Theorem Y. For each positive integer n, let  $F_n$  denote the set of all  $x \in I$  such that  $|(f(y) - f(x))/(y - x)| \leq n$  whenever  $y \in I$  and 0 < |y - x| < 1/n. Also, let F denote the set of all points at which each Dini derivate of f is finite. Then it is geometrically

clear (and not difficult to prove analytically) that  $F = \bigcup_{n=1}^{\infty} F_n$ . Once we prove that each  $F_n$  is closed in I we shall be done. Suppose that n is a positive integer and that x is a limit point of  $F_n$  in I. We want to prove that  $x \in F_n$ . Let y be a point of I such that 0 < |y - x| < 1/n. We want to prove that

(1) 
$$\left|\frac{f(y)-f(x)}{y-x}\right| \leq n$$
.

Since x is a limit point of  $F_n$ , there exists a sequence  $z_1, z_2, z_3, \cdots$  of points of  $F_n \setminus \{x, y\}$  such that  $z_k \to x$  as  $k \to \infty$ . Next, note that, for each positive integer k,

$$(2) \quad \frac{f(y) - f(z_k)}{y - z_k} = \frac{f(y) - f(x)}{y - x} \frac{y - x}{y - z_k} + \frac{f(x) - f(z_k)}{x - z_k} \frac{x - z_k}{y - z_k}$$

Since  $z_k \to x$  as  $k \to \infty$  and  $z_k \in F_n$  for each k, it follows that

$$\left|\frac{f(x)-f(z_k)}{x-z_k}\right| \leq n$$

for all sufficiently large k. From  $\lim_{k \to \infty} (x - z_k)/(y - z_k) = 0$ , we conclude that

$$\lim_{k o\infty}rac{f(x)-f(z_k)}{x-z_k}rac{x-z_k}{y-z_k}=0\;.$$

Finally, since  $\lim_{k\to\infty}(y-x)/(y-z_k)=1$ , we see from (2) that

(3) 
$$\lim_{k\to\infty} \frac{f(y) - f(z_k)}{y - z_k} = \frac{f(y) - f(x)}{y - x}.$$

Since  $z_k \in F_n$  for each k and  $\lim_{k \to \infty} |y - z_k| = |y - x| < 1/n$ , it follows that

(4) 
$$\left| \frac{f(y) - f(z_k)}{y - z_k} \right| \leq n$$
 for all sufficiently large  $k$ .

From (3) we obtain

(5) 
$$\lim_{k\to\infty} \left| \frac{f(y) - f(z_k)}{y - z_k} \right| = \left| \frac{f(y) - f(x)}{y - x} \right|$$

We conclude from (4) and (5) that (1) holds, as desired.

Thus,  $F = \bigcup_{n=1}^{\infty} F_n$  is an  $F_{\sigma}$  subset of *I*, and  $I \setminus F$  is a  $G_{\delta}$  subset of *I*, that is, the set of all points at which at least one of the Dini derivates of *f* is infinite is a  $G_{\delta}$  subset of *I*. This completes the proof of Theorem Y.

Next, we shall prove that Theorem F is a simple consequence

of Theorem Y. As we noted above, the set of discontinuities of f is a subset of the set of all points at which at least one of the Dini derivates of f is infinite. Since the former set is dense in I, so is the latter. By Theorem Y, the latter set is a  $G_{\delta}$  subset of I. Since a dense  $G_{\delta}$  subset is co-meager (see [8], p. 135), it follows that the set of points at which all four Dini derivates are finite is meager in I. Finally, the set of points at which f has a (finite) derivative is meager in I because it is a subset of the latter set.

3. An extension. Next, we shall prove a theorem that has Theorem 2 as a direct corollary. If the domain of a real-valued function f contains an open interval containing a real number x, we define the set D(f;x) of derivates of f at x to consist of all  $A \in \overline{R}$ for which there exists a sequence  $x_1, x_2, x_3, \cdots$  of real numbers distinct from x and converging to x such that  $\lim_{n\to\infty}(f(x_n) - f(x))/(x_n - x) = A$  (see [7], pp. 115-116). The set  $D^+(f;x)$  of right derivates of f at x and the set  $D_-(f;x)$  of left derivates of f at xare defined in the obvious way. Clearly,  $D(f;x) = D^+(f;x) \cup$  $D_-(f;x)$ . One can prove that D(f;x) is a closed subset of  $\overline{R}$  and, if f is continuous in a neighborhood of x, that D(f;x) is an interval. The usual Dini derivates are extreme unilateral derivates (see [7], p. 116). For example, the upper right (Dini) derivate of f at x is just the largest element of  $D^+(f;x)$ , that is

$$f^+(x) = \limsup_{u \to x^+} \frac{f(u) - f(x)}{u - x} = \max D^+(f; x)$$
.

Of course, f has a derivative at x in the extended sense if and only if D(f; x) consists of just one point of  $\overline{R}$ .

THEOREM 3. Let  $f: I \to R$  where I is an open interval, and let  $A \in \overline{R}$ . Then the set of x such that D(f; x) contains at least one element of  $\{A, +\infty, -\infty\}$  is a  $G_{\tilde{s}}$  subset of I.

*Proof.* If  $A = +\infty$  or  $A = -\infty$ , the desired conclusion follows from Theorem Y, which we just proved.

Suppose that  $A \in R$ . Let F denote the set of all points at which each derivate of f is finite; let  $D_A$  denote the set of all  $x \in I$  such that  $A \in D(f; x)$ ; and, for each positive integer n, let  $E_n$  denote the set of all  $x \in I$  such that

$$\left| rac{f(y)-f(x)}{y-x} - A 
ight| \geqq rac{1}{n}$$

whenever  $y \in I$  and 0 < |y - x| < 1/n.

First, let us prove that  $I \setminus D_A = \bigcup_{n=1}^{\infty} E_n$ . Suppose that  $x \in \bigcup_{n=1}^{\infty} E_n$ . Then  $x \in E_n$  for some positive integer n. If  $x_k \to x$  as  $k \to \infty$  where  $x_k \in I \setminus \{x\}$  for each k, then  $0 < |x_k - x| < 1/n$  for all sufficiently large k; hence, since  $x \in E_n$ ,

$$\left|rac{f(x_k)-f(x)}{x_k-x}-A
ight| \geq rac{1}{n}$$

for all sufficiently large k. Thus,  $(f(x_k) - f(x))/(x_k - x)$  cannot converge to A as  $k \to \infty$ , that is,  $x \in I \setminus D_A$ . Next, suppose that  $x \in I \setminus \bigcup_{n=1}^{\infty} E_n$ . Then, for each positive integer  $n, x \in I \setminus E_n$ ; and, hence, there exists  $y_n \in I$  such that  $0 < |y_n - x| < 1/n$  and

$$\left|rac{f(y_n)-f(x)}{y_n-x}-A
ight|<rac{1}{n}$$

Then  $y_n \to x$  as  $n \to \infty$ ,  $y_n \in I \setminus \{x\}$  for each n, and  $(f(y_n) - f(x))/(y_n - x) \to A$  as  $n \to \infty$ ; consequently,  $x \in D_A$ , that is,  $x \notin I \setminus D_A$ , as desired.

Next, let us prove that, for each positive integer  $n, F \cap E_n$  is closed in F. Let  $x_0 \in F$  be a limit point of  $F \cap E_n$ . We want to prove that  $x_0 \in E_n$ . Given  $y \in I$  such that  $0 < |y - x_0| < 1/n$ , it will suffice to prove that

$$(6) \qquad \left|\frac{f(y)-f(x_0)}{y-x_0}-A\right| \geq \frac{1}{n}$$

Since  $x_0$  is a limit point of  $F \cap E_n$ , there exists a sequence  $x_1, x_2, x_3, \cdots$  of points of  $E_n \setminus \{x_0, y\}$  such that  $x_k \to x_0$  as  $k \to \infty$ . Now, clearly, f is continuous at  $x_0$  since  $x_0 \in F$ . Hence,

$$(7) f(x_k) \longrightarrow f(x_0) as k \longrightarrow \infty$$

Since  $x_k \to x_0$  as  $k \to \infty$ , it follows that  $0 < \lim_{k \to \infty} |y - x_k| = |y - x_0| < 1/n$ . Thus, there exists a positive integer  $k_1$ , such that  $0 < |y - x_k| < 1/n$  if  $k > k_1$ . Since  $x_k \in E_n$  for each k, it follows that

$$(8) \qquad \quad \left|rac{f(y)-f(x_k)}{y-x_k}-A
ight| \geq rac{1}{n} ext{whenever } k>k_1 \ .$$

From (7) we obtain

(9) 
$$\lim_{k\to\infty} \left| \frac{f(y) - f(x_k)}{y - x_k} - A \right| = \left| \frac{f(y) - f(x_0)}{y - x_0} - A \right|;$$

and (9) combined with (8) yields (6). Thus, each  $F \cap E_n$  is closed in F. Since  $F \cap (I \setminus D_A) = F \cap \bigcup_{n=1}^{\infty} E_n = \bigcup_{n=1}^{\infty} (F \cap E_n)$ , it follows that

 $F \cap (I \setminus D_A)$  is an  $F_{\sigma}$  subset of F. By Theorem Y, F is an  $F_{\sigma}$  subset

of I. Moreover, if U is an  $F_{\sigma}$  subset of V, and V is an  $F_{\sigma}$  subset of W, then U is an  $F_{\sigma}$  subset of W (see [8], p. 63). Hence,  $F \cap$  $(I \setminus D_A)$  is an  $F_{\sigma}$  subset of I. Finally, by De Morgan's law,  $I \setminus \{F \cap$  $(I \setminus D_A)\} = \{I \setminus F\} \cup D_A$  is a  $G_{\delta}$  subset of I, that is, the set of x such that D(f; x) contains at least one element of  $\{A, +\infty, -\infty\}$  is a  $G_{\delta}$ subset of I. This completes the proof of the theorem.

Next, let us prove a corollary of Theorem 3 that, in turn, has Theorem 2 as a direct corollary.

COROLLARY 2. Let  $f: I \to R$  where I is an interval, and let  $A \in \overline{R}$ . If, at each point of a dense subset of I, A is a derivate of f, then the set of those points at which f has a (finite) derivative different from A is meager in I.

*Proof.* Without loss of generality we may, and do, assume that I is open.

Since  $D_A = \{x \in I: A \in D(f; x)\}$  is, by hypothesis, dense in I and  $D_A \subset D_A \cup (I \setminus F)$  where F is the set of all points at which each Dini derivate of f is finite, it follows that  $D_A \cup (I \setminus F)$  is dense in I. According to Theorem 3,  $D_A \cup (I \setminus F)$  is a  $G_\delta$  subset of I. Since  $D_A \cup (I \setminus F)$  is a dense  $G_\delta$  subset of I, it is co-meager in I, that is,  $I \setminus \{D_A \cup (I \setminus F)\} = \{I \setminus D_A\} \cap F$  is meager in I. Since the subset of I where f'(x) exists (finite) and  $f'(x) \neq A$  is a subset of  $\{I \setminus D_A\} \cap F$ , it, too, must be meager in I.

4. Conclusion. We note that a trivial modification of the proof of Theorem 2 yields Corollary 2 directly. Also, "finite" may be deleted in the statements of Theorems 1 and 2.

When this investigation was in the final stages, we discovered that it overlaps some recent research of Garg [5]. In particular, our Theorem 1 follows from Garg's Proposition 3.9 and also from his Corollary 5.2.

While this paper was in press, we learned of Filipczak's paper [3a]. Our Theorem 2 is a corollary of his lemma (p. 74). However, our Theorem 3 is in some sense stronger than that lemma since it asserts that a potentially smaller set is residual.

Finally, it should be pointed out that our observation that Fort's theorem is an easy consequence of Young's theorem was anticipated by Garg [6] in 1962.

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Received September 8, 1978.

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Printed in Japan by International Academic Printing Co., Ltd., Tokyo, Japan

# Pacific Journal of MathematicsVol. 83, No. 1March, 1979

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