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TAKESAKI'S DUALITY FOR REGULAR EXTENSIONS OF VON NEUMANN ALGEBRAS

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We extend Takesaki's duality to regular extensions, and hence twisted crossed products, of von Neumann algebras by locally compact groups.

Introduction. For a von Neumann algebra M, ε denotes the canonical map of the automorphism group $\operatorname{Aut}(M)$ of M to the quotient $\operatorname{Aut}(M)/\operatorname{Int}(M) = \operatorname{Out}(M)$ of $\operatorname{Aut}(M)$ by the normal subgroup of inner automorphisms. When M_* is separable, and G is a separable locally compact group (always endowed with a right Haar measure and modular function Δ), we can associate to certain Borel mappings $\alpha_{(\cdot)}: t \mapsto \alpha_t \in \operatorname{Aut}(M)$ with $t \mapsto \varepsilon(\alpha_t)$ a homomorphism, a family of extensions of M by G, known as regular extensions, or, in special cases, twisted crossed products, [7, 10, 12, 13, 15]. Indeed, since $\varepsilon(\alpha_s)\varepsilon(\alpha_t) = \varepsilon(\alpha_{st})$ there is a Borel family $(s, t) \in G \times G \mapsto u(s, t) \in M$ of unitaries such that

(1)
$$\begin{cases} \alpha_s \circ \alpha_t = \operatorname{Ad} u(s, t) \circ \alpha_{st} \\ (\operatorname{or} (\alpha \otimes t) \circ \alpha = \operatorname{Ad} u \circ (t \otimes \delta) \circ \alpha) \end{cases}$$

where δ is the isomorphism of $L^{\infty}(G)$ into $L^{\infty}(G) \otimes L^{\infty}(G)$ determined by $(\delta f)(s, t) \equiv f(st), f \in L^{\infty}(G); \alpha: M \to M \otimes L^{\infty}(G)$ is given by $(\alpha(x))(t) \equiv \alpha_t(x), x \in M$ and $(u\xi)(s, t) \equiv u(s, t)\xi(s, t)$ for $\xi \in \mathscr{H} \otimes L^2(G) \otimes L^2(G)$ (where M acts on \mathscr{H}).

Since $t \mapsto \varepsilon(\alpha_t)$ is a homomorphism, we see

$$\alpha_r(u(s, t))u(r, st) = f_u(r, s, t)u(r, s)u(rs, t)$$

for some Borel map $f_u: G \times G \times G \to M$ with unitary values in the center of M. Also, f_u is a 3-cocycle for the natural action of G on the center of M. If f_u cobounds, we may assume, by modifying by unitaries in the center of M, that

(2)
$$\alpha_r(u(s, t))u(r, st) = u(r, s)u(rs, t)$$

on $G \times G \times G$. Hence we may construct the regular extension $M \bigotimes_{\alpha,u} G$ of M by G, as the von Neumann algebra on $\mathscr{H} \otimes L^2(G)$ generated by the operators

$$(lpha(x)\xi)(t)\equiv lpha_t(x)\xi(t)$$
 , $(\lambda^u(r)\xi)(t)\equiv u(t,\,r)\xi(tr)$

for $x \in M$, $r \in G$ and $\xi \in \mathscr{H} \otimes L^2(G)$. (See [13, Theorem 3.1.6] for

further details on regular extensions and the significance of f_u cobounding.)

In order to formulate Takesaki's duality for a general locally compact group, we introduce the concept of a dual action of G on a von Neumann algebra N; this is an isomorphism β of N into $N \otimes R(G)$ satisfying

$$(\beta \otimes \iota) \circ \beta = (\iota \otimes \gamma) \circ \beta$$

where R(G) is the von Neumann algebra generated by the right regular representation λ of G and γ is the isomorphism of R(G) into $R(G) \otimes R(G)$ determined by $\gamma(\lambda(t)) = \lambda(t) \otimes \lambda(t), t \in G$. The crossed dual product N by $G, N \bigotimes_{\beta}^{d} G$, is the von Neumann algebra generated by $\beta(N)$ and $1 \otimes L^{\infty}(G)$, [3, 6, 8, 9, 11, 14]. Our main result, Theorem 2 extends Takesaki's duality to regular extensions, thus answering a question raised in [13, §1].

Duality for regular extensions. Before beginning our discussion, we define unitaries U, V, V' and W on $L^2(G) \otimes L^2(G)$ by

$$\xi_{\lambda}(U\xi)(s,\,t)\equiv\xi(t,\,s)\;,\quad (V\xi)(s,\,t)\equiv\xi(st,\,t)\;,\quad (V'\xi)(s,\,t)\equiv\varDelta(t)^{1/2}\xi(t^{-1}s,\,t)\;,$$

and $W \equiv UVU$, so $(W\xi)(s, t) = \xi(s, ts)$. Note that AdU is the symmetry $\sigma: x \otimes y \mapsto y \otimes x$, $\delta f = \operatorname{Ad} V(f \otimes 1_G)$, $f \in L^{\infty}(G)$, and

$$\gamma(\lambda(t)) = \operatorname{Ad} W^*(\lambda(t) \otimes \mathbf{1}_G) .$$

LEMMA 1. If $\hat{\alpha}$ is defined on $M \bigotimes_{\alpha, u} G$ by

 $\widehat{lpha}(y) \equiv \operatorname{Ad} \mathbf{1} \otimes W^*(y \otimes \mathbf{1}_{\scriptscriptstyle G})$,

then it is a dual action of G on $M\bigotimes_{\alpha,u} G$.

Proof. Direct computations easily show

$$(4) \qquad \qquad \begin{cases} \operatorname{Ad} \mathbf{1} \otimes W^*(\alpha(x) \otimes \mathbf{1}_G) = \alpha(x) \otimes \mathbf{1}_G \\ \operatorname{Ad} \mathbf{1} \otimes W^*(\lambda^u(r) \otimes \mathbf{1}_G) = \lambda^u(r) \otimes \lambda(r) \end{cases}$$

The identity $(\hat{\alpha} \otimes \iota) \circ \hat{\alpha} = (\iota \otimes \gamma) \circ \hat{\alpha}$ now follows trivially on the generators of $M \bigotimes_{\alpha,u} G$, and hence on all of $M \bigotimes_{\alpha,u} G$.

Following [6, 8], we say that actions α^{j} of a group G on von Neumann algebras M_{j} , j = 1, 2 are equivalent if

$$(\rho \otimes \iota) \circ \alpha^{\scriptscriptstyle 1} = \alpha^{\scriptscriptstyle 2} \circ \rho$$

¹ An action α of G on M means a homomorphism of G into Aut(M) such that $t \mapsto \alpha_t(x)$ is σ -weakly continuous for each $x \in M$.

for some isomorphism ρ of M_1 onto M_2 ; we denote this relation by $\{M_1, \alpha^1\} \sim \{M_2, \alpha^2\}.$

THEOREM 2. Let $\tilde{\alpha} \equiv \operatorname{Ad} 1 \otimes V' \circ (\iota \otimes \sigma) \circ \operatorname{Ad} u^* \circ (\alpha \otimes \iota)$, and

$$\widehat{lpha}(x) \equiv \operatorname{Ad} \mathbf{1} \otimes \mathbf{1}_G \otimes V'(x \otimes \mathbf{1}_G) \qquad \left(x \in \left(M \bigotimes_{\alpha \ u} G \right) \bigotimes_{\widehat{\alpha}}^d G \right),$$

so that $\widehat{\alpha}$ is the action² of G on $(M \bigotimes_{\alpha,u} G) \bigotimes_{\alpha}^{d} G$ dual to $\widehat{\alpha}$. Then $\widetilde{\alpha}$ is an action of G on $M \otimes B(L^{2}(G))$ and we have

$$\left\{\left(M\bigotimes_{\alpha, u}G\right)\bigotimes_{\widehat{\alpha}}^{d}G, \widehat{\alpha}\right\} \sim \{M\otimes B(L^{2}(G)), \widetilde{\alpha}\}$$

Proof. We note first that the operators $\alpha(x), x \in M, \lambda^u(r), r \in G$ and $1 \otimes f, f \in L^{\infty}(G)$ generate $M \otimes B(L^2(G))$. Indeed, if N is the von Neumann algebra generated by the above operators, then $N' \subset B(\mathscr{H}) \otimes L^{\infty}(G)$. If $x \in N'$, then for all $y \in M$ we see that

$$\alpha_t(y)x(t)\xi(t) = (\alpha(y)x\xi)(t) = (x\alpha(y)\xi)(t) = x(t)\alpha_t(y)\xi(t)$$

a.e. on G, so that $x(t) \in M'$ a.e. Since also $\lambda^u(r)x = x\lambda^u(r)$ for all $r \in G$, we obtain x(t)u(t, r) = u(t, r)x(tr) a.e. in t for each $r \in G$. A routine argument now shows $x \in M' \otimes 1_G$, and $N = M \otimes B(L^2(G))$. Note that in fact we have shown that $\alpha(x), x \in M$ and $1 \otimes L^{\infty}(G)$ generate $M \otimes L^{\infty}(G)$.

Now define a map $\rho: M \otimes B(L^2(G)) \to M \otimes B(L^2(G)) \otimes B(L^2(G))$ by $\rho \equiv \operatorname{Ad} 1 \otimes V^* \circ \operatorname{Ad} u^* \circ (\alpha \otimes \iota)$. We have then

$$(5) \qquad egin{array}{c}
ho(lpha(x)) = lpha(x) \otimes \mathbf{1}_G \
ho(\lambda^u(r)) = \lambda^u(r) \otimes \lambda(r) \
ho(\mathbf{1} \otimes f) = \mathbf{1} \otimes \mathbf{1}_G \otimes f \end{array}$$

Of these, the last is trivial, the first follows from (1), and the second is checked as follows. Since, from (2),

$$\alpha_{st^{-1}}(u(t, r))u(st^{-1}, tr) = u(st^{-1}, t)u(s, r)$$

we have, for $\xi \in \mathscr{H} \otimes L^2(G) \otimes L^2(G)$,

$$\begin{split} &((1\otimes V^*)u^*(\alpha\otimes\iota(\lambda^u(r)))u(1\otimes V)\xi)(s,t)\\ &=u(st^{-1},t)^*(\alpha\otimes\iota(\lambda^u(r))u(1\otimes V)\xi)(st^{-1},t)\\ &=u(st^{-1},t)^*\alpha_{st^{-1}}(u(t,r))u(st^{-1},tr)((1\otimes V)\xi)(st^{-1},tr)\\ &=u(st^{-1},t)^*\alpha_{st^{-1}}(u(t,r))u(st^{-1},tr)\xi(sr,tr) \end{split}$$

² α is an action of G on M if and only if α is a normal isomorphism of M into $M \otimes L^{\infty}(G)$ with $(\alpha \otimes \iota) \circ \alpha = (\iota \otimes \delta) \circ \alpha$, [8, Theorem 2.1].

$$= u(s, r)\xi(sr, tr)$$

= $((\lambda^u(r) \otimes \lambda(r))\xi)(s, t)$.

Since, from (4), the right hand sides of (5) generate

$$\left(M \bigotimes_{lpha,\,u} G
ight) \bigotimes_{\hat{lpha}}^{d} G$$
 ,

 ρ is an isomorphism of $M \otimes B(L^2(G))$ onto $(M \bigotimes_{\alpha,u} G) \bigotimes_{\alpha}^d G$.

It remains to check the identity $(\rho \otimes \iota) \circ \tilde{\alpha} = \hat{\alpha} \circ \rho$. Notice that $\tilde{\alpha} = \operatorname{Ad} (1 \otimes V' U V) \circ \rho$, and that

$$(V'UV\xi)(s,t)=arDelta(t)^{1/2}\xi(s,t^{-1}s)\;,\;\;\;((V'UV)^*\xi)(s,t)=arDelta(ts^{-1})^{1/2}\xi(s,st^{-1})\;.$$

Thus we obtain

$$egin{aligned} &(
ho\otimes\iota)\circ\widetilde{lpha}(lpha(x))=(
ho\otimes\iota)\circ\mathrm{Ad}(\mathbf{1}\otimes V'UV)(lpha(x)\otimes\mathbf{1}_{G})\ &=(
ho\otimes\iota)(lpha(x)\otimes\mathbf{1}_{G})\ &=lpha(x)\otimes\mathbf{1}_{G}\otimes\mathbf{1}_{G}\ , \end{aligned}$$

and

$$egin{aligned} &(
ho\otimes\iota)\circ\widetilde{lpha}(\lambda^u(r))\,=\,(
ho\otimes\iota)\circ\mathrm{Ad}(1\otimes V'UV)(\lambda^u(r)\otimes\lambda(r))\ &=\,(
ho\otimes\iota)(\lambda^u(r)\otimes\mathbf{1}_G)\ &=\,\lambda^u(r)\otimes\lambda(r)\otimes\mathbf{1}_G\ . \end{aligned}$$

Also

$$\widetilde{lpha}(1\otimes f) = \operatorname{Ad}(1\otimes V')\circ(\iota\otimes\sigma)\circ\operatorname{Ad}u^*(1\otimes 1_G\otimes f)$$

= Ad $(1\otimes V')(1\otimes f\otimes 1_G) = 1\otimes \kappa f$,

where $(\kappa f)(s, t) = f(t^{-1}s)$, by direct computation.

Finally, noticing that $\operatorname{Ad} V'(\lambda(r) \otimes 1_G) = \lambda(r) \otimes 1_G$, and that $\operatorname{Ad} V'(f \otimes 1_G) = \kappa f$, we obtain also

$$egin{aligned} &\widehat{lpha} \circ
ho(lpha(x)) = lpha(x) \otimes \mathbf{1}_{G} \otimes \mathbf{1}_{G} \ , \ &\widehat{lpha} \circ
ho(\lambda^{u}(r)) = \widehat{lpha}(\lambda^{u}(r) \otimes \lambda(r)) \ &= \operatorname{Ad}(1 \otimes \mathbf{1}_{G} \otimes V')(\lambda^{u}(r) \otimes \lambda(r) \otimes \mathbf{1}_{G}) \ &= \lambda^{u}(r) \otimes \lambda(r) \otimes \mathbf{1}_{G} \ , \end{aligned}$$

and

~

$$egin{aligned} \widehat{lpha} \circ
ho(\mathbf{1} \otimes f) &= \widehat{lpha}(\mathbf{1} \otimes \mathbf{1}_{G} \otimes f) \ &= \operatorname{Ad}(\mathbf{1} \otimes \mathbf{1}_{G} \otimes V')(\mathbf{1} \otimes \mathbf{1}_{G} \otimes f \otimes \mathbf{1}_{G}) \ &= \mathbf{1} \otimes \mathbf{1}_{G} \otimes \kappa f \ ; \end{aligned}$$

the equality $(\rho \otimes \iota) \circ \tilde{\alpha} = \hat{\alpha} \circ \rho$ is verified.

COROLLARY 3. If ${}^{u}\lambda(r)$ is defined on $\mathscr{H} \otimes L^{2}(G)$ by

 $({}^u\lambda(r)\xi)(s)\equiv {\it \Delta}(r)^{1/2}u(r,\,r^{-1}s)^*\xi(r^{-1}s)$,

then $\tilde{\alpha}_t = \operatorname{Ad} {}^{u}\lambda(t) \circ (\alpha_t \otimes \mathfrak{c}).$

Proof. It suffices to show the indicated equality on the generators $\alpha(x)$, $\lambda^{u}(r)$ and $1 \otimes f$ of $M \otimes B(L^{2}(G))$. We compute

$$egin{aligned} &(^{u}\lambda(t)lpha_{t}\otimes\iota(lpha(x))^{u}\lambda(t)^{*}\xi)(s)\ &=arphi(t)^{1/2}u(t,\,t^{-1}s)^{*}(lpha_{t}\otimes\iota(lpha(x))^{u}\lambda(t)^{*}\xi)(t^{-1}s)\ &=u(t,\,t^{-1}s)^{*}lpha_{t}(lpha_{t^{-1}s}(x))u(t,\,t^{-1}s)\xi(s)\ &=lpha_{s}(x)\xi(s) \end{aligned}$$

for $\xi \in \mathscr{H} \otimes L^{\scriptscriptstyle 2}(G)$ and

$$(\widetilde{lpha}(lpha(x))\xi)(s,\,t)=((lpha(x)\otimes 1_{\scriptscriptstyle G})\xi)(s,\,t)=lpha_s(x)\otimes 1_{\scriptscriptstyle G}\xi(s,\,t)\;,$$

for $\xi \in \mathscr{H} \otimes L^2(G) \otimes L^2(G)$. Similarly, we have

$$egin{aligned} &(\mathrm{Ad}\;^u\lambda(t)\circ(lpha_t\otimes t)(\lambda^u(r))\xi)(s)\ &= arphi(t)^{1/2}u(t,\,t^{-1}s)^*lpha_t(u(t^{-1}s,\,r))(^u\lambda(t)^*\xi)(t^{-1}sr)\ &= u(t,\,t^{-1}s)^*lpha_t(u(t^{-1}s,\,r))u(t,\,t^{-1}sr)\xi(sr)\ &= u(s,\,r)\xi(sr)\ &= u(s,\,r)\xi(sr)\ &= (\lambda^u(r)\xi)(s)\ , \end{aligned}$$

and

$$egin{aligned} (\operatorname{Ad}^u\lambda(t)\circ(lpha_t\otimes\iota)(1\otimes f)\xi)(s)&=(\operatorname{Ad}^u\lambda(t)(1\otimes f)\xi)(s)\ &=u(t,\,t^{-1}s)^*f(t^{-1}s)u(t,\,t^{-1}s)\xi(s)\ &=f(t^{-1}s)\xi(s) \end{aligned}$$

for $\xi \in \mathscr{H} \otimes L^{2}(G)$. Since

$$(\widetilde{lpha}(\lambda^{u}(r))\xi)(s,\,t)=((\lambda^{u}(r)\otimes 1_{G})\xi)(s,\,t)$$

and

$$(\widetilde{lpha}(1\otimes f)\xi)(s,t) = ((1\otimes\kappa f)\xi)(s,t) \ = f(t^{-1}s)\xi(s,t)$$

for $\xi \in \mathscr{H} \otimes L^2(G) \otimes L^2(G)$, the verification is complete.

This result is a partial clarification of [13, Proposition 2.1.3] asserting that the 2-cocycle $u \otimes 1_G$ cobounds with respect to $\alpha_t \otimes c$ in $M \otimes B(L^2(G))$. Indeed, it is trivially checked that

$$u(s, t) \otimes \mathbf{1}_{G} = (\alpha_{s} \otimes \iota)({}^{u}\lambda(t)^{*}){}^{u}\lambda(s)^{*u}\lambda(st)$$

as required.

For a given action θ of G on a von Neumann algebra N, we write $N^{\theta} \equiv \{x \in N : \theta_t(x) = x, \forall t \in G\}$, the fixed point subalgebra of N.

COROLLARY 4. $M \bigotimes_{\alpha,u} G = (M \otimes B(L^2(G)))^{\widetilde{\alpha}}$.

Proof. Since $((M \bigotimes_{\alpha,u} G) \bigotimes_{\alpha}^{d} G)^{\hat{\alpha}} = \hat{\alpha}(M \bigotimes_{\alpha,u} G)$ by [8, Proposition 6.4], Takesaki's duality (Theorem 2) tells us that

$$\widehat{lpha}\Big(M \bigotimes_{{}^{lpha},{}^{u}} G\Big) =
ho((M \otimes B(L^{\scriptscriptstyle 2}(G)))^{\widetilde{lpha}}) \; .$$

From (4) and (5), we see that $\hat{\alpha}$ and ρ agree on $M\bigotimes_{\alpha,u} G$, so that $M\bigotimes_{\alpha,u} G = (M\bigotimes B(L^2(G)))^{\tilde{\alpha}}$ as claimed.

Corollary 4 gives some information on when regular extensions $M \bigotimes_{\alpha^1, u} G$ and $M \bigotimes_{\alpha^2, v} G$ of M by G, with $\varepsilon \circ \alpha^1 = \varepsilon \circ \alpha^2$, are isomorphic. For if $\tilde{\alpha}^1$ and $\tilde{\alpha}^2$ denote the actions of G on $\overline{M} \equiv M \otimes B(L^2(G))$ with fixed point algebras $M \bigotimes_{\alpha^1, u} G$ and $M \bigotimes_{\alpha^2, v} G$ respectively, then $\overline{M} \bigotimes_{\tilde{\alpha}^1} G$ and $\overline{M} \bigotimes_{\tilde{\alpha}^2} G$ will be isomorphic whenever there is a Borel map $t \in G \to u_t$ with $\tilde{\alpha}^1_t = \operatorname{Ad} u_t \circ \tilde{\alpha}^2_t$ and $u_t \tilde{\alpha}^2_t(u_s) = u_{ts}$ for $t, s \in G$, [14]. On the other hand these crossed products are isomorphic respectively to $(M \bigotimes_{\alpha^1, u} G) \otimes B(L^2(G))$ and $(M \bigotimes_{\alpha^2, v} G) \otimes B(L^2(G))$, [8].

Also, note that $\varepsilon \circ \tilde{\alpha}^1 = \varepsilon \cdot \tilde{\alpha}^2$ whenever $\varepsilon \circ \alpha^1 = \varepsilon \circ \alpha^2$, so it is necessary only to provide conditions under which the "comparison cocycle" $\omega_{\tilde{\alpha}^1,\tilde{\alpha}^2}$ associated to $\tilde{\alpha}^1$ and $\tilde{\alpha}^2$ is trivial, [13]. The hypothesis of the next result are two situations in which this is known to happen, [1, 4].

COROLLARY 5. Let $M \bigotimes_{\alpha^1, u} G$ and $M \bigotimes_{\alpha^2, v} G$ be regular extensions of M by G with $\varepsilon \circ \alpha^1 = \varepsilon \circ \alpha^2$. If either

(1) G is discrete, acts freely on the center of M, and is a locally finite extension of a solvable group; or

(2) G is a compact, abelian and connected group K, or $K \times R$, and acts trivially on the center of M, then $(M \bigotimes_{a^1, u} G) \otimes B(L^2(G))$ and $(M \bigotimes_{a^2, v} G) \otimes B(L^2(G))$ are isomorphic.

Just as in the case of ordinary crossed products, regular extensions may be characterized by the existence of a dual action and of a distinguished family of unitaries.

THEOREM 6. Let N be a von Neumann algebra with N_* separable and β a dual action of G on N. Then the following two conditions are equivalent: (ii) there is a Borel map $t = G \mapsto v(t) \in N$ with unitary values such that $\beta(v(t)) = v(t) \otimes \lambda(t), t \in G$.

The proof goes the same way as in the proof [5, 8, 11] except the following lemma.

LEMMA 7. Assume the condition (ii) in Theorem 6. Then, N is generated by $N^{\beta} \equiv \{y \in N: \beta(y) = y \otimes \mathbf{1}_{g}\}$ and $v(t), t \in G$.

Proof (Takesaki). Let $\bar{N} \equiv N \otimes F_{\infty}$, $\bar{\beta} \equiv (\iota \otimes \sigma) \circ (\beta \otimes \iota)$ and $\bar{v}(t) \equiv v(t) \otimes 1$, where F_{∞} is a factor of type I_{∞} . Then $\bar{\beta}$ is a dual action of G on \bar{N} , $\bar{N}^{\beta} = N^{\beta} \otimes F_{\infty}$ is properly infinite and $\bar{\beta}(\bar{v}(t)) = \bar{v}(t) \otimes \lambda(t)$ for all t. Therefore $\bar{\beta}$ is dominant³, because $\bar{\beta}(v) = (v \otimes 1_G)(1 \otimes W)$ for a unitary v in $N \otimes L^{\infty}(G)$ defined by $(v\xi)(t) \equiv v(t)\xi(t)$, [2, 9]. Therefore there exists a strongly continuous unitary representation u of G in \bar{N} such that $\bar{\beta}(u(t)) = u(t) \otimes \lambda(t)$ by [5, 8, 11]. In this case \bar{N} is generated by $N^{\beta} \otimes F_{\infty}$ and $u(t), t \in G$. If e is a projection in \bar{N} of the form $1 \otimes p$ with dim p = 1, then $\{N, \beta\}$ is identified with $\{\bar{N}_{e}, \bar{\beta}^{e}\}$. Since $\bar{\beta}(v(t)) = v(t) \otimes \lambda(t)$, $t \in G$, $v(t)u(t)^{*} \in N^{\beta} \otimes F_{\infty}$ and hence v(t) = ew(t)u(t)e for some $w(t) \in N^{\beta} \otimes F_{\infty}$. Here we may assume that $w(t) = ew(t)\alpha_{t}(e)$. So, w(t) is a partial isometry. If x is an arbitrary element in $N^{\beta} \otimes F_{\infty}$, then

$$exu(t)e = exw(t)^*w(t)u(t)e = exw(t)^*v(t)$$

and hence $e(N^{\beta} \otimes F_{\infty})u(t)e = N^{\beta}v(t)$. It remains to show that $e(N^{\beta} \otimes F_{\infty})u(t)e, t \in G$ generate $e\overline{N}e = N$. Since the set L of all finite linear combinations of xu(t) with $x \in N^{\beta} \otimes F_{\infty}$ and $t \in G$ is a σ -weakly dense *-subalgebra of \overline{N} , eLe is σ -weakly dense in $e\overline{N}e = N$. Consequently, $N^{\beta}v(t), t \in G$ generate N.

Proof of Theorem 6. That $(i) \Rightarrow (ii)$ has already verified in Lemma 1.

(ii) \Rightarrow (i). Let $M \equiv N^{\beta}$. Since $\beta(v(t)xv(t)^*) = v(t)xv(t)^* \otimes 1_{\alpha}$ for $x \in M$, v(t) normalizes M. Also with $u(s, t) \equiv v(s)v(t)v(st)^*$, we see $\beta(u(s, t)) = u(s, t) \otimes 1_{\alpha}$, so $u(s, t) \in M$ for $s, t \in G$.

Set $\alpha_s \equiv \operatorname{Ad} v(s) \upharpoonright M$. Then $\alpha_s \circ \alpha_t = \operatorname{Ad} u(s, t) \circ \alpha_{st}$ and $\alpha_r(u(s, t))u(r, st) = u(t, s)u(rs, t)$. Then α and u determine a regular extension $M \bigotimes_{\alpha,u} G$ of M by G, with generators $\alpha(M)$ and $\lambda^u(s), s \in G$. Define a unitary v in $N \otimes L^{\infty}(G)$ by $(v\xi)(t) = v(t)\xi(t)$. Then, by di-

³ A dual action β of G on N is said to be *dominant*, if N^{β} is properly infinite and $\{\overline{N}, \overline{\beta}\} \sim \{\overline{N}, \widetilde{\beta}\}$, where $\overline{N} = N \otimes B(L^2(G)), \overline{\beta} = (\iota \otimes \sigma) \circ (\beta \otimes \iota)$ and $\overline{\beta} = (\operatorname{Ad} 1 \otimes W) \circ \overline{\beta}$. If β is dominant, then $\{N, \beta\} \sim \{\overline{N}, \beta\} \sim \{(N \otimes_{\beta}^{d} G) \otimes_{\beta}^{\beta} G, \overline{\beta}\}$.

rect computation,

$$v^*\lambda^u(s)v = \beta(v(s))$$
 and $v^*\alpha(x)v = \beta(x)$

for $s \in G$ and $x \in M$. Thus $v^*(M \bigotimes_{\alpha,u} G)v = \beta(N)$ by Lemma 7.

According to the above theorem we know the relation between [2, Theorem III. 3.1] and [5, Theorem].

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