# Pacific Journal of Mathematics

#### AXIOMS FOR CLOSED LEFT IDEALS IN A C\*-ALGEBRA

JEN-CHUNG CHUAN

Vol. 87, No. 1

January 1980

### AXIOMS FOR CLOSED LEFT IDEALS IN A $C^*$ -ALGEBRA

#### JEN-CHUNG CHUAN

#### A set of axioms is formulated to describe the conditions under which a Banach algebra may be embedded as a closed left ideal in a $C^*$ -algebra.

In this paper we attempt to characterize the class of all closed left ideals in a  $C^*$ -algebra as a class of Banach algebras equipped with a certain (nonassociative) multiplication structure. To describe such a multiplication, we formulate a set of axioms which extracts the essential properties of the binary operation

 $(x, y) \longrightarrow y^*x$ 

taking place in a closed left ideal of a  $C^*$ -algebra. Following the notion of centralizers of  $C^*$ -algebras introduced by B. E. Johnson [3] and R. C. Busby [1] we are able to show that the axioms are indeed suitable for our purpose: in order that a Banach algebra L fulfills the conditions of the axiom, it is necessary and sufficient that L can be identified with a closed left ideal of some  $C^*$ -algebra. This paper is taken from parts of author's Ph. D. thesis under the supervision of C. Akemann.

AXIOM 1. Let (L, || ||) be a complex Banach algebra which contains a closed subalgebra B that has a  $C^*$ -algebra structure, i.e., besides the algebraic and the norm structures inherited from L, Bhas an involution \* so that B is a Banach \*-algebra satisfying  $||x^*x|| = ||x||^2$  for  $x \in B$ . Suppose that

 $[\cdot, \cdot]: L \times L \longrightarrow B$ 

is a function such that for elements x, y, z in L and for each complex scalar  $\lambda$  the following rules hold:

(i) 
$$[x, y] = [y, x]^*$$

(ii) [x + y, z] = [x, z] + [y, z]

(iii) 
$$[\lambda x, y] = \lambda [x, y]$$

(iv) [x, x] is a positive element of the C<sup>\*</sup>-algebra B

 $(\mathbf{v}) ||[x, x]|| = ||x||^2$ 

$$(vi) ||[x, y]|| \leq ||x|| ||y||$$

$$(vii) [xy, z] = [y, [z, x]]$$

(viii)  $[x, y] = y^*x$  for x, y in B.

We now exhibit a situation in which the conditions stated in the

above axiom hold naturally.

PROPOSITION 2. Suppose that L is a closed left ideal of a C<sup>\*</sup>algebra A. Set  $B = L \cap L^*$ . Let  $[\cdot, \cdot]: L \times L \to B$  be defined by  $[x, y] = y^*x$ . Then L, B,  $[\cdot, \cdot]$  satisfy the conditions of Axiom 1.

**Proof.** Clearly B is a C\*-algebra and condition (viii) is satisfied. Conditions (i) ~ (v) reflect the basic properties of A. For x, y in  $A, ||y^*x|| \leq ||y|| ||x||$ , thus (vi). Condition (vii) is the consequence of the associative law of multiplication: for x, y, z in L, we have

$$z^*(xy) = (z^*x)y = (x^*z)^*y$$
.

We remark that the binary operation  $[\cdot, \cdot]$  is not required to be associative. As in the situation of Proposition 2, for arbitrary elements x, y, z in A,  $[[x, y], z] = z^*(y^*x)$  is usually not the same as  $(y^*z)^*x = [x, [y, z]].$ 

Conditions (i)  $\sim$  (v) resemble rules of the scalar product defined on vector spaces. Indeed we are able to derive consequences similar to those of the inner product.

PROPOSITION 3. Under Axiom 1 the following hold: (1) [x, y + z] = [x, y] + [x, z] for x, y, z in L. (2)  $[x, \lambda y] = \overline{\lambda}[x, y]$  for complex scalars  $\lambda$  and elements x, yin L. (3)  $||x|| = \sup\{||[x, y]||: y \in L, ||y|| \leq 1\}$  for  $x \in L$ . (4) The condition " $x \in L$  and [x, y] = 0 for all  $y \in L$ " implies x = 0. (5) For a, b, x, y in L, we have [ax, by] = [[a, b]x, y]. (6) For  $x \in L$ , we have

 $(\mathbf{0})$  **FOT**  $x \in \mathbf{D}$ , we have

 $||x|| = \sup\{||[xy, z]||: y, z \in L, ||y|| \le 1, ||z|| \le 1\}.$ 

*Proof.* For x, y, z in L and for each complex scalar  $\lambda$ , we have

$$\begin{split} [x, y + z] &= [y + z, x]^* = [y, x]^* + [z, x]^* \\ &= [x, y] + [x, z] , \\ [x, \lambda y] &= [\lambda y, x]^* = \overline{\lambda} [y, x]^* \\ &= \overline{\lambda} [x, y] . \end{split}$$

Thus (1) and (2). Now for x, y in L with  $||y|| \leq 1$ , we have

50

 $||[x, y]|| \leq ||x|| ||y|| \leq ||x||.$ 

Thus if x = 0 then clearly ||[x, y]|| = 0. If  $x \neq 0$ , then

$$\left\| \left[ x, \frac{x}{\|x\|} \right] \right\| = \frac{\|x\|^2}{\|x\|} = \|x\|.$$

Therefore (3).

Condition (4) follows from (3). To see condition (5), we repeat rule (vii) to obtain:

$$[ax, by] = [x[by, a]] = [x, [y, [a, b]]]$$
$$= [[a, b]x, y] .$$

To see (6), notice that if  $x \neq 0$ , then

$$\left\| \left[ x \frac{[x, x]}{||x||^2}, \frac{x}{||x||} \right] \right\| = \frac{[x, x]}{||x||^2}, \frac{[x, x]}{||x||} \right] \|$$
$$= ||x||^4 / ||x||^3 = ||x|| .$$

The following definition is a slight modification of a concept first introduced by B. E. Johnson and was later investigated by R. C. Busby in the case of  $C^*$ -algebras (see [3], [1]).

L is assumed to satisfy Axiom 1 from now on.

DEFINITION 4. A bracket centralizer on L is a pair (T', T'') of functions from L to L such that [T'x, y] = [x, T''y] for x, y in L. We denote the set of all bracket centralizers of L by M(L).

PROPOSITION 5. Let  $(T', T'') \in M(L)$ . Then (1)  $(T'', T') \in M(L)$ . (2) T' and T'' are continuous linear maps from L to L. (3) T'(xy) = T'(x)y, T''(xy) = T''(x)y for all x, y in L.

*Proof.* (1) For all x, y in L, we have

$$[T''x, y] = [y, T''x]^*$$

and

$$[T'y, x]^* = [x, T'y]$$
.

Hence  $(T', T'') \in M(L)$  iff  $(T'', T') \in M(L)$ .

(2) Fix  $z \in L$  and for each x, y in L and complex scalars  $\alpha, \beta$ , we have

$$[T'(\alpha x + \beta y), z] = [\alpha x + \beta y, T''z] = \alpha[x, T''z] + \beta[y, T''z]$$
$$= \alpha[T'x, z] + \beta[T'y, z]$$
$$= [\alpha T'x + \beta T'y, z].$$

Hence  $\alpha T'x + \beta T'y = T'(\alpha x + \beta y)$ , by Proposition 3 (4). Consequently T' is a linear map. Since T'' plays the same role as T' by (1), we conclude that T'' is also a linear map.

Suppose that  $\{x_n\}$  is a sequence in L and y is an element of L such that

$$\lim_n ||x_n - x|| = 0 = \lim_n ||T'x_n - y||$$
.

Then for each fixed z in L, we have

$$\begin{aligned} ||[T'x - y, z]|| &= ||[T'x, z] - [y, z]|| \\ &\leq ||[T'x, z] - [T'x_n, z]|| + ||[T'x_n, z] - [y, z]|| \\ &= ||[x, T''z] - [x_n, T''z]|| + ||[T'x_n - y, z]|| \\ &\leq ||x - x_n|| ||T''z|| + ||T'x_n - y|| ||z|| \\ &\longrightarrow 0 \quad \text{as} \quad n \longrightarrow \infty . \end{aligned}$$

As a result of Proposition 3 (4), we have T'x = y. By the closed graph theorem, T' is continuous. By symmetry, T'' is continuous.

(3) Let  $x, y, z \in L$ . Then

$$egin{aligned} [T'(xy),\,z] &= [xy,\,T''z] = [y,\,[T''z,\,x]] = [y,\,[z,\,T'x]] \ &= [T'(x)y,\,z] \ , \end{aligned}$$

by condition (vii) of Axiom 1. Therefore, T'(xy) = T'(x)y.

The above proposition has the following interesting byproduct:

COROLLARY 6 (see [4; p. 296]). Let H be a Hilbert space with inner product  $\langle \cdot, \cdot \rangle$ . A function  $T: H \to H$  is a bounded linear operator on H iff there exists some function  $T^*: H \to H$  so that  $\langle Tx, y \rangle = \langle x, T^*y \rangle$  holds for all x, y in H.

*Proof.* The "only if" part follows from the fact that corresponding to each bounded linear operator T there exists an adjoint operator  $T^*$  such that  $\langle Tx, y \rangle = \langle x, T^*y \rangle$  for all x, y in H.

We now prove the "if" part. Fix an orthonormal basis  $\{\xi_{\alpha}\}_{\alpha \in \Gamma}$ Imagine H as being embedded in some fixed column of for H. matrices of the size  $\Gamma \times \Gamma$ , i.e., we fix an index  $\gamma_0$  in  $\Gamma$  and identify  $\sum_{\alpha \in \Gamma} c_{\alpha} \xi_{\alpha} \in H$  with the complex matrix  $(b_{\beta\gamma})$ , where  $b_{\beta\gamma} = 0$  for  $\gamma \neq \gamma_0$ and  $b_{\beta\gamma_0} = c_{\beta}$  for  $\beta \in \Gamma$ . Notice that the norm is preserved under this identification. H is stable under the matrix multiplication so induced and thus becomes a Banach algebra which contains a onedimensional  $C^*$ -subalgebra B, where

$$B = \{(b_{eta au}): b_{eta au} = 0 ext{ if } eta 
eq \gamma_0 ext{ or } \gamma 
eq \gamma_0\} \ = ext{the scalar multiples of the matrix } e = (e_{eta au}) \ , \ ext{where } e_{eta au} = 0 ext{ if } eta 
eq \gamma_0 ext{ or } \gamma 
eq \gamma_0, e_{\gamma_0 \gamma_0} = 1 ext{ .}$$

Then the binary operation

$$[\cdot, \cdot]: H \times H \longrightarrow B$$

defined by

$$[x, y] = y^* x = \langle x, y \rangle e$$

 $(y^* \text{ is the conjugate transpose of } y)$  satisfies all the conditions listed in Axiom 1. (Notice that condition (vii) is a result of the associative law of matrix multiplication.) Thus if  $T: H \to H$  is a function with the property that there is some function  $T^*: H \to H$  such that  $\langle Tx, y \rangle = \langle x, T^*y \rangle$  for all x, y in H, then

$$[Tx, y] = \langle Tx, y \rangle e = \langle x, T^*y \rangle e$$
$$= [x, T^*y].$$

It follows from Proposition 5 (2) that  $T: H \to H$  is a bounded linear operator.

The next corollary is a slight generalization of the previous one.

COROLLARY 7. Let H be a Hilbert space with inner product  $\langle \cdot, \cdot \rangle$ . Suppose that T, T<sup>\*</sup>:  $H \rightarrow H$  is a pair of functions such that

 $\{\langle Tx, y \rangle - \langle x, T^*y \rangle : x, y \in H\}$ 

is a bounded subset of complex numbers. Then T and  $T^*$  are bounded linear operators on H. Furthermore,  $T^*$  is indeed the adjoint of T.

*Proof.* Assume that M is a positive real number such that

 $|\langle Tx, y \rangle - \langle x, T^*y \rangle| \leq M$ 

for all x, y in H. Replacing x by  $\lambda x$  ( $\lambda \in C$ ) we have

$$|\langle T(\lambda x), y 
angle - \langle \lambda x, T^*y 
angle| \leq M$$
 .

Thus

$$rac{1}{\lambda}\langle T(\lambda x), y 
angle - \langle x, T^*y 
angle \Big| \leq rac{M}{\lambda}$$

for  $\lambda > 0$  and x, y in H. Let  $\varepsilon > 0$  be given. Fix  $\beta > 0$ . Choose  $\lambda > 0$  so that

$$eta M/\lambda < arepsilon/2 \ \ ext{and} \ \ M/\lambda < arepsilon/2 \ .$$

Hence

$$\left|\frac{1}{\lambda}\langle T(\lambda x), \beta y \rangle - \langle x, T^*(\beta y) \rangle \right| \leq \frac{M}{\lambda}.$$

In view of the equality

$$rac{1}{\lambda}\langle T(\lambda x),\ eta y
angle = rac{eta}{\lambda}\langle T(\lambda x),\ y
angle$$

and the inequality

$$\left|rac{eta}{\lambda}\langle T(\lambda x),\ y
angle-\langle x,\ eta T^*y
angle
ight|\leq rac{eta M}{\lambda}$$
 ,

we have

$$|\langle x,\,T^*(eta y)-eta T^*y
angle|\leq rac{M}{\lambda}+rac{eta M}{\lambda} .$$

This shows  $T^*(\beta y) = \beta T^* y$  for all y in H and for all  $\beta > 0$ . Now for all x, y in H, we have

$$|\langle Tx, \beta y \rangle - \langle x, T^*(\beta y) \rangle| \leq M$$

and so

$$|\langle Tx,\,y
angle-\langle x,\,T^*y
angle|\leq M/eta$$

for all x, y in H and all  $\beta > 0$ . Hence

$$\langle Tx, y 
angle = \langle x, T^*y 
angle$$

for all x, y in H. The desired conclusion follows from Corollary 6.

**PROPOSITION 8.** Let (T', T'') be in M(L). If we regard T' and T'' as bounded linear operators on the Banach space L, then

$$||T'|| = ||T''||$$
.

*Proof.* Let  $x \in L$ , ||x|| < 1. Considering Proposition 3 (3), we have

$$||T'x|| = \sup_{\substack{y \in L \\ ||y|| \le 1}} ||[T'x, y]|| = \sup_{\substack{y \in L \\ ||y|| \le 1}} ||[x, T''y]||$$
  
$$\leq \sup_{\substack{y \in L \\ ||y|| \le 1}} ||T''y|| = ||T''||.$$

Hence  $||T'|| \le ||T''||$ . By symmetry, we also have  $||T''|| \le ||T'||$ . Thus ||T'|| = ||T''||.

PROPOSITION 9. If (T', T''),  $(S', S'') \in M(L)$ , then  $(T'S', S''T'') \in M(L)$ .

*Proof.* For  $x, y \in L$ ,

$$[T'S'x, y] = [S'x, T''y] = [x, S''T''y].$$

THEOREM 10. M(L) equipped with the norm and algebraic operations defined as follows becomes a C<sup>\*</sup>-algebra with identity. For (T', T''),  $(S', S'') \in M(L)$  and complex scalar  $\alpha$ , set

- (1) (T', T'') + (S', S'') = (T' + S', T'' + S'')
- $(2) \quad \alpha(T', T'') = (\alpha T', \overline{\alpha} T'')$
- (3) (T', T'')(S', S'') = (T'S', S''T'')
- $(4) (T', T'')^* = (T'', T')$

(5) ||(T', T'')|| = the operator norm of T' on L(=the operator norm of T'' on L, by Proposition 8).

*Proof.* It is clear that M(L) is an involutive normed algebra with respect to the above operations. We now show M(L) is complete under the norm given by (5). Let  $\{(T'_n, T''_n)\}_{n\geq 1}$  be a Cauchy sequence in M(L). Then  $\{T'_n\}_{n\geq 1}$  and  $\{T''_n\}_{n\geq 1}$  are Cauchy sequences in the Banach space B(L) of all bounded linear transformations on L. Thus there are elements  $T'_{\infty}$  and  $T''_{\infty}$  in B(L) such that  $T'_{\infty}$  and  $T''_{\infty}$ are the uniform limits of  $\{T'_n\}_{n\geq 1}$  and  $\{T''_n\}_{n\geq 1}$  respectively. If  $x, y \in L$ , then

$$egin{aligned} [T'_{\infty}y,\,x] &= \lim_n \, [T'_ny,\,x] = \lim_n \, [y,\,T''_nx] \ &= [y,\,T''_{\infty}x] \ . \end{aligned}$$

Hence  $(T'_{\infty}, T''_{\infty}) \in M(L)$  and  $(T'_n, T''_n)$  is convergent to  $(T'_{\infty}, T''_{\infty})$ . It remains to check the C\*-norm condition:

$$\begin{split} ||(T', T'')^*(T', T'')|| &= ||(T''T', T'T'')|| = ||T''T'|| \\ &= \sup\{||[T''T'x, y]||: ||x|| \leq 1, ||y|| \leq 1, x, y \in L\} \\ &= \sup\{||[T'x, T'y]||: ||x|| \leq 1, ||y|| \leq 1, x, y \in L\} \\ &\geq \sup\{||[T'x, T'x]||: ||x|| \leq 1, x \in L\} \\ &= ||T'||^2 = ||T'|| ||T''|| \geq ||T''T'|| \\ &= ||(T''T', T'T'')|| = ||(T', T'')^*(T', T'')|| . \end{split}$$

Therefore

$$||(T', T'')^*(T', T'')|| = ||(T', T'')||^2$$
.

We are now ready to define an embedding of L satisfying Axiom 1 onto a closed left ideal of the  $C^*$ -algebra M(L). For each a in L, let  $\pi'(a)$  (respectively  $\pi''(a)$ ) be the function from L to Ldefined by  $\pi'(a)x = ax$  (respectively  $\pi''(a)x = [x, a]$ ) for x in L. Condition (vii) of Axiom 1 guarantees that the pair  $(\pi'(a), \pi''(a))$ belongs to M(L) for each a in L.

THEOREM 11. There is a closed left ideal J of the  $C^*$ -algebra

M(L) and an isometric linear map  $\pi$  from L onto J with the following properties:

- $(1) \quad \pi(B) = J \cap J^*.$
- (2)  $\pi|_B$  is a \*-isomorphism of C\*-algebras.
- (3)  $\pi(xy) = \pi(x)\pi(y)$  for x, y in L.
- $(4) \quad \pi([x, y]) = \pi(y)^* \pi(x) \text{ for } x, y \text{ in } L.$

*Proof.* As noticed above, the pair  $\pi(a) = (\pi'(a), \pi''(a)) \in M(L)$  for  $a \in L$ . We shall show that

$$J = \{\pi(a) \colon a \in L\}$$

is a closed left ideal of M(L) and  $\pi: L \to J$  indeed fulfills conditions  $(1) \sim (4)$ .

First we observe that, when regarded as a map from L into M(L),  $\pi$  is linear. Thus J is a linear subspace of M. As a result of Proposition 3(6), we have

$$||\pi(a)|| = \sup_{\substack{||x|| \leq 1, x \in L \\ ||y|| \leq 1, y \in L}} ||[ax, y]|| = ||a||.$$

Therefore  $\pi(L) = J$  is a complete linear subspace of M(L) and so is uniformly closed. Suppose that  $(T', T'') \in M(L)$ . Then for a in L, b = T'a is an element of L. Thus for x in L we have

$$[T' \circ \pi'(a)](x) = T'(ax) = T'(a)x = bx = \pi'(b)(x);$$
  
$$[\pi''(a) \circ T''](x) = \pi''(a)(T''x) = [T''x, a] = [x, T'a]$$
  
$$= [x, b] = \pi''(b)(x).$$

Consequently,  $(T', T'')\pi(a) = \pi(b)$ . This shows that  $\pi(L) = J$  is a surjective linear isometry.

For x, y, v, w in L, by Proposition 3 (5), we have

$$[[x, y]v, w] = [xv, yw] = [[xv, y], w]$$
.

Therefore  $\pi([x, y]) = \pi(y)^*\pi(x)$ , so (4) is proved. In particular,  $\pi([x, x])$  is a positive element of J. Since every positive element of B is of the form [x, x] for some x in L (by condition (vii)) and since every element of B is a linear combination of positive ones, we conclude that  $\pi(B) \subset J \cap J^*$ . On the other hand, every positive element of  $J \cap J^*$  is of the form  $\pi(x)^*\pi(x) = \pi(x^*x)$  for some x in L, we see that  $\pi(B) = J \cap J^*$ . Thus (1) holds.

Condition (3) is clear. Condition (2) follows from conditions (1) and (3) and the fact that  $\pi$  is isometric. This completes the proof.

There is an alternative method of embedding a Banach algebra satisfying Axioms 1 into a  $C^*$ -subalgebra of B(H), the  $C^*$ -algebra

of all bounded linear operators on some Hilbert space H [2; p. 41]. Based on the characterization of Jordan and von Neumann, it is shown in [2; p. 45] that parts of Axiom 1 may be formulated differently.

We conclude with the following summary of the main result:

THEOREM 12. A Banach algebra can be isometrically embedded as a closed left ideal of a  $C^*$ -algebra if and only if conditions of Axiom 1 hold.

#### References

1. R. C. Busby, Double centralizers and extensions of C\*-algebras, Trans. Amer. Math. Soc., 132 (1968), 79-99.

2. J. C. Chuan, One-sided ideals in a C\*-algebra, dissertation, University of California, Santa Barbara, 1977.

3. B. E. Johnson, An introduction to the theory of centralisers, Proc. London Math. Soc., (3), 14 (1964), 299-320.

4. F. Riesz and B. Sz.-Nagy, Functional Analysis, Frederick Ungar, New York, 1955.

Received December 11, 1978 and in revised form April 25, 1979.

TSING HUA UNIVERSITY HSINCHU, TAIWAN 300, REPUBLIC OF CHINA

#### PACIFIC JOURNAL OF MATHEMATICS

#### EDITORS

DONALD BABBITT (Managing Editor) University of Galifornia Los Angeles, California 90024

HUGO ROSSI University of Utah Salt Lake City, UT 84112 J. Dugundji

Department of Mathematics University of Southern California Los Angeles, California 90007

R. FINN AND J. MILGRAM Stanford University Stanford, California 94305

C. C. MOORE AND ANDREW OGG

University of California Berkeley, CA 94720

#### ASSOCIATE EDITORS

E. F. BECKENBACH

B. H. NEUMANN F. WOLF

K. YOSHIDA

#### SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA CALIFORNIA INSTITUTE OF TECHNOLOGY UNIVERSITY OF CALIFORNIA MONTANA STATE UNIVERSITY UNIVERSITY OF NEVADA, RENO NEW MEXICO STATE UNIVERSITY OREGON STATE UNIVERSITY UNIVERSITY OF OREGON UNIVERSITY OF SOUTHERN CALIFONIA STANFORD UNIVERSITY UNIVERSITY OF HAWAII UNIVERSITY OF TOKYO UNIVERSITY OF UTAH WASHINGTON STATE UNIVERSITY UNIVERSITY OF WASHINGTON

Printed in Japan by International Academic Printing Co., Ltd., Tokyo, Japan

## Pacific Journal of MathematicsVol. 87, No. 1January, 1980

Spiros Argyros, <i>A decomposition of complete Boolean algebras</i>	1
Gerald A. Beer, <i>The approximation of upper semicontinuous multifunctions</i> <i>by step multifunctions</i>	11
Ehrhard Behrends and Richard Evans <i>Multiplicity theory for Boolean</i>	
algebras of $L^p$ -projections	21
Man-Duen Choi, The full C*-algebra of the free group on two	
generators	41
Jen-Chung Chuan, Axioms for closed left ideals in a C*-algebra	49
Jo-Ann Deborah Cohen, <i>The strong approximation theorem and locally</i>	
bounded topologies on $F(X)$	59
Eugene Harrison Gover and Mark Bernard Ramras, <i>Increasing sequences of</i>	
Betti numbers	65
Morton Edward Harris, <i>Finite groups having an involution centralizer with</i>	
a 2-component of type PSL(3, 3)	69
Valéria Botelho de Magalhães Iório, <i>Hopf C*-algebras and locally compact</i>	
groups	75
Roy Andrew Johnson, Nearly Borel sets and product measures	97
Lowell Edwin Jones, Construction of $Z_p$ -actions on manifolds	111
Manuel Lerman and Robert Irving Soare, <i>d-simple sets, small sets, and</i>	
degree classes	135
Philip W. McCartney, Neighborly bushes and the Radon-Nikodým property	
for Banach spaces	157
Robert Colman McOwen, Fredholm theory of partial differential equations	1.50
on complete Riemannian manifolds	169
Ernest A. Michael and Carl Preston Pixley, <i>A unified theorem on continuous</i>	107
selections	187
Ernest A. Michael, <i>Continuous selections and finite-dimensional sets</i>	189
Vassili Nestoridis, Inner functions: noninvariant connected	100
components	199
Bun Wong, A maximum principle on Clifford torus and nonexistence of	011
proper holomorphic map from the ball to polydisc	211
Steve Wright, Similarity orbits of approximately finite C <sup>*</sup> -algebras	223
Kenjiro Yanagi, On some fixed point theorems for multivalued	222
mappings	233
wiesiaw Zelazko, A characterization of LC-nonremovable ideals in	2.41
commutative Banach algebras	241