# Pacific Journal of Mathematics

## FINITE GROUPS HAVING AN INVOLUTION CENTRALIZER WITH A 2-COMPONENT OF TYPE PSL(3, 3)

MORTON EDWARD HARRIS

Vol. 87, No. 1

January 1980

### FINITE GROUPS HAVING AN INVOLUTION CENTRALIZER WITH A 2-COMPONENT OF TYPE PSL (3, 3)

MORTON E. HARRIS

A finite group L is said to be quasisimple if L=L'and L/Z(L) is simple and is said to be 2-quasisimple if L=L'and L/O(L) is quasisimple. Let G denote a finite group. Then E(G) is the subgroup of G generated by all subnormal quasisimple subgroups of G and  $F^*(G)=E(G)F(G)$  where F(G) is the Fitting subgroup of G. Also a subnormal quasisimple subgroup of G is called a component of G and a subnormal 2-quasisimple subgroup of G is called a 2-component of G.

We can now state the main result of this paper:

THEOREM A. Let G be a finite group with  $F^*(G)$  simple. Assume that G contains an involution t such that  $H=C_{d}(t)$  possesses a 2-component L with  $L/O(L) \cong PSL(3, 3)$  and such that  $C_{H}(L/O(L))$ has cyclic Sylow 2-subgroups. Then  $|F^*(G)|_2 \leq 2^{10}$ .

In order to state an important consequence of Theorem A, we require two more definitions. A subgroup K of a finite group G is said to be tightly embedded (in G) if |K| is even and  $|K \cap K^{g}|$  is odd for every  $g \in G - N_{G}(K)$ . A quasisimple subgroup L of a finite group G is said to be standard (in G) if  $[L, L^{g}] \neq 1$  for all  $g \in G$ ,  $C_{G}(L)$  is tightly embedded in G and  $N_{G}(L) = N_{G}(C_{G}(L))$ .

THEOREM B. Let G be a finite group with O(G) = 1 and containing a standard subgroup L with  $L \cong PSL(3, 3)$ . Then either  $L \cong G$ or  $L \neq \langle L^{a} \rangle = F^{*}(G)$  and one of the following five conditions hold:

(a)  $F^*(G) \cong PSL(3, 9);$ 

(b)  $F^*(G) \cong PSL(4, 3);$ 

(c)  $F^*(G) \cong PSL(5, 3);$ 

(d)  $F^*(G) \cong PSp(6, 3);$ 

(e)  $F^*(G) = H_1 \times H_2$  with  $H_1 \cong H_2 \cong L$  and  $C_G(L) = \langle t \rangle$  where t is an involution such that  $H_1^t = H_2$  and  $L = \langle h_1 h_1^t | h_1 \in H_1 \rangle$ .

Note that Theorem B is a step toward the verification of Hypothesis  $\theta^*$  of [13] and is therefore of import for completing a proof of the Unbalanced Group Conjecture and the B(G)-Conjecture and for completing an inductive characterization of all Chevalley groups over finite fields of characteristic 3 (cf. [13, § 1]). Also by applying [13, Lemma 2.9], [3, Theorem], [1, Corollary II], [8, Theorem 5.4.10 (ii)], [3, Table 1] and [6, Tables 3 and 4], it suffices, in proving Theorem B, to assume, in addition to O(G) = 1, that  $L \neq F^*(G) = \langle L^{G} \rangle$ ,  $F^*(G)$  is simple and that  $C_{\sigma}(L)$  has cyclic Sylow 2-subgroups. But then Theorem A and the classification of all finite simple groups whose Sylow 2-subgroups have order dividing  $2^{10}$  (cf. [4] and [7]) yield Theorem B. Consequently Theorem B is a consequence of Theorem A.

The remainder of this paper is devoted to demonstrating that the analysis of [12] and [14] can be applied to prove Theorem A.

All groups in this paper are finite. Our notation is standard and tends to follow the notation of [8], [12] and [14]. In particular, if X is a (finite) group, then S(X) denotes the solvable radical of  $X, O^2(X)$  is the subgroup of X generated by all elements of X of odd order and is consequently the intersection of all normal subgroups Y of X such that X/Y is a 2-group and  $\mathscr{C}(X)$  denotes the set of elementary abelian 2-subgroups of X. Also, if n is a positive integer, then  $\mathscr{C}_n(X)$  denotes the set of elementary abelian 2-subgroups of order n of X. Finally  $m_2(X)$  denotes the maximal rank of the elements of  $\mathscr{C}(X)$ ,  $r_2(X)$  denotes the minimal integer k such that every 2-subgroup of X can be generated by k elements and if  $Y \subseteq X$ , then  $\mathscr{I}(Y)$  denotes the set of involutions contained in Y.

Clearly, if X is a group, then  $m_2(X) \leq r_2(X)$  and  $r_2(X) \leq r_2(Y) + r_2(X/Y)$  for every normal subgroup Y of X.

2. A proof of Theorem A. Throughout the remainder of this paper, we shall let G, t, H and L be as in the hypotheses of Theorem A and we shall assume that  $|F^*(G)|_2 > 2^{10}$ .

Then [9, Main Theorem], [15, Four Generator Theorem], [3, Table 1], [6, Tables 3 and 4] and [2] imply that  $4 < r_2(F^*(G)) \leq r_2(G)$  and that Sylow 2-subgroups of G and  $F^*(G)$  contain normal elementary abelian subgroups of order 8.

Clearly  $C_{\rm H}(L/O(L))$  has a normal 2-complement by [8, Theorem 7.6.1], every 2-component K of H with  $K \neq L$  lies in  $C_{\rm H}(L/O(L))$  and  $O(H) \leq C_{\rm H}(L/O(L))$  (cf. [10, §2]). Thus L is the unique 2-component of H, L char H,  $S(H) \cap L = O(L)$  and  $S(H) = C_{\rm H}(L/O(L))$  by [10, Lemma 2.3].

Since H/S(H) is isomorphic to a subgroup of Aut (PSL (3, 3)) with (LS(H))/S(H) corresponding to  $\Im nn(PSL (3, 3))$  and since  $|Aut (PSL (3, 3))/\Im nn(PSL (3, 3))| = 2$ , we have  $|H/(S(H)L)| \leq 2$  and  $H^{(\infty)} = L$ .

Let  $S \in \operatorname{Syl}_2(H)$  and  $T = S \cap L$ . Then  $T \triangleleft S$ ,  $T \in \operatorname{Syl}_2(L)$ ,  $|T| = 2^4$ , T is semidihedral and  $T = \langle \lambda, y | |y| = 8$ , |y| = 2 and  $\lambda^y = \lambda^s \rangle$  for suitable elements  $\lambda$ , y of T. Also  $\Phi(T) = T' = \langle \lambda^2 \rangle \cong Z_4$  and  $\Omega_1(T') =$   $Z(T) = \langle z \rangle$  for an involution z of T. Also  $D = \langle \lambda^2, y \rangle \cong D_8$ ,  $Q = \langle \lambda^2, \lambda y \rangle \cong Q_8$  and  $\langle \lambda \rangle \cong Z_8$  are the three distinct maximal subgroups of T. Let  $P = S \cap S(H)$ . Then  $P \trianglelefteq S$ , P is cyclic,  $P \cap T = 1$  and  $\Omega_1(P) = \langle t \rangle$ . Also  $\mathscr{I}(L) = z^L$ ,  $C_{L/O(L)}(z) \cong \operatorname{GL}(2, 3)$  and S(H) = O(H)P. Since  $r_2(S) \le 1 + r_2(S/P) \le 2 + r_2(T) = 4$ , we have  $S \notin \operatorname{Syl}_2(G)$ .

LEMMA 2.1. The following four conditions hold:

(a) |H/(S(H)L)| = 2 and  $H/S(H) \cong \text{Aut}(\text{PSL}(3, 3));$ 

(b) there is an involution  $u \in S - (P \times T)$  such that  $D = C_T(u) \in Syl_2(C_L(u))$ ,  $L\langle u \rangle / O(L) \cong Aut (PSL(3, 3))$ ,  $\mathscr{I}(uL) = u^L$ ,  $C_{L/O(L)}(u) = (O(L)C_L(u))/O(L)$ ,  $O(C_L(u)) = O(L) \cap C_L(u)$ ,  $C_L(u)/O(C_L(u)) \cong PGL(2, 3)$ ,  $O^2(C_G(\langle t, u \rangle))/O(C_G(\langle t, u \rangle)) \cong PSL(2, 3)$ ,  $S = (P \times T)\langle u \rangle$ ,  $\lambda^u = \lambda z$  and  $C_{T\langle u \rangle}(\langle z, y, u \rangle) = \langle z, y, u \rangle$ ;

(c)  $Z(S) = \langle t, z \rangle$ ,  $P\langle u \rangle$  is dihedral or semidihedral and  $S \in Syl_2(C_d(t, z))$ ; and

(d)  $Q = \langle \lambda^2, \lambda y \rangle \in \operatorname{Syl}_2(O^2(C_G(\langle t, z \rangle)), C_{O(H)}(z) = O(C_G(\langle t, z \rangle)) = O(O^2(C_G(\langle t, z \rangle))) \text{ and } O^2(C_G(\langle t, z \rangle))/O(C_G(\langle t, z \rangle)) \cong \operatorname{SL}(2, 3).$ 

*Proof.* Assume that H = S(H)L. Then  $S = P \times T$  and Z(S) = $P \times \langle z \rangle$ . Since  $S \notin \operatorname{Syl}_2(G)$ , we have  $P = \langle t \rangle$ . Then  $\langle t, y, z \rangle \in$  $\operatorname{Syl}_2(C_G\langle t, y, z \rangle)$  and [11, Theorem 2] implies that  $r_2(G) \leq 4$ . This contradiction implies that (a) holds. For the proofs of (b) and (c) of this lemma, it clearly suffices to assume that O(H) = 1. Then  $P = O_2(H) = C_H(L), H/P \cong \text{Aut}(\text{PSL}(3, 3))$  and there is an element  $v \in S - (P \times T)$  such that  $v^2 \in P$ ,  $C_T(v) = D$  and  $C_L(v) \cong \sum 4$  by [6, Table 4]. Thus  $S = (P \times T) \langle v \rangle$ . Suppose that  $\Omega_1(S) \leq P \times T$ . Then  $\Omega_1(S) = \langle t \rangle \times D \text{ char } S, C_s(\Omega_1(S)) = (P \times \langle z \rangle) \langle v \rangle \text{ char } S \text{ and } \langle t \rangle \text{ char } S.$ Since this is impossible, there is an involution  $w \in S - (P \times T)$ . Then  $L\langle w \rangle \cong$  Aut (PSL(3, 3)) since  $(T\langle w \rangle) \cap P = 1$  and  $T\langle w \rangle \in$  $\operatorname{Syl}_2(L\langle w \rangle)$ . Then, as is well known  $\mathscr{I}(wL) = w^L$  and there is an involution  $u \in Tw$  such that  $C_T(u) = D \in Syl_2(C_L(u)), C_L(u) \cong \sum 4, S =$  $(P \times T)\langle u \rangle$  and  $C_{T \langle u \rangle}(\langle z, y, u \rangle) = \langle z, y, u \rangle$ . Also  $u \in N_G(\langle \lambda \rangle)$  and  $C_{\langle \lambda \rangle}(u) = \langle \lambda^2 \rangle$ . Thus  $\lambda^u = \lambda z$  and (b) holds. Hence  $Z(T\langle u \rangle) = \langle z \rangle$ .  $\langle t, z \rangle \leq Z(S) = C_P(u) \times \langle z \rangle$  and (c) holds since  $\langle t \rangle$  is not characteristic in S. For (d) observe that  $C_{d}(\langle t, z \rangle) = C_{H}(z)$  and set  $\overline{H} = H/O(H)$ . Then  $C_{\overline{H}}(\overline{z}) = \overline{C_H(z)}$  and  $\overline{z} \in O^{\circ}(\overline{H}) = \overline{L} \cong PSL(3, 3)$ . But  $O^{\circ}(C_{\overline{H}}(\overline{z})) =$  $O^2(C_{\overline{L}}(\overline{z})) \cong \operatorname{SL}(2, 3), \ \overline{Q} \in \operatorname{Syl}_2(O^2(C_{\overline{H}}(\overline{z}))) \text{ and } O^2(C_{\overline{H}}(\overline{z})) = O^2(\overline{C_H(z)}) =$  $\overline{O^2(C_H(z))} \cong \mathrm{SL}(2,3).$  Hence  $O(H)Q \leq O(H)O^2(C_H(z)).$ 

$$Q \leqq C_{{\scriptscriptstyle O}({\scriptscriptstyle H})}({\it z}) O^{\scriptscriptstyle 2}(C_{{\scriptscriptstyle H}}({\it z})) = O^{\scriptscriptstyle 2}(C_{{\scriptscriptstyle H}}({\it z}))$$
 ,

(d) holds and we are done.

LEMMA 2.2.  $P = \langle t \rangle$ , t is not a square in  $G, S = \langle t \rangle \times \langle T \langle u \rangle \rangle$ ,

 $|S| = 2^{6}, S' = \langle \lambda^{2} \rangle, \langle z \rangle \leq N_{G}(S) \text{ and } t \not\sim z \text{ in } G.$ 

**Proof.** Assume that  $P \neq \langle t \rangle$  and let  $w \in \mathscr{I}(S - Z(S))$ . Suppose that  $w \in P \times T$ . Then w is conjugate in  $P \times T$  to an element of  $y\langle t \rangle$ . Since  $C_s(y) = C_s(yt) = (P \times \langle z, y \rangle) \langle u \rangle$ , we have  $\mathcal{Q}_1(C_s(w)') = \langle t \rangle$ . Suppose that  $w \notin P \times T$ . Then  $C_P(w) = \langle t \rangle$ ,  $C_s(w) = \langle t \rangle \times C_T(w) \times \langle w \rangle$ and  $\mathcal{Q}_1(C_s(w)') \leq \langle z \rangle$ . Since  $Z(S) = \langle t, z \rangle$ , we have  $Q \leq N_G(S)$  by Lemma 2.1 (d),  $\langle z \rangle \leq N_G(S)$  and  $t^{N_G(S)} = t \langle z \rangle$ . However  $\langle z \rangle \leq N_G(S)$ implies  $\langle t \rangle \leq N_G(S)$  and we have a contradiction. Thus  $P = \langle t \rangle$  and the lemma is clear.

Since  $\mathscr{I}(uL) = u^L$ , we immediately conclude:

COROLLARY 2.3.  $\{t, z, tz, u, tu\}$  is a complete set of representatives for the H-conjugacy classes of involutions in H. Also  $u\mathscr{I}(D) \subseteq u^{H}$ .

Note that  $T\langle u \rangle = \langle \lambda, yu, u | | yu | = | u | = 2$ , [yu, u] = 1,  $|\lambda| = 2^3$ ,  $\lambda^{yu} = \lambda^{-1}$  and  $\lambda^u = \lambda z$  where  $z = \lambda^4 \rangle$  and hence [12, Lemma 2.1] lists various facts about  $T\langle u \rangle$ .

Let  $x = \lambda^2 y$ . Then  $\mathscr{I}(T) = \mathscr{I}(D) = \{z\} \cup y \langle z \rangle \cup x \langle z \rangle$  and  $y \langle z \rangle \cup x \langle z \rangle = y^T$ . Also  $C_s(y) = \langle t, u \rangle \times \langle z, y \rangle$ ,  $C_s(x) = \langle t, u \rangle \times \langle z, x \rangle$ ,  $m_2(\langle t \rangle \times T) = 3$  and  $\mathscr{C}_s(\langle t \rangle \times T) = \{\langle t, z, y \rangle, \langle t, z, x \rangle\}$ . Hence  $m_2(S) = 4$  and  $\mathscr{C}_{16}(S) = \{\langle t, u, z, y \rangle, \langle t, u, z, x \rangle\}$ . Note also that  $u^s = u^T = u \langle z \rangle$  and  $\exp(S) = 8$ .

Set  $A = \langle t, u, z, y \rangle$  and  $B = \langle t, u, z, x \rangle$ . Then  $\mathscr{C}_{16}(S) = \{A, B\}$ ,  $A \sim B \operatorname{via} T, \langle A, B \rangle = \langle t, u \rangle \times D \operatorname{char} S, N_{S}(A) = N_{S}(B) = \langle t, u \rangle \times D,$  $C_{G}(A) = O(C_{G}(A)) \times A, C_{G}(B) = O(C_{G}(B)) \times B \text{ and } N_{G}(S) = S(N_{G}(S) \cap N_{G}(A) \cap N_{G}(B)).$ 

Let  $X = \langle t, u, z \rangle$ . Clearly  $C_s(X) = \langle t, u \rangle \times D$ .

LEMMA 2.4. X is the unique element Y of  $\mathscr{C}(S)$  such that  $Y \leq S$  and |Y| > 4.

*Proof.* Let  $Y \in \mathscr{C}(S)$  satisfy  $Y \leq S$  and |Y| > 4. Then we may assume that  $Z(S) = \langle t, z \rangle \leq Y$  and  $|Y| = 2^3$ . Then  $E_4 \simeq Y \cap (T\langle u \rangle) = \langle z, \tau \rangle$  where  $\tau \in \mathscr{I}(T\langle u \rangle)$  and  $[\langle \lambda \rangle, \tau] \leq \langle z \rangle$ . This forces  $Y \cap (T\langle u \rangle) = \langle z, u \rangle$  and we are done.

Set  $M = N_{d}(A)$  and  $\overline{M} = M/O(M)$ . Clearly  $C_{d}(A) = O(M) \times A$ and, interchanging u and uz if necessary, there is a 3-element  $\rho \in C_{H}(u) \cap N_{L}(A)$  such that x inverts  $\rho$ ,  $C_{A}(\rho) = \langle t, u \rangle$ ,  $[A, \rho] = \langle z, y \rangle$ and  $\rho^{3} \in O(M)$ . Also  $C_{\overline{u}}(\overline{t}) = \overline{C_{u}(t)} = \overline{A} \langle \overline{\rho}, \overline{x} \rangle = \langle \overline{t}, \overline{u} \rangle \times \langle \overline{y}, \overline{z}, \overline{\rho}, \overline{x} \rangle$ with  $\langle \overline{y}, \overline{z}, \overline{\rho}, \overline{x} \rangle \cong \sum 4$ ,  $C_{\overline{u}}(\overline{A}) = \overline{A}$  and  $\overline{M}/\overline{A} \hookrightarrow \operatorname{Aut}(A) \cong \operatorname{GL}(4, 2) \cong A_{s}$ . Moreover, it is clear that  $O^{2}(C_{d}(\langle t, u \rangle)) = O(C_{d}(\langle t, u \rangle)) \langle y, z, \rho \rangle$ ,  $\langle y, z \rangle \in \operatorname{Syl}_{2}(O^{2}(C_{d}(\langle t, u \rangle)))$  and  $O^{2}(C_{d}(\langle t, u \rangle))/O(C_{d}(\langle t, u \rangle)) \cong \operatorname{PSL}(2, 3)$ . LEMMA 2.5.  $M = N_G(A)$  controls the G-fusion of elements in  $t^{\sigma} \cap A$ .

**Proof.** Assume that  $t^g \in A$  for  $g \in G$ . Let  $A < S_1 \in \operatorname{Syl}_2(C_G(t^g))$ . Since  $S^g \in \operatorname{Syl}_2(C_G(t^g))$ , we may assume that  $S^g = S_1$ . If  $A^g = A$ , then  $g \in M$ . Suppose that  $A^g \neq A$ . Then  $\mathscr{C}_{16}(S_1) = \{A, A^g\}$  and there is an element  $h \in S_1$  such that  $A^{gh} = A$ . Then  $gh \in M$ ,  $t^g = t^{gh}$  and the lemma holds.

Let  $S \leq \mathscr{S} \in \operatorname{Syl}_2(G)$ . Then  $S \neq \mathscr{S}$ ,  $|\mathscr{S}| > 2^{10}$  and  $S < N_{\mathscr{S}}(S)$ . Since  $Z(S) \leq N_G(S)$  and  $\langle z \rangle \leq N_G(S)$ , we have  $|N_{\mathscr{S}}(S)/S| = 2$ ,  $t^{N_{\mathscr{S}}(S)} = t\langle z \rangle$  and  $Z(N_{\mathscr{S}}(S)) = \langle z \rangle = Z(\mathscr{S})$ .

Clearly  $O(C_{d}(S)) = O(N_{d}(S)) \times \langle t, z \rangle$  and if  $\pi$  is an element of odd order of  $N_{d}(S)$ , then  $\pi \in C_{d}(\langle t, z \rangle), \pi \in C_{d}(X), \pi \in C_{d}(\langle t, u \rangle \times D)$  and hence  $\pi \in O(N_{d}(S))$ . Thus  $N_{d}(S) = O(N_{d}(S))N_{\mathscr{S}}(S)$ .

As in [12, § 4], we have  $SCN_{5}(\mathscr{S}) = \phi$  and there is an element  $E \in \mathscr{C}_{8}(\mathscr{S})$  such that  $E \trianglelefteq \mathscr{S}$ . Clearly  $z \in E$ ,  $|C_{E}(t)| \ge 4$  and  $z \in C_{E}(t) \trianglelefteq S = C_{\mathscr{S}}(t)$ . Suppose that  $\tau \in t^{g} \cap E$ . Then  $|\mathscr{S}| = |\tau^{\mathscr{S}}| |C_{\mathscr{S}}(\tau)| \le 2^{2} \cdot |S| = 2^{8}$ . Thus  $t^{g} \cap E = \phi$ ,  $t \notin C_{E}(t)$ ,  $|C_{E}(t)| = 4$ ,  $\langle t, C_{E}(t) \rangle = X = \langle t, y, z \rangle$ ,  $[S, E] \le E \cap S = C_{E}(t)$ ,  $N_{\mathscr{S}}(S) = SE$  and  $t^{E} = t \langle z \rangle$ . Interchanging u and tu if necessary, it follows that we may assume that  $C_{E}(t) = \langle u, z \rangle$ .

Set  $F = \langle y, z \rangle$ . Then  $A = F \cup tF \cup uF \cup tuF$ ,  $tF \subseteq t^{c} \cap A$ ,  $t^{d} \cap (F \cup uF) = \phi$  and  $tF \subseteq t^{c} \cap A \subseteq tF \cup tuF$ . Consequently:

COROLLARY 2.6. Either  $t^{\scriptscriptstyle M} = t^{\scriptscriptstyle G} \cap A = tF$  and  $|\bar{M}/\bar{A}| = 24$  or  $t^{\scriptscriptstyle M} = t^{\scriptscriptstyle G} \cap A = tF \cup tuF$  and  $|\bar{M}/\bar{A}| = 48$ .

Now the analyses of [12, §5-11], with the obvious slight changes, shows that  $|O^{2}(G)|_{2} \leq 2^{10}$ . Since  $F^{*}(G) \leq O^{2}(G)$ , our proof of Theorem A is complete.

#### References

1. M. Aschbacher, A characterization of Chevalley groups over fields of odd order, Ann. of Math., **106** (1977), 353-398.

2. M. Aschbacher and G. M. Seitz, Involutions in Chevalley groups over fields of even order, Nagoya Math. J., 63 (1976), 1-91.

3. \_\_\_\_\_, On groups with a standard component of known type, Osaka J. Math., 13 (1976), 439-482.

4. B. Beisiegel, Über einfache gruppen mit Sylow 2-gruppen der ordnung höchstens  $2^{10}$ , Comm. in Alg., 5 (1977), 113-170.

5. H. Bender, On groups with Abelian Sylow 2-subgroups, Math. Z., 117 (1970), 164-176.

6. N. Burgoyne and C. Williamson, Centralizers of involutions in Chevalley groups of odd characteristic, unpublished dittoed notes.

7. F. J. Fritz, On centralizers of involutions with components of 2-rank two, I, II, J. Alg., 47 (1977), 323-399.

8. D. Gorenstein, Finite Groups, Harper and Row, New York, 1968.

9. D. Gorenstein and K. Harada, Finite groups whose 2-subgroups are generated by at most 4-elements, Mem. Amer. Math. Soc. No. 147, Amer, Math. Soc., Providence, R. I., 1974.

10. D. Gorenstein and J. H. Walter, Balance and generation in finite groups, J. Alg., 33 (1975), 224-287.

11. K. Harada, On finite groups having self-centralizing 2-subgroups of small order, J. Alg., 33 (1975), 144-160.

12. M. E. Harris, Finite groups having on involution centralizer with a 2-component of dihedral type, II, Illinois J. Math., 21 (1977), 621-647.

13. M. E. Harris, On PSL (2, q)-type 2-components and the unbalanced group conjecture, to appear.

14. M. E. Harris and R. Solomon, Finite groups having an involution centralizer with a 2-component of dihedral type, I, Illinois J. Math., 21 (1977), 575-620.

15. A. MacWilliams, On 2-groups with no normal abelian subgroups of rank 3, and their occurrence as Sylow 2-subgroups of finite simple groups, Trans. Amer. Math. Soc., **150** (1970), 345-408.

Received June 21, 1978. This research was partially supported by a National Science Foundation Grant.

UNIVERSITY OF MINNESOTA MINNEAPOLIS, MN 55455

#### PACIFIC JOURNAL OF MATHEMATICS

#### EDITORS

DONALD BABBITT (Managing Editor) University of Galifornia Los Angeles, California 90024

HUGO ROSSI University of Utah Salt Lake City, UT 84112 J. Dugundji

Department of Mathematics University of Southern California Los Angeles, California 90007

R. FINN AND J. MILGRAM Stanford University Stanford, California 94305

C. C. MOORE AND ANDREW OGG

University of California Berkeley, CA 94720

#### ASSOCIATE EDITORS

E. F. BECKENBACH

B. H. NEUMANN F. WOLF

K. YOSHIDA

#### SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA CALIFORNIA INSTITUTE OF TECHNOLOGY UNIVERSITY OF CALIFORNIA MONTANA STATE UNIVERSITY UNIVERSITY OF NEVADA, RENO NEW MEXICO STATE UNIVERSITY OREGON STATE UNIVERSITY UNIVERSITY OF OREGON UNIVERSITY OF SOUTHERN CALIFONIA STANFORD UNIVERSITY UNIVERSITY OF HAWAII UNIVERSITY OF TOKYO UNIVERSITY OF UTAH WASHINGTON STATE UNIVERSITY UNIVERSITY OF WASHINGTON

Printed in Japan by International Academic Printing Co., Ltd., Tokyo, Japan

# Pacific Journal of MathematicsVol. 87, No. 1January, 1980

Spiros Argyros, <i>A decomposition of complete Boolean algebras</i>	1
Gerald A. Beer, <i>The approximation of upper semicontinuous multifunctions</i> <i>by step multifunctions</i>	11
Ehrhard Behrends and Richard Evans, <i>Multiplicity theory for Boolean</i>	
algebras of $L^p$ -projections	21
Man-Duen Choi, The full C*-algebra of the free group on two	
generators	41
Jen-Chung Chuan, Axioms for closed left ideals in a C*-algebra	49
Jo-Ann Deborah Cohen, <i>The strong approximation theorem and locally</i>	
bounded topologies on $F(X)$	59
Eugene Harrison Gover and Mark Bernard Ramras, <i>Increasing sequences of</i>	
Betti numbers	65
Morton Edward Harris, Finite groups having an involution centralizer with	
a 2-component of type PSL(3, 3)	69
Valéria Botelho de Magalhães Iório, <i>Hopf C*-algebras and locally compact</i>	
groups	75
Roy Andrew Johnson, Nearly Borel sets and product measures	97
Lowell Edwin Jones, <i>Construction of Z<sub>p</sub>-actions on manifolds</i>	111
Manuel Lerman and Robert Irving Soare, <i>d-simple sets, small sets, and</i>	
degree classes	135
Philip W. McCartney, <i>Neighborly bushes and the Radon-Nikodým property</i>	
for Banach spaces	157
Robert Colman McOwen, Fredholm theory of partial differential equations	1.50
on complete Riemannian manifolds	169
Ernest A. Michael and Carl Preston Pixley, <i>A unified theorem on continuous</i>	107
selections	187
Ernest A. Michael, <i>Continuous selections and finite-dimensional sets</i>	189
Vassili Nestoridis, Inner functions: noninvariant connected	100
components	199
Bun Wong, A maximum principle on Clifford torus and nonexistence of	011
proper holomorphic map from the ball to polydisc	211
Steve Wright, <i>Similarity orbits of approximately finite C</i> *-algebras	223
Kenjiro Yanagi, On some fixed point theorems for multivalued	222
mappings	233
Wieslaw Zelazko, A characterization of LC-nonremovable ideals in	2.41
commutative Banach algebras	241