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A general Tauberian theorem is given that can be applied to any regular matrix summability method. The Tauberian condition is determined by the lengths of the blocks of consecutive terms that dominate the rows of the matrix.

The Tauberian theorems given in [1] are given for real matrices only, when if fact they hold for general matrices with complex entries. Using the notation and terminology of [1], we recall that the matrix A is $\{B_n\}$-dominated if

$$(1)\qquad \liminf_n\{|\sum_{k\in B_n} a_{nk}| - \sum_{k\notin B_n} |a_{nk}|\} > 0\ ,$$

where each B_n is a block of consecutive column indices in the nth row of A. Let L_n denote the length of B_n.

THEOREM. *Suppose that A is a regular matrix that is $\{B_n\}$-dominated; if x is a bounded sequence such that Ax is convergent and*

$$\max_{k\in B_n} |(\Delta x)_k| = o(L_n^{-1})\ ,$$

then x is convergent.

Proof. We assume that x is bounded but nonconvergent, and we shall show that no complex number r can be the limit of Ax. Define $R = \limsup_k|x_k - r|$ and proceed as in the proof of Theorem 1 of [1] to show that if $0 < \varepsilon < R$, then

$$(2)\qquad |(Ax)_n - r| \geqq o(1) + |\sum_{k\in B_n} a_{nk}(x_k - r)| - R\sum_{k\notin B_n} |a_{nk}| - \varepsilon\,||A||\ .$$

Next select a subsequence $\{x_{k(i)}\}$ such that $\lim_i(x_{k(i)} - r) = \rho$, where $|\rho| = R$. We now assert that for every j there is an $n(j)$ such that

$$(3)\qquad k\in B_{n(j)} \quad \text{implies} \quad |x_k - r - \rho| < 1/j\ .$$

To prove this, first choose N so that $L_n \max_{i\in B_n} |(\Delta x)_i| < 1/2j$ whenever $n > N$; then select some $n(j)$ greater than N for which $B_{n(j)}$ contains an integer $k(i)$ satisfying $|x_{k(i)} - r - \rho| < 1/2j$. For any k in $B_{n(j)}$, we have

$$|x_k - x_{k(i)}| \leqq L_n \max_{p \in B_n(j)} |(\Delta x)_p| < 1/2j ,$$

so (3) follows from the triangle inequality. We can also assume that $\{n(j)\}$ is chosen so that the block sums converge, say

$$\lim_j \{\sum_{k \in B_n(j)} a_{n(j),k}\} = S . \tag{4}$$

Consider the matrix C given by

$$c_{jk} = \begin{cases} a_{n(j),k}, & \text{if } k \in B_{n(j)} , \\ 0, & \text{otherwise} . \end{cases}$$

This is a multiplicative matrix, so (3) and (4) yield

$$\lim_j \{\sum_{k \in B_n(j)} a_{n(j),k}(x_k - r)\} = \rho S . \tag{5}$$

Now it follows from (2) and (5) that for j sufficiently large,

$$\begin{aligned} |(Ax)_{n(j)} - r| &\geqq o(1) + (R - \varepsilon)| \sum_{k \in B_n(j)} a_{n(j),k}| \\ &\quad -R \sum_{k \in B_n(j)} |a_{n(j),k}| - \varepsilon ||A|| \\ &\geqq o(1) - 2\varepsilon ||A|| + R\{| \sum_{k \in B_n(j)} a_{n(j),k}| - \sum_{k \notin B_n(j)} |a_{n(j),k}|\} . \end{aligned}$$

Since ε is an arbitrarily small positive number, it follows from (1) that $\limsup_n |(Ax)_n - r| > 0$. Hence, Ax cannot have limit r, so it is nonconvergent.

From the preceding theorem, one can immediately deduce that all of the Tauberian results of [1, § 2] can be extended to the more general case of complex matrices.

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REFERENCE

1. J. A. Fridy, *Tauberian theorems via block dominated matrices,* Pacific J. Math., 81, No. 2 (1979).

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