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The main theorem of this paper is that topological manifolds modelled on $R^\infty = \varinjlim R^n$ are stable. Combined with previous work this theorem enables us to embed R^∞-manifolds as open subsets of R^∞, classify R^∞-manifolds by homotopy type, and triangulate R^∞-manifolds.

The results established here were announced in the [8].

1. Definitions and results. Let R^n be the cartesian product of n copies of R, where R denotes the reals. Define $i_n: R^n \to R^{n+1}$ by $i_n((x_1, \cdots, x_n)) = (x_1, \cdots, x_n, 0)$. Then $R^\infty = \varinjlim \{R^n; i_n\}$. We regard R^∞ as the set $\{(x_1, x_2, x_3, \cdots,) \mid x_i \in R$, all i, and $x_i \neq 0$ for at most finitely many $i\}$. We identify R^n with $R^n \times \{(0, 0, \cdots, 0)\} \subset R^{n+k}$, $k \geqq 1$, and with $R^n \times \{(0, 0, \cdots)\} \subset R^\infty$. With this identification, a set $\mathscr{O} \subset R^\infty$ is open if and only if $\mathscr{O} \cap R^n$ is open in R^n, $n \geqq 1$. In the terminology of [14], for example, R^∞ is thus the strict inductive limit of $\{R^n\}$. As such it is a locally convex [14, Prop. 1, p. 127], nonmetrizable [14, Prop. 5, p. 129] topological vector space having a natural simplicial structure.

A topological manifold modelled on R^∞, or, more simply, an R^∞-manifold, is a Hausdorff space in which each point has a neighborhood homeomorphic to an open subset of R^∞. By way of example we note that countable direct limits of finite-dimensional manifold are often R^∞-manifolds. Also by [9, Corollary 2], if X is a locally finite polyhedron (more generally, a locally compact, locally finite-dimensional ANR) then $X \times R^\infty$ is an R^∞-manifold. Our main result is Theorem S, below, which asserts that R^∞-manifolds are stable with respect to multiplication by R^∞. We remark that because R^∞ is nonmetrizable and not a countable product (one can show that R^∞ is not homeomorphic to $R^\infty \times R^\infty \times R^\infty \times \cdots$) many of the arguments used in establishing stability of Hilbert space and Hilbert cube manifolds as, for example, in [1] and [16] do not apply here. Our proof uses an inductive argument on finite-dimensional subsets.

By "$\cong$" we denote "is homeomorphic to". We let $I = [0, 1]$. If $\mathscr{U}$ is an open cover of the space Y, two maps $f, g: X\ \ Y$ are *$\mathscr{U}$-close* if for each $x \in X$ there is a $U \in \mathscr{U}$ such that $\{f(x), g(x)\} \subset U$. A map $f: X \to Y$ is a *near homeomorphism* if for each open cover $\mathscr{U}$ of Y there is a homeomorphism $h: X \to Y$ such that f and h are $\mathscr{U}$-close.

For the remainder of this section let M and N denote paracompact, connected R^∞-manifolds.

THEOREM S (Stability). *The projection map $M \times R^\infty \to M$ is a near-homeomorphism. In particular $M \times R^\infty \cong M$.*

The proof of the stability theorem is given in § 3 of this paper.

In [7] it was shown that $M \times R^\infty$ embeds as an open subset of R^∞. Combined with Theorem S this immediately implies the open embedding theorem for R^∞-manifolds.

THEOREM $\mathscr{O}$ (Open Embedding). *There is an open embedding $f: M \to R^\infty$.*

Using Theorem $\mathscr{O}$, regard M as an open subset of R^∞. Let $\mathscr{C}$ be an open cover of M consisting of convex sets. By Theorem S there is a homeomorphism $h: M \times R^\infty \to M$ which is $\mathscr{C}$-close to the projection. Clearly, then, $H: M \times R^\infty \times I \to M$ defined by $H((m, x, t)) = (1 - t)h((m, x)) + tm$ is a homotopy in M, and the following corollary results.

COROLLARY 1. *There is a homeomorphism $h: M \times R^\infty \to M$ which is homotopic to the projection map.*

Let $f: M \to N$ be a homotopy equivalence. By [7, Theorem II-9] $(f \times \mathrm{id}): M \times R^\infty \to N \times R^\infty$ is homotopic to a homeomorphism g. Let $h_M: M \times R^\infty \to M$ and $h_N: N \times R^\infty \to N$ be homeomorphisms homotopic to the corresponding projection maps. Then $h_N g h_M^{-1}$ is a homeomorphism homotopic to f, and we have proven the following classification theorem.

THEOREM C (Classification by Homotopy Type). *If $f: M \to N$ is a homotopy equivalence, then f is homotopic to a homeomorphism $h: M \to N$.*

Since R^∞-manifolds have the homotopy type of ANR's [7, Theorem II-10], Theorem C has the following corollary.

COROLLARY 2. *If M and N have the same weak homotopy type, then they are homeomorphic.*

In [4] Dobrowolski obtains a special case of Corollary 2; namely, that $R^\infty \cong \varinjlim S^n$, where S^n is the n-sphere. He obtains this result by first showing that compact subsets of $\varinjlim S^n$ are negligible.

Using Theorems $\mathscr{O}$ and C we can now triangulate M. By Theorem $\mathscr{O}$ we may regard M as an open subset of R^∞. Since open subsets of R^∞ are Lindelöf [7, Propositions III. 1 and III. 2] M has the homotopy type of a countable, locally finite, simplicial complex K [13, Theorem 1 and Proposition 2]. By [9, Corollary 2] $|K| \times R^\infty$ is an R^∞-manifold, and clearly, $|K| \times R^\infty$ has the same homotopy type as M. By Theorem C, $M \cong |K| \times R^\infty$. This establishes the triangulation theorem.

THEOREM T (Triangulation). $M \cong |K| \times R^\infty$, *where K is a countable, locally-finite simplicial complex.*

We remark that Theorems S and T answer affirmatively two questions in the Appendix "Open problems of infinite-dimensional topology" in [3].

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§2. **Lemmas.** Recall that we identify R^n with $R^n \times \{0, 0, \cdots, 0\} \subset R^{n+k}$ and with $R^n \times \{0, 0, \cdots\} \subset R^\infty$. If $A \subset R^\infty$, let $A^n = A \cap R^n$. Let d_n be the metric induced on R^n by the norm $\|x\| = (\sum x_i^2)^{1/2}$. If $\mathscr{U}$ is an open cover of Y, a homotopy $H: X \times I \to Y$ is *limited* by $\mathscr{U}$ if for every $x \in X$, $H(\{x\} \times I) \subset U$, some $U \in \mathscr{U}$. We abbreviate "finite-dimensional" by f.d. and "piecewise linear" by p.l. If $A \subset X$, by $\bar{A}$ we denote the closure of A in X.

LEMMA 1. *Let A and B be* f.d. *compact metric spaces with $A \subset B$. Let $f: B \to R^n$ be a continuous map such that f/A is an embedding. Then if m is sufficiently large, for every $\varepsilon > 0$ there is an embedding $g_\varepsilon: B \to R^m$ such that $g_\varepsilon/A = f/A$ and $d_m(f, g_\varepsilon) < \varepsilon$.*

Proof. We may assume that $2(\dim B) + 1 \leqq n$ so that there is an embedding $\alpha: B \to R^n$. Let $\beta: R^n \to R^n$ be a continuous extension of $\alpha f^{-1}: f(A) \to R^n$. Define $h: B \to R^n \times R^n$ by $h(b) = (f(b), \alpha(b))$ and $T: R^n \times R^n \to R^n \times R^n$ by $T(x, y) = (x, y - \beta(x))$. Then $g = Th$: $B \to R^n \times R^n$ is an embedding extending f/A. Choose $r > 0$ such that $g(B) \subset R^n \times \{z \in R^n \mid \|z\| \leqq r\}$. If $e(x, y) = (x, (\varepsilon/r)y)$, then $g_\varepsilon = eg$ is the desired embedding.

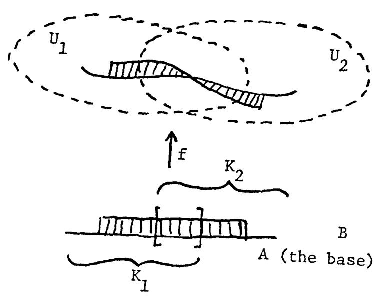
LEMMA 2. *Let X be a* f.d. *locally compact metric space and A and B closed sets in X such that $X = A \cup B$ and B is compact. Let U be an open subset of R^∞, $\mathscr{U}$ an open cover of U. Let $f: X \to U$ be a continuous map such that f/A is a closed embedding. Then there is an embedding g: $X \to U$ such that $g/A = f/A$ and such that*

$H: X \times I \to U$ defined by $H(x, t) = (1-t)f(x) + tg(x)$ is limited by $\mathscr{U}$.

Proof. If C is a compact subset of U, then $f^{-1}(C)$ is contained in the compact set $(f/A)^{-1}(C) \cup B$. Hence f is proper. Thus, we can choose a relatively compact neighborhood V of the compact set $f^{-1}(f(B))$ in locally compact X.

Let n be such that $f(\bar{V}) \subset U \cap R^n$. By Lemma 1 there is an $m > n$ and an embedding $g_\varepsilon: \bar{V} \to R^m$ with $g_\varepsilon(x) = f(x)$ for $x \in A \cap \bar{V}$ and $d_m(g_\varepsilon, f/\bar{V}) < \varepsilon$, where $\varepsilon > 0$ is chosen smaller than $d_m(f(B), f(A \backslash V) \cap R^m)$ and such that the ε-neighborhood in R^m of any point of $f(\bar{V})$ is contained in a member of $\{W \cap R^m \mid W \in \mathscr{U}\}$. Define $g: X \to U$ by $g(x) = f(x)$ for $x \in A$ and $g(x) = g_\varepsilon(x)$ for $x \in \bar{V}$. Thus, g is one-to-one. Moreover, g is proper, for the same reason for which f is. It follows that g is the desired embedding.

LEMMA 3. *Let A and B be* f.d. *compact metric spaces with $A \subset B$. Let M be a paracompact space such that $M = U_1 \cup U_2$, where U_i, $i = 1, 2$, is an open subset of M homeomorphic to an open subset of R^∞. Let $f: B \to M$ be a continuous map such that f/A is an embedding. Then there is an embedding $f': B \to M$ such that $f'/A = f/A$.*



Let $\{K_1, K_2\}$ be a cover of B by compact sets such that $K_i \subset f^{-1}(U_i)$, $i = 1, 2$. By Lemma 2 there is an embedding $g_1: K_1 \cup [A \cap f^{-1}(U_1)] \to U_1$ such that $g_1(x) = f(x)$ for $x \in A \cap f^{-1}(U_1)$ and such that $f/K_1 \cup [A \cap f^{-1}(U_1)]$ is homotopic to g_1 by a homotopy H fixed on $A \cap f^{-1}(U_1)$ and limited by $\{U_1 \cap U_2, M \backslash f(K_1 \cap K_2)\}$. Note that $H[(K_1 \cap K_2) \times I] \subset U_1 \cap U_2$. Define $H': [(K_1 \cap K_2) \cup (A \cap K_2)] \to U_2$ by $H'(x, t) = H(x, t)$ for $x \in K_1 \cap K_2$, $t \in I$, and $H'(x, t) = f(x)$, $x \in A \cap K_2$, $t \in I$.

By Dugundji's theorem [5, p. 188], R^∞, and, hence, [10, p. 42], U_2 is an absolute neighborhood extensor for the class of metrizable spaces. It follows, as in the proof of [10, Theorem 2.2, p. 117], that U_2 has the homotopy extension property with respect to metric spaces. Since $H_0' = f/A \cap K_2$ extends by f to all of K_2, H' has an extension $\bar{H}: K_2 \times I \to U_2$. Define $g: B \to U_1 \cup U_2$ by $g/K_1 = g_1/K_1$ and $g/K_2 = \bar{H}_1$. Then g extends f/A.

By Lemma 2 there is an embedding $g_2: g^{-1}(U_2) \to U_2$ such that $g_2(x) = g(x)$ for $x \in g^{-1}(U_2) \cap (A \cup K_1)$. Define $f': B \to U_1 \cup U_2$ by $f'(x) = g_2(x)$ if $x \in g^{-1}(U_2)$ and $f'(x) = g(x)$ otherwise. Then $f'/K_1 = g_1/K_1$ and $f'/K_2 = g_2/K_2$ so that f' is continuous. If $f'(x) = f'(y)$, then either both x and y or neither x nor y is in $(f')^{-1}(U_2) = g^{-1}(U_2)$. In the first case $x = y$ since g_2 is one-to-one. In the latter case $x = y$ since g/K_1 is one-to-one. Clearly $f'/A = f/A$. Thus, f' is the desired embedding.

The last lemma is probably known. We include a proof for completeness.

LEMMA 4. *Let X be a finite polyhedron and M a compact* p.l. *manifold with boundary. If $f, g: X \to \operatorname{Int} M$ are homotopic topological embeddings, then for sufficiently large k there is an ambient isotopy H on $M \times [-1, 1]^k$ such that $H_1(f, 0) = (g, 0): X \to M \times [-1, 1]^k$.*

Proof. Let $H: X \times I \to \operatorname{Int} M$ be a homotopy with $H_0 = f$ and $H_1 = g$. Define $\bar{H}: X \times I \to \operatorname{Int}(M \times [-1, 1]^k)$ by $\bar{H}(x, t) = (H(x, t), t/2, 0, 0, \cdots, 0)$. Then $\bar{H}_0 = (f, 0)$ and $\bar{H}_1 = (g, 1/2, 0, \cdots, 0)$. Clearly it is sufficient to show that $(f, 0)$ and $\bar{H}_1$ are ambient isotopic.

Since $\bar{H}/X \times \{0, 1\}$ is an embedding, Theorem 1 of [2] implies that for sufficiently large k, $\bar{H}/X \times \{0, 1\}$ is ε-tame in $\operatorname{Int}(M \times [-1, 1]^k)$. Thus, there is an ambient isotopy $K_t: M \times [-1, 1]^k \to M \times [-1, 1]^k$ such that $K_t(\bar{H}(X \times I)) \subset \operatorname{Int}(M \times [-1, 1]^k)$, $t \in I$, and such that $K_1\bar{H}/X \times \{0, 1\}$ is a p.l. embedding. Using general position [15, 5.4, p. 61] there is, for sufficiently large k, a p.l. embedding $h: X \times I \to \operatorname{Int}(M \times [-1, 1]^k)$ such that $h/X \times \{0, 1\} = K_1\bar{H}/X \times \{0, 1\}$. By [11, Theorem 1.1, p. 426] there is an ambient isotopy $E_t: M \times [-1, 1]^k \to M \times [-1, 1]^k$ such that $E_1 h_0 = h_1$. Then $K_t^{-1}E_tK_t$ is an ambient isotopy on $M \times [-1, 1]^k$ with $K_1^{-1}E_1K_1(f, 0) = K_1^{-1}E_1K_1\bar{H}_0 = K_1^{-1}E_1h_0 = K_1^{-1}h_1 = \bar{H}_1$, as required.

3. Proof of Theorem S.

We first prove the following weaker version of the stability theorem.

THEOREM S′. *Let M be a paracompact R^∞-manifold such that $M = U \cup V$, where U and V are homeomorphic to open subsets of R^∞. Then there is a homeomorphism $M \to M \times R^\infty$.*

Proof. We first show that M can be suitably expressed as the direct limit of topological manifolds. Let $\gamma\colon U \to U'$ and $\delta\colon V \to V'$ be homeomorphisms onto open subsets of R^∞. Then $U' = \varinjlim C'_n$ where C'_n is a compact metric subspace of $U' \cap R^n$ and where $C'_n \subset \mathrm{Int}_{n+1} C_{n+1}$. Express $V' = \varinjlim D'_n$ similarly. Let $C_n = \delta^{-1}(C'_n)$ and $D_n = \delta^{-1}(D'_n)$.

Fix $n \geqq 1$. Since $C_n \cup D_n$ is a compact f.d. metric space, there is an embedding $\alpha\colon C_n \cup D_n \to R^k$, some k. Since M is an absolute neighborhood extensor for metric spaces ([5, p. 188] and [10, p. 45]), $\alpha^{-1}\colon \alpha(C_n \cup D_n) \to M$ has a continuous extension β to a compact p.l. submanifold N of R^k containing $\alpha(C_n \cup D_n)$. Let $\pi\colon N \times I \to N$ be the projection. Then $\beta\pi\colon N \times I \to M$ is an embedding on $\alpha(C_n \cup D_n) \times \{0\}$. By Lemma 4 there is an embedding $\beta'\colon N \times I \to M$ such that $\beta'(\alpha(C_n \cup D_n) \times \{0\}) = \beta\pi(\alpha(C_n \cup D_n) \times \{0\}) = C_n \cup D_n$. Let $X = \partial(N \times I)$, a closed p.l. manifold. Let $X_n = \beta'(X)$. Note that $X_n \supset C_n \cup D_n$ and, since $M = \varinjlim (C_n \cup D_n)$, $M = \varinjlim X_n$.

Let $A \subset M$ be a compact subspace. Choose an open cover $\{Y_1, Y_2\}$ of M such that $\bar{Y}_1 \subset U$ and $\bar{Y}_2 \subset V$. Then $A = (A \cap \bar{Y}_1) \cup (A \cap \bar{Y}_2)$. The compactness of $A \cap \bar{Y}_1$ and $A \cap \bar{Y}_2$ implies that for some n, $\gamma(A \cap \bar{Y}_1) \subset C'_n$ and $\delta(A \cap \bar{Y}_2) \subset D'_n$ so that $A \subset C_n \cup D_n$. Thus, every compact subspace of M is contained in some X_n.

Now, let $B_n = [-n, n]^n$, $n \geqq 1$. Then $R^\infty = \varinjlim B_n$. Define $j'_{n,k}\colon X_n \times B_k \to M$ by $j'_{n,k}(x, t) = x$. By Lemma 4 there is an embedding $j_{n,k}\colon X_n \times B_k \to M$ such that $j_{n,k}(x, 0) = x$ for each $x \in X_n$.

Let $j_1 = j_{1,1}$. Choose $n_2 > 1$ such that $j_1(X_1 \times B_1) \subset X_{n_2}$. Consider

$$\begin{tikzcd} X_1 \times B_1 \arrow[r, "\alpha_1"] \arrow[d, "j_1"] & X_{n_2} \times B_{k_2} \\ j_1(X_1 \times B_1) \arrow[ur, "i_1"] & \end{tikzcd}$$

where $k_2 > 1$ is yet to be chosen, $i_1(y) = (y, 0)$ and $\alpha_1(x, t) = (x, (t, 0))$. Since B is contractible $i_1 j_1 \sim \alpha_1$ ("$\sim$" denotes "is homotopic to") with the homotopy taking place in Int $(X_{n_2} \times B_{k_2})$. Choose k_2 so large that, by Lemma 4, there is an ambient isotopy F_2 on $X_{n_2} \times B_{k_2}$ such that $(F_2)_1 \alpha_1 = i_1 j_1$. Let $j_2 = j_{n_2,k_2}\colon X_{n_2} \times B_{k_2} \to j_2(X_{n_2} \times B_{k_2})$. Let $h_1 = j_1$ and $h_2 = j_2(F_2)_1$. Consider

$$\begin{tikzcd} X_1 \times B_1 \arrow[r, "\alpha_1"] \arrow[d, "h_1"] & X_{n_2} \times B_{k_2} \arrow[d, "h_2"] \\ j_1(X_1 \times B_1) \arrow[r, "\beta_1"] & j_2(X_{n_2} \times B_{k_2}) \end{tikzcd}$$

where $\beta_1(y) = y$. Since $h_2\alpha_1 = j_2(F_2)_1\alpha_1 = j_2 i_1 j_1 = \beta_1 j_1$, the square commutes. Also, $((y, t), s) \to j_2 F_2((y, t), s)$ defines a homotopy from

j_2 to h_2.

Choose $n_3 > n_2$ such that $j_2(X_{n_2} \times B_{k_2}) \subset X_{n_3}$. Consider

$$\begin{tikzcd} X_{n_2} \times B_{k_2} \arrow[r, "\alpha_2"] \arrow[d, "h_2"] & X_{n_3} \times B_{k_3} \\ j_2(X_{n_2} \times B_{k_2}) \arrow[ur, "i_2"'] & \end{tikzcd}$$

where $k_3 > k_2$ is yet to be chosen, $\alpha_2(x, t) = (x, (t, 0))$ and $i_2(y) = (y, 0)$. Since $j_2 \sim h_2$, we obtain homotopies $i_2h_2 \sim i_2j_2 \sim \alpha_2$ taking place in $\mathrm{Int}\,(X_{n_2} \times B_{k_2})$. By Lemma 4 we may choose k_3 so large that there is an ambient isotopy F_3 on $X_{n_3} \times B_{k_3}$ such that $(F_3)_1\alpha_2 = i_2h_2$. Let $j_3 = j_{n_3,k_3}$ and $h_3 = j_3(F_3)_1$.

Continuing, by induction we obtain for every $r \geqq 1$ a commutative diagram

$$\begin{tikzcd} X_{n_r} \times B_{k_r} \arrow[r, "\alpha_r"] \arrow[d, "h_r"] & X_{n_{r+1}} \times B_{k_{r+1}} \arrow[d, "h_{r+1}"] \\ j_r(X_{n_r} \times B_{k_r}) \arrow[r, "\beta_r"] & j_{r+1}(X_{n_{r+1}} \times B_{k_{r+1}}) \end{tikzcd}$$

where $\alpha_r(x, t) = (x, (t, 0))$, $\beta_r(y) = y$ and h_r is a homeomorphism. Let $D = \varinjlim \{X_{n_r} \times B_{k_r}; \alpha_r\}$ and $E = \varinjlim \{j_r(X_{n_r} \times B_{k_r}); \beta_r\}$. The h_r's induce a homeomorphism $h: D \to E$. As sets clearly $D \equiv M \times R^\infty$ and $E \equiv M$. Since $j_r(X_{n_r} \times B_{n_r}) \supset X_{n_r}$ and $M = \varinjlim X_n$ it follows immediately that $E \cong M$. Also, $M \times R^\infty$ is homeomorphic to an open subset of R^∞ [7, Corollary II-7] and is therefore the direct limit of its compact subsets. If $C \subset M \times R^\infty$ is compact, then $C \subset \pi_1(C) \times \pi_2(C) \subset X_{n_r} \times B_{k_r}$ some r. (Here $\pi_1: M \times R^\infty \to M$ and $\pi_2: M \times R^\infty \to R^\infty$ are the projections.) It follows that $D \cong M \times R^\infty$. Thus, $M \cong M \times R^\infty$, and Theorem S′ is proved.

Theorem S now follows quickly. Let M be a paracompact, connected R^∞-manifold. As shown in [7, Proposition III. 1] every subset of M is paracompact. Say that a paracompact space Z has property P if for every open subset U of Z there is an open embedding $U \to R^\infty$. Then M has property P locally. Let $X = U \cup V \subset M$ where U and V are open in M having property P. By Theorem S′ $X \cong X \times R^\infty$. By [7, Corollary II.7] X has property P. Let $Y = \bigcup_i Y_i$ where Y_i is open in M and has property P, and where $\{Y_i\}$ is discrete. Since M is Lindelöf [7, Proposition III. 1], $\{Y_i\}$ is at most countable, indexed, say, by a subset of the integers. Let $f_i: Y_i \to R^\infty$ be an open embedding. Let $\rho_i: R^\infty \to [(i - 1/3, i + 1/3) \times R \times R \times \cdots] \cap R^\infty$ be a homeomorphism. Then $f: Y \to R^\infty$ defined by $f/Y_i = \rho_i \circ f_i$ is an open embedding, showing that Y has property P. By a theorem of Michael [12, Theorem 3.6] M has property P. That is, there is an open embedding $M \to R^\infty$. By [6, Theorem 1] the projection $\pi: M \times R^\infty \to M$ is then a near homeomorphism. This completes the proof of Theorem S.

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