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**ON LOCAL ISOMETRIES OF FINITELY COMPACT METRIC  
SPACES**

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By local isometries we mean mappings which locally preserve distances. Local isometries which do not increase distances are called nonexpansive local isometries. A few of the main results are:

1. Let  $f$  be a local isometry (nonexpansive local isometry) of a finitely compact metric space  $(M, \rho)$  into itself. If for each (some)  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then there exists a unique decomposition of  $M$  into disjoint open sets,  $M = M'_0 \cup M'_1 \cup \dots$ , such that (i)  $f$  maps  $M'_0$  injectively into itself, and (ii)  $f(M'_{i+1}) \subset M'_i$  for each  $i = 0, 1, \dots$ . Moreover,  $f$  maps  $M'_0$  homeomorphically (isometrically) onto itself.

2. Let  $f$  be a nonexpansive local isometry (local isometry) of a connected (convex) finitely compact metric space  $(M, \rho)$  into itself. If for some  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then  $f$  is an isometry onto.

1. Introduction. Let  $f$  be a mapping of a metric space  $(M, \rho)$  into a metric space  $(N, \sigma)$ . We will call  $f$  a *local isometry* if for each  $z \in M$  there is a neighborhood  $U_z$  of  $z$  such that  $\sigma(f(x), f(y)) = \rho(x, y)$  for all  $x, y \in U_z$ . If  $f$  is a local isometry and also a nonexpansive mapping (i.e.,  $\sigma(f(x), f(y)) \leq \rho(x, y)$  for all  $x, y \in M$ ), we will say that  $f$  is a *nonexpansive local isometry*.

A metric space  $(M, \rho)$  is said to be *finitely compact* [2] if each bounded and closed subset of  $M$  is compact.

The purpose of this paper is to extend the results of the author's paper [4] to those local isometries  $f$  of a finitely compact metric space  $(M, \rho)$  into itself which have the property that for each  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded. In §2 we give some more notation and preliminary lemmas. Section 3 contains the main results. Roughly speaking, the main theorem is: Let  $f$  be a local isometry (nonexpansive local isometry) of a finitely compact metric space  $(M, \rho)$  into itself. If for each (for some)  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then there exists a unique decomposition of  $M$  into disjoint open sets,  $M = M'_0 \cup M'_1 \cup \dots$ , such that (i)  $f$  maps  $M'_0$  injectively into itself, (ii)  $f(M'_i) \subset M'_{i-1}$  for each  $i \geq 1$ . Moreover,  $f$  maps  $M'_0$  homeomorphically (isometrically) onto itself.

It should be noted that open surjective local isometries were studied by Busemann [2], [3], Kirk [5], [6], [7] and Szenthe [8], [9], [10], in the special case where  $(M, \rho)$  is a  $G$ -space (Busemann [2] called them "locally isometric mappings"). In [5] Kirk proved that

if an open local isometry  $f$  of a  $G$ -space  $(M, \rho)$  onto itself has a fixed point, then  $f$  is an isometry (from which it follows that if the isometries of  $(M, \rho)$  onto itself form a transitive group, then each open surjective local isometry is an isometry). Later Kirk [6] proved that if an open local isometry  $f$  of a  $G$ -space  $(M, \rho)$  onto itself has the property that for some  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then  $f$  is an isometry.

In § 4 and § 5 of the present paper, by using the results of § 3, we extend the above results of Kirk to the case of general local isometries of finitely compact metric spaces.

## 2. Preliminaries.

(2.1) DEFINITION. Let  $\rho_i$ ,  $i = 0, 1$ , be metrics on a set  $M$ . We shall say that  $\rho_1$  is *locally identical* with  $\rho_0$  if the identity mapping,  $\text{id}_M$ , of  $M$  is a local isometry of  $(M, \rho_0)$  into  $(M, \rho_1)$ . We shall say that  $\rho_1$  and  $\rho_0$  are *locally identical* if  $\rho_i$  is locally identical with  $\rho_j$ , for all  $i, j = 0, 1$ .

(2.2) DEFINITION. Let  $f$  be a mapping of a metric space  $(M, \rho)$  into itself. Then the function  $\rho_f$  defined by

$$\rho_f(x, y) = \sup_{n \geq 0} \rho(f^n(x), f^n(y)) \quad \text{for all } x, y \in M,$$

(where  $f^0 = \text{id}_M$ ,  $f^{n+1} = f \circ f^n$ ) is called the *induced metric* on  $M$ .

(2.3) REMARKS. (i) Let  $\rho_i$ ,  $i = 0, 1$ , be metrics on a set  $M$  such that  $\rho_1$  and  $\rho_0$  are locally identical. Then  $\rho_1$  and  $\rho_0$  are topologically equivalent. If  $(M, \rho_0)$  is finitely compact and  $\rho_1 \geq \rho_0$ , then  $(M, \rho_1)$  is also finitely compact. If  $f$  is a local isometry of  $(M, \rho_0)$  into itself, then  $f$  is also a local isometry of  $(M, \rho_1)$  into itself.

(ii) Let  $f$  be a mapping of a metric space  $(M, \rho)$  into itself such that for each  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded. Then for each  $x, y \in M$ ,  $\rho_f(x, y) < \infty$ , and hence the induced metric,  $\rho_f$ , is a metric on the set  $M$  such that

- (1)  $\rho_f \geq \rho$ ,
- (2)  $f$  is a nonexpansive mapping of the metric space  $(M, \rho_f)$  into itself, and
- (3)  $\rho_f = \rho$  if and only if  $f$  is a nonexpansive mapping of  $(M, \rho)$  into itself.

In [4] we proved the following theorem ((4.3) of [4]).

(2.4) THEOREM. *Let  $f$  be a local isometry of a compact metric*

space  $(M, \rho)$  into itself. Then there exists a unique decomposition of  $M$  into disjoint open sets,

$$M = M_0^f \cup \cdots \cup M_n^f \quad (0 \leq n),$$

such that (i)  $f(M_0^f) = M_0^f$ , (ii)  $f(M_i^f) \subset M_{i-1}^f$  and  $M_i^f \neq \emptyset$  for each  $i$ ,  $1 \leq i \leq n$ . Moreover, the induced metric  $\rho_f$  is a metric on  $M$  such that  $\rho_f$  and  $\rho$  are locally identical and  $f$  is a nonexpansive local isometry of  $(M, \rho_f)$  into itself which maps  $M_0^f$  isometrically onto itself.

From this theorem we have

(2.5) COROLLARY. Let  $f$  be a one-to-one local isometry of a compact metric space  $(M, \rho)$  into itself. Then  $f(M) = M$ .

*Proof.* If  $f$  is one-to-one, then by (2.4),  $M = M_0^f$  and hence  $f(M) = M$ .

REMARK. If  $f$  is a local isometry of a compact metric space  $(M, \rho)$  into itself and if  $N$  is a compact subset of  $M$  such that  $f(N) \subset N$ , then the restriction of  $f$  to  $N$ ,  $f|_N$ , is also a local isometry. For convenience,  $N = N_0^f \cup \cdots \cup N_{n(N)}^f$  will denote the decomposition of  $N$  defined by (2.4) for  $f|_N$ .

(2.6) PROPOSITION. Let  $f$  be a local isometry of a compact metric space  $(M, \rho)$  into itself. If  $N$  is a compact subset of  $M$  such that  $f(N) \subset N$ , then

$$N_i^f = N \cap M_i^f \quad \text{for each } i = 0, \dots, n(N),$$

where  $n(N) = \max \{i \geq 0: N \cap M_i^f \neq \emptyset\}$ .

*Proof.* By (2.4), it is sufficient only to show that  $f(N \cap M_0^f) = N \cap M_0^f$ . However, it follows from (2.4) that  $f$  maps  $N \cap M_0^f$  isometrically into itself. Hence, by (2.5),  $f(N \cap M_0^f) = N \cap M_0^f$  as desired.

We will need the following.

(2.7) LEMMA. Let  $f$  be a local isometry of a metric space  $(N, \rho)$  into itself. If  $N$  is a compact subset of  $M$ , then there exists a number  $\delta > 0$  such that for each  $z \in N$ ,

$$(4) \quad \rho(f(x), f(y)) = \rho(x, y),$$

for all  $x, y \in S_\rho(z, \delta) = \{p \in M: \rho(z, p) < \delta\}$ .

The straightforward verification of (2.7) is omitted.

The convexity in this paper is to be understood in the sense of Menger (cf. [1, p. 40]). A subset  $N$  of a metric space  $(M, \rho)$  is, accordingly, convex if for each two distinct points  $x, y \in N$ , there exists a point  $z \in N$ ,  $z \neq x, y$ , such that  $\rho(x, y) = \rho(x, z) + \rho(z, y)$ .

Also, we will use

(2.8) LEMMA. *If  $f$  is a local isometry of a convex and complete metric space  $(M, \rho)$  into itself, then  $f$  is a nonexpansive local isometry.*

*Proof.* Let  $x$  and  $y$  be given points of  $M$  such that  $x \neq y$ . Since  $M$  is convex and complete, by a theorem of Menger (cf. [1, p. 41]) there exists a metric segment  $L \subset M$  whose extremities are  $x$  and  $y$ ; that is, a subset isometric to an interval of length  $\rho(x, y)$ . Since  $L$  is compact, it follows that there exists a finite sequence  $z_0, z_1, \dots, z_k$  of points of  $L$  such that  $z_0 = x$ ,  $z_k = y$  and

$$\rho(f(z_i), f(z_{i+1})) = \rho(z_i, z_{i+1}) \quad \text{for each } i = 0, \dots, k-1$$

and

$$\rho(x, y) = \sum_{i=0}^{k-1} \rho(z_i, z_{i+1}).$$

Thus,

$$\rho(f(x), f(y)) \leq \sum_{i=0}^{k-1} \rho(f(z_i), f(z_{i+1})) = \sum_{i=0}^{k-1} \rho(z_i, z_{i+1}) = \rho(x, y).$$

This proves that  $f$  is a nonexpansive mapping, and hence a nonexpansive local isometry.

**3. Local isometries and decomposition theorems.** We shall now prove the following extension of (2.4).

(3.1) THEOREM. *Let  $f$  be a local isometry of a finitely compact metric space  $(M, \rho)$  into itself. If for each  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then there exists a unique decomposition of  $M$  into disjoint open sets,*

$$(5) \quad M = M_0^f \cup M_1^f \cup \dots,$$

such that

$$(6) \quad f \text{ maps } M_0^f \text{ injectively into itself,}$$

$$(7) \quad f(M_i^f) \subset M_{i-1}^f \quad \text{for each } i = 1, 2, \dots.$$

Moreover, the induced metric,  $\rho_f$ , is a metric on  $M$  such that  $\rho_f$  and  $\rho$  are locally identical,  $(M, \rho_f)$  is a finitely compact metric space and  $f$  is a nonexpansive local isometry of  $(M, \rho_f)$  into itself which maps  $M_0^f$  isometrically onto itself.

*Proof.* In the proof, for each  $A \subset M$  and  $\delta > 0$ ,  $S_\rho(A, \delta)$  is the  $\delta$ -ball in  $M$  about  $A$  and  $\text{cl } A$  ( $\text{Int } A$ ) is the closure (interior) of  $A$ . For each  $z \in M$  we denote:  $c(z) = \text{cl } \{f^n(z) : n \geq 0\}$ .

We first define a sequence  $A_n$ ,  $n = 0, 1, \dots$ , of compact subsets of  $M$  such that

$$(8) \quad f(A_n) \subset A_n \quad \text{for each } n = 0, 1, \dots,$$

$$(9) \quad A_n \subset \text{Int } A_{n+1} \quad \text{for each } n = 0, 1, \dots,$$

$$(10) \quad \bigcup_{n=0}^{\infty} A_n = M.$$

For each  $z \in M$ , let  $\delta_z > 0$  be a number defined by (2.7) for the compact set  $c(z)$  and let  $V_z = S_\rho(c(z), \delta_z)$ . Thus, for each  $z \in M$ ,  $V_z$  is an open and bounded subset of  $M$  and using (4) and the fact that  $f(c(z)) \subset c(z)$ , we have  $f(V_z) \subset V_z$ . Since  $(M, \rho)$  has a countable base of neighborhoods, there exists a sequence  $z_n$ ,  $n = 0, 1, \dots$ , of points of  $M$  such that  $\bigcup_{n=0}^{\infty} V_{z_n} = M$ . Define the sets  $A_n$ ,  $n = 0, 1, \dots$ , inductively, as follows:  $A_0 = \text{cl } V_{z_0}$  and  $A_{n+1} = \bigcup_{i=0}^{k(n)} \text{cl } V_{z_i}$ , where  $k(n)$  is an integer such that  $k(n) > n$  and  $A_n \subset \bigcup_{i=0}^{k(n)} V_{z_i}$ . Clearly, the sets  $A_n$ ,  $n = 0, 1, \dots$ , satisfy conditions (8), (9) and (10), and are compact.

It follows now from (2.4), that for each  $n \geq 0$ , there exists a sequence  $(A_n)_i^f$ ,  $i = 0, 1, \dots$ , of disjoint subsets of  $A_n$  such that

$$(11) \quad (A_n)_i^f \cap \text{Int } A_n \text{ is open, for each } i = 0, 1, \dots,$$

$$(12) \quad \bigcup_{i=0}^{\infty} (A_n)_i^f = A_n,$$

$$(13) \quad f \text{ maps } (A_n)_i^f \text{ injectively into itself,}$$

$$(14) \quad f((A_n)_i^f) \subset (A_n)_{i-1}^f, \quad \text{for each } i = 1, 2, \dots$$

By (2.6), we have

$$(15) \quad (A_n)_i^f = A_n \cap (A_{n+1})_i^f, \quad \text{for all } n, i = 0, 1, \dots$$

Now, for each  $i = 0, 1, \dots$ , we define the set  $M_i^f$  as follows:

$$M_i^f = \bigcup_{n=0}^{\infty} (A_n)_i^f.$$

Then, by (15) and the fact that  $(A_n)_i^f$ ,  $i \geq 0$ , are disjoint, the sets  $M_i^f$ ,  $i \geq 0$ , are disjoint. By (9) and (15),

$$(A_n)_i^f \subset (A_{n+1})_i^f \cap \text{Int } A_{n+1} \subset (A_{n+1})_i^f ,$$

hence,

$$M_i^f = \bigcup_{n=0}^{\infty} ((A_{n+1})_i^f \cap \text{Int } A_{n+1}) , \quad \text{for each } i = 0, 1, \dots ,$$

and therefore, by (11), the sets  $M_i^f$ ,  $i \geq 0$ , are open. By (10) and (12),

$$\bigcup_{i=0}^{\infty} M_i^f = \bigcup_{i, n=0}^{\infty} (A_n)_i^f = \bigcup_{n=0}^{\infty} A_n = M ,$$

and it follows from (13), (14) and (15) that the sets  $M_i^f$ ,  $i \geq 0$ , satisfy conditions (6) and (7). This proves the existence of the desired decomposition of  $M$ .

In order to prove the uniqueness, it is sufficient only to show that for each decomposition of  $M$  into disjoint open sets,  $M = \bigcup_{i=0}^{\infty} M_i$ , conditions (6) and (7) imply

$$(16) \quad M_0 = \{z \in M: f(c(z)) = c(z)\} .$$

Let us assume,  $M = \bigcup_{i=0}^{\infty} M_i$  is a decomposition of  $M$  into disjoint open sets, satisfying conditions (6) and (7). If  $z \in M_0$ , then (6) implies that the restriction of  $f$  to  $c(z)$  is a one-to-one local isometry of  $c(z)$  into itself. Since  $c(z)$  is compact, it follows from (2.5) that  $f(c(z)) = c(z)$ . Conversely, if  $z \notin M_0$ , then  $z \in M_n$  for some  $n \geq 1$ . Using (7) and the fact that  $M_i$ ,  $i \geq 0$ , are disjoint and open, we obtain

$$f(c(z)) \subset c(f(z)) \subset M_0 \cup \dots \cup M_{n-1} ,$$

hence  $z \in c(z) \setminus c(f(z))$ , i.e.,  $c(z) \neq c(f(z))$ . Therefore (16) follows as desired.

Finally, by (ii) of (2.3), the induced metric,  $\rho_f$ , is a metric on  $M$  and it follows from (8), (9), (10) and (2.4) that  $\rho_f$  and  $\rho$  are locally identical (cf. also (1)). Hence, by (1) and (i) of (2.3), the metric space  $(M, \rho_f)$  is finitely compact and, by (2),  $f$  is a nonexpansive local isometry of  $(M, \rho_f)$  into itself. It follows from (2.4) and (15) and the definition of  $M_i^f$  that  $f$  maps  $M_i^f$  isometrically onto itself with respect to the metric  $\rho_f$ . This completes the proof.

(3.2) REMARK. Let  $f$  be a nonexpansive mapping of a metric space  $(M, \rho)$  into itself. If for some  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then for each  $x \in M$  the sequence  $\{f^n(x)\}$  is bounded.

Indeed, since  $f$  is nonexpansive, then for all  $x, z \in M$  and each  $i = 0, 1, \dots$ , we have

$$\rho(f^i(x), \{f^n(z)\}) \leq \rho(f^i(x), f^i(z)) \leq \rho(x, z) ,$$

hence, if  $\{f^n(z)\}$  is bounded, then also  $\{f^n(x)\}$  is bounded.

The following theorem is an immediate consequence of (3.1), (3.2) and (3).

(3.3) THEOREM. *Let  $f$  be a nonexpansive local isometry of a finitely compact metric space  $(M, \rho)$  into itself. If for some  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then there exists a unique decomposition of  $M$  into disjoint open sets,*

$$M = M_0^f \cup M_1^f \cup \dots,$$

*such that (i)  $f$  maps  $M_0^f$  injectively into itself, (ii)  $f(M_i^f) \subset M_{i-1}^f$  for each  $i = 1, 2, \dots$ . Moreover,  $f$  maps  $M_0^f$  isometrically onto itself.*

We have the following corollaries

(3.4) COROLLARY. *Let  $f$  be a local isometry of a finitely compact metric space  $(M, \rho)$  into itself. If for each  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then the following are equivalent:*

- (i)  *$f$  is one-to-one,*
- (ii)  *$f$  is a homeomorphism of  $M$  onto itself,*
- (iii)  *$f$  is an isometry with respect to the induced metric  $\rho_f$ .*

*Proof.* The proof follows from (3.1), since each of (i)–(iii) is equivalent to  $M_0^f = M$ .

(3.5) COROLLARY. *Let  $f$  be a nonexpansive local isometry of a finitely compact metric space  $(M, \rho)$  into itself. If for some  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then the following are equivalent:*

- (i)  *$f$  is one-to-one,*
- (ii)  *$f$  is a homeomorphism of  $M$  onto itself,*
- (iii)  *$f$  is an isometry onto.*

*Proof.* This follows from (3.3) (or from (3.4) and (3)).

4. Some consequences. As an immediate consequence of (3.1), we get

(4.1) THEOREM. *Let  $f$  be a local isometry of a connected finitely compact metric space  $(M, \rho)$  into itself. If for each  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then the induced metric,  $\rho_f$ , is a metric on  $M$  such that  $\rho_f$  and  $\rho$  are locally identical,  $(M, \rho_f)$  is a finitely compact metric space and  $f$  is an isometry of  $(M, \rho_f)$  onto itself. In particular,  $f$  is a homeomorphism of  $M$  onto itself.*

As an immediate consequence of (3.3), we get



(4.2) THEOREM. *Let  $f$  be a nonexpansive local isometry of a connected finitely compact metric space  $(M, \rho)$  into itself. If for some  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then  $f$  is an isometry onto.*

The corresponding statement concerning local isometries of convex finitely compact metric spaces is stated next.

(4.3) THEOREM. *Let  $f$  be a local isometry of a convex finitely compact metric space  $(M, \rho)$  into itself. If for some  $z \in M$  the sequence  $\{f^n(z)\}$  is bounded, then  $f$  is an isometry onto.*

*Proof.* Since  $(M, \rho)$  is convex and complete, by (2.8),  $f$  is a nonexpansive local isometry. Hence, our assertion follows from (4.2).

Finally, we note the following special cases of (4.2) and (4.3).

(4.4) COROLLARY. *Let  $f$  be a nonexpansive local isometry of a connected finitely compact metric space  $(M, \rho)$  into itself. If  $f$  has a fixed (periodic) point, then  $f$  is an isometry onto.*

(4.5) COROLLARY. *Let  $f$  be a local isometry of a convex finitely compact metric space  $(M, \rho)$  into itself. If  $f$  has a fixed (periodic) point, then  $f$  is an isometry onto.*

REMARK. Theorems (4.2) and (4.3) extend the result of [6]; Corollaries (4.4) and (4.5) extend Theorem 1 of [5] to the case of general local isometries of finitely compact metric spaces.

5. A condition on  $(M, \rho)$  under which local isometries are isometries. In this section, by using (3.3), we extend Theorem 3 of [5]. First, we shall prove

(5.1) PROPOSITION. *Let  $f$  be a nonexpansive local isometry of a finitely compact metric space  $(M, \rho)$  into itself. If  $(M, \rho)$  has a transitive group of isometries, then there exists a sequence  $N_n$ ,  $n = 0, 1, \dots$ , of open and closed subsets of  $M$  such that  $M = \bigcup_{n=0}^{\infty} N_n$  and for each  $n \geq 0$ ,  $f$  maps  $N_n$  isometrically onto an open closed subset of  $M$ .*

*Proof.* Let  $z \in M$ . Then, by assumption, there exists an isometry  $g_z$  of  $(M, \rho)$  onto itself such that  $g_z(f(z)) = z$ . Since  $g_z \circ f$  is a nonexpansive local isometry, it follows from (3.3) that there is an open and closed set  $N_z$  such that  $z \in N_z$  and  $g_z \circ f$  maps  $N_z$  isometrically onto itself. Hence  $g_z^{-1}(N_z)$  is open and closed, and  $f$  maps  $N_z$  iso-

metrically onto  $g_z^{-1}(N_z)$ . Since  $(M, \rho)$  is separable, our assertion follows.

The next two results follow immediately from (5.1) and (2.8) (or, in a direct fashion, from (4.4) and (4.5)).

(5.2) THEOREM. *If a connected finitely compact metric space  $(M, \rho)$  has a transitive group of isometries, then each nonexpansive local isometry of  $(M, \rho)$  into itself is an isometry onto.*

(5.3) THEOREM. *If a convex finitely compact metric space  $(M, \rho)$  has a transitive group of isometries, then each local isometry of  $(M, \rho)$  into itself is an isometry onto.*

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