Pacific Journal of Mathematics

CHARACTERS OF INDUCED REPRESENTATIONS AND WEIGHTED ORBITAL INTEGRALS

Rebecca A. Herb

Vol. 114, No. 2 June 1984

PACIFIC JOURNAL OF MATHEMATICS
Vol. 114, No. 2, 1984

CHARACTERS OF INDUCED REPRESENTATIONS AND WEIGHTED ORBITAL INTEGRALS

REBECCA A. HERB

The main result of this paper is a formula relating characters of principal series representations of a reductive Lie group to weighted orbital integrals of wave packets.

1. Introduction. Let G be a reductive Lie group satisfying Harish-Chandra's general assumptions [**2**]. Let $P = MAN$ be the Langlands decomposition of a cuspidal parabolic subgroup of G. Denote by $\varepsilon_2(M)$ the set of equivalence classes of irreducible unitary square integrable representations of M. For $\omega \in \varepsilon_2(M)$ and $\nu \in \mathcal{F} = \mathfrak{a}^*$, the real dual of the Lie algebra of A, let $\pi_{\omega,\nu}$ be the corresponding unitary representation of G induced from P. Let f be a wave packet corresponding to ω. Then the integral of f over any regular (semisimple) orbit of G which can be represented by an element of $L = MA$ has been evaluated by Harish-Chandra in terms of the character $\Theta_{\omega,\nu}$ of $\pi_{\omega,\nu}$ [**4**].

Let γ be a regular element of G contained in a Cartan subgroup H of L. Write $H = H_K H_p$ where H_K is compact, H_p is split, and $A \subseteq H_p$. Then for suitable normalizations of the G-invariant measure $d\dot{x}$ on $H_p \setminus G$ and Haar measure $d\nu$ on $\mathcal{F}$.

$$(1.1)\quad \int_{H_p\setminus G} f(x^{-1}\gamma x)\, d\dot{x} = \varepsilon(A, H)[W(\omega)]^{-1}\int_{\mathcal{F}} \langle \Theta_{\omega,\nu}, f\rangle \Theta_{\omega,\nu}(\gamma)\, d\nu$$

where $W(\omega) = \{s \in N_G(A)/L \mid s\omega = \omega\}$ and $\varepsilon(A, H)$ is 1 if $H_p = A$ and is 0 otherwise. This formula can be interpreted as giving the value of $\Theta_{\omega,\nu}$ on regular elements γ of a fundamental Cartan subgroup of L in terms of the integral of a wave packet for ω over the orbit of γ. It also gives the Fourier inversion formula for the tempered invariant distribution

$$f \to \langle \Lambda(\gamma), f\rangle = \int_{H_p\setminus G} f(x^{-1}\gamma x)\, d\dot{x}$$

restricted to the subspace of $\mathcal{C}(G)$, the Schwartz space of G, spanned by wave packets corresponding to representations induced from cuspidal parabolic subgroups $P = MAN$ with $A \subseteq H_p$. The complete Fourier inversion formula for $\Lambda(\gamma)$ is much more complicated. (See [**5**].)

In the case that $P = G$ is cuspidal and $\omega \in \varepsilon_2(G)$, then Θ_ω is a discrete series character of G, and f is a matrix coefficient corresponding to ω. Formula (1.1) becomes

$$\text{(1.2)} \qquad \int_{H_p \backslash G} f(x^{-1}\gamma x)\, d\dot{x} = \varepsilon(1, H)\langle \Theta_\omega, f\rangle \Theta_\omega(\gamma).$$

Arthur has obtained the following generalization of (1.2) [**1**]. Let A be the split component of a parabolic subgroup of G. Let L be the centralizer in G of A. Corresponding to A, Arthur defines a function v_A on G which is left L-invariant. Let γ be a regular element of G contained in a Cartan subgroup $H = H_K H_p$ of L. Let $\omega \in \varepsilon_2(G)$, and let f be a matrix coefficient for ω. Then Arthur's formula is

$$\text{(1.3)} \qquad \int_{H_p \backslash G} f(x^{-1}\gamma x) v_A(x)\, d\dot{x} = (-1)^p \varepsilon(A, H)\langle \Theta_\omega, f\rangle \Theta_\omega(\gamma)$$

where p is the dimension of A. This formula gives the value of the character Θ_ω on the nonelliptic element γ in terms of a weighted orbital integral of a matrix coefficient of ω. It also gives the Fourier inversion formula for the tempered distribution

$$f \to \langle r_A(\gamma), f\rangle = \int_{H_p \backslash G} f(x^{-1}\gamma x) v_A(x)\, d\dot{x}$$

restricted to the space ${}^0\mathscr{C}(G)$ of cusp forms on G. The distributions $r_A(\gamma)$ occur in the Selberg trace formula for $\Gamma \backslash G$, Γ a discrete subgroup of G for which $\Gamma \backslash G$ has finite volume but is not compact. As formula (1.3) shows, $r_A(\gamma)$ is invariant on ${}^0\mathscr{C}(G)$. However, $r_A(\gamma)$ is not an invariant distribution on $\mathscr{C}(G)$, and the full Fourier inversion formula for $r_A(\gamma)$ is not known.

Authur's formula can be generalized to the setting of induced representations and wave packets. Let $P = MAN$ be a cuspidal parabolic subgroup of G, and let A_1 be the split component of a parabolic subgroup of $L = MA$, L_1 its centralizer in G. Let γ be a regular element of G contained in a Cartan subgroup $H = H_K H_p$ of L_1. We will define a left L_1-invariant function $v_{A_1}^P$ on G with the following properties.

If f' is a wave packet coming from a cuspidal parabolic subgroup $P' = M'A'N'$ of G with $\dim A' \leq \dim A$ and A not conjugate to A', then

$$\text{(1.4)} \qquad \int_{H_p \backslash G} f'(x^{-1}\gamma x) v_{A_1}^P(x)\, d\dot{x} = 0.$$

Now let f be a wave packet corresponding to $\omega \in \varepsilon_2(M)$. Then

$$\int_{H_p\backslash G} f(x^{-1}\gamma x) v^P_{A_1}(x)\, d\dot{x} = 0 \quad \text{if } H_p \neq A_1. \tag{1.5}$$

If $H_p = A_1$, let $\gamma = \gamma_1, \gamma_2, \ldots, \gamma_k$ be a complete set of elements of L for which $\gamma_i = x_i \gamma x_i^{-1}$ for some $x_i \in G$, but γ_i and γ_j are not conjugate in L for $1 \leq i \neq j \leq k$. Let $A_i = x_i A_1 x_i^{-1}$. Then

$$\begin{aligned} &\int_{H_p\backslash G} f(x^{-1}\gamma x) \sum_{i=1}^{k} v^P_{A_i}(x_i x)\, d\dot{x} \\ &\quad = (-1)^{p_1}[W(\omega)]^{-1} \int_{\mathscr{F}} \langle \Theta_{\omega,\nu}, f\rangle \Theta_{\omega,\nu}(\gamma)\, d\nu \end{aligned} \tag{1.6}$$

where p_1 is the dimension of $A_1 \cap M$.

Formulas (1.4)–(1.6) are proved by using Arthur's formula and results of Harish-Chandra relating characters and orbital integrals on G to those on M and L. Any unexplained notation follows that of Harish-Chandra [**2, 3, 4**].

2. Background material. Let G be a real reductive Lie group, $\mathfrak{g}$ the Lie algebra of G. Let K be a maximal compact subgroup of G, θ the Cartan involution of G corresponding to K, and B a real symmetric bilinear form on $\mathfrak{g}$. Assume that (G, K, θ, B) satisfy the general assumptions of Harish-Chandra in [**2**] and that Haar measures are normalized as in [**2**]. Given a θ-stable Cartan subgroup H of G, we will write $H = H_K H_p$ where $H_K = H \cap K$ and H_p is a vector subgroup with Lie algebra $\mathfrak{h}_p$ contained in the -1 eigenspace for θ. Let G' be the set of regular semisimple elements of G, $H' = H \cap G'$. If J is any subgroup of G, we will write $N_G(J)$ and $C_G(J)$ for the normalizer and centralizer of J in G, respectively, and $W(G, J) = N_G(J)/C_G(J)$.

We will first review some definitions and formulas of Harish-Chandra from [**2, 3, 4**]. Fix a double unitary representation τ of K on a finite-dimensional Hilbert space V. Let $\mathscr{C}(G, \tau)$ and ${}^0\mathscr{C}(G, \tau)$ denote the τ-spherical functions in the spaces of V-valued Schwartz functions $\mathscr{C}(G, V)$ and V-valued cusp forms ${}^0\mathscr{C}(G, V)$ respectively. Let F_0 be the operator on V given by

$$F_0 v = \int_K \tau(k^{-1}) v \tau(k)\, dk, \qquad v \in V.$$

For $f \in \mathscr{C}(G, V)$ and $x \in G$, define $\bar{f}(x) = \int_K f(k^{-1}xk)\, dk$. Then if $f \in \mathscr{C}(G, \tau)$, $\bar{f}(x) = F_0 f(x)$, $x \in G$.

Fix a cuspidal parabolic subgroup $P = MAN$, that is, a parabolic subgroup of G with $\varepsilon_2(M) \neq \varnothing$. Let τ_M be the restriction of τ to $K_M = K \cap M$. For any $f \in \mathscr{C}(G, V)$, $m \in M$, and $a \in A$, let

$$f^{(P)}(ma) = f_a^{(P)}(m) = \delta_P^{1/2}(a) \int_N f(man)\, dn \tag{2.1}$$

where δ_P is the module of P. Then $f_a^{(P)} \in \mathscr{C}(M, V)$, $f^{(P)} \in \mathscr{C}(MA, V)$, and the following relationships between f and $f^{(P)}$ can be found in or easily derived from results in [**2, 3, 4**].

Let H be a θ-stable Cartan subgroup of L. for $f \in \mathscr{C}(G, V)$ and $h \in H'$,

$$\int_N f(n^{-1}hn)\, dn = \Delta_+^G(h)^{-1} \Delta_+^L(h) f^{(P)}(h) \tag{2.2}$$

where Δ_+^L and Δ_+^G are the functions Δ_+ on H, considered as a Cartan subgroup of L and G respectively, defined by Harish-Chandra in [**2**].

For $\nu \in \mathscr{F} = \mathfrak{a}^*$ and $m \in M$, define

$$f_\nu^{(P)}(m) = \int_A f^{(P)}(ma) e^{-i\nu(\log a)}\, da. \tag{2.3}$$

Then because $d\nu$ is the dual measure to da on A and $f^{(P)}$ is rapidly decreasing in the A variable,

$$f^{(P)}(ma) = \int_{\mathscr{F}} f_\nu^{(P)}(m) e^{i\nu(\log a)}\, d\nu. \tag{2.4}$$

For $\omega \in \varepsilon_2(M)$ and $\nu \in \mathscr{F}$, let $\pi_{\omega,\nu}$ be the tempered unitary representation of G induced from $\omega \otimes e^{i\nu} \otimes 1$ on MAN. Let $\Theta_{\omega,\nu}$ and Θ_ω denote the characters of $\pi_{\omega,\nu}$ and ω considered as functions on G' and M' respectively. For $f \in \mathscr{C}(G, \tau)$, $g \in \mathscr{C}(M, \tau_M)$, define

$$\langle \Theta_{\omega,\nu}, f \rangle = \int_G f(x) \overline{\Theta_{\omega,\nu}(x)} dx \quad \text{and} \quad \langle \Theta_\omega, g \rangle = \int_M g(m) \overline{\Theta_\omega(m)} dm.$$

Then, for $f \in \mathscr{C}(G, \tau)$, $\nu \in \mathscr{F}$, $f_\nu^{(P)} \in \mathscr{C}(M, \tau_M)$ and

$$\langle \Theta_{\omega,\nu}, f \rangle = F_0 \langle \Theta_\omega, f_\nu^{(P)} \rangle. \tag{2.5}$$

For $\omega \in \varepsilon_2(M)$, let $L(\omega) = {}^0\mathscr{C}(M, \tau_M) \cap \mathfrak{h}_\omega \otimes V$ where $\mathfrak{h}_\omega$ is the closed subspace of $L^2(M)$ spanned by matrix coefficients for ω. For $\psi \in L(\omega)$, $\alpha \in C_c^\infty(\mathscr{F})$, and $x \in G$, define

$$\varphi_\alpha(x) = \int_{\mathscr{F}} \alpha(\nu) E(P : \psi : \nu : x) \mu(\omega : \nu)\, d\nu \tag{2.6}$$

where $E(P:\psi:\nu)$ is the Eisenstein integral defined in [2], and $\mu(\omega:\nu)$ is the Plancherel factor corresponding to $\pi_{\omega,\nu}$. Then $\varphi_\alpha \in \mathscr{C}(G,\tau)$ is called a wave packet for $\omega \in \varepsilon_2(M)$, and for $\nu \in \mathscr{F}$, $(\varphi_\alpha)_\nu^{(P)}$ belongs to $\sum_{s\in W(G,A)} L(s\omega)$ and is supported on a compact subset of $\mathscr{F}$.

We now turn to Arthur's results. Let A be a special vector subgroup of G, that is, the split component of a parabolic subgroup of G. Write $\mathscr{P}(A)$ for the (finite) set of all parabolic subgroups of G having A as split component. For $P \in \mathscr{P}(A)$ let Φ_P denote the set of simple roots of (P, A). We identify $\mathfrak{a}$, the Lie algebra of A, and its dual via the bilinear form B. A set $\mathscr{Y} = \{Y_P | P \in \mathscr{P}(A)\}$ of points in $\mathfrak{a}$ is called A-orthogonal if for any pair of adjacent parabolic subgroups $P, P' \in \mathscr{P}(A)$, $Y_P - Y_{P'} = r\alpha$, $r \in \mathbf{R}$, where α is the unique element of Φ_P with $-\alpha \in \Phi_{P'}$. Let

$$\mathfrak{a}^0 = \{H \in \mathfrak{a} | \langle \alpha, H\rangle = 0 \text{ for every root } \alpha \text{ of } (\mathfrak{g}, \mathfrak{a})\},$$

$\mathfrak{a}^1$ its orthogonal complement in A. Let p be the dimension of $\mathfrak{a}^1$, and let $c_A = |\det C|^{1/2}$ where C is the Cartan matrix for the roots of $(\mathfrak{g}, \mathfrak{a})$. For any $P = MAN \in \mathscr{P}(A)$ and $x \in G$, write

$$x = m(x)\exp(H_P(x))n(x)k(x)$$

where $m(x) \in M$, $n(x) \in N$, $k(x) \in K$, and $H_P(x) \in \mathfrak{a}$. For any A-orthogonal set $\mathscr{Y}$ and $\lambda \in \mathfrak{a}^1_{\mathbf{C}}$, define

$$v(x:\mathscr{Y}) = c_A(p!)^{-1} \sum_{P\in\mathscr{P}(A)} \frac{\langle \lambda, Y_P - H_P(x)\rangle^p}{\prod_{\alpha\in\Phi_P}\langle\lambda,\alpha\rangle}. \tag{2.7}$$

Then $v(x:\mathscr{Y})$ is independent of λ and is left-invariant under $L = C_G(A)$. It is also clearly right K-invariant. If $v_A(x) = v(x:\mathscr{Y})$ for any A-orthogonal set $\mathscr{Y}$, then (1.3) is valid.

3. The distributions. Fix a cuspidal parabolic subgroup $P = MAN$ of G. Let A_1^M be a special vector subgroup of M, $A_1 = A_1^M A$. Let $\mathscr{Y}_1$ be an A_1^M-orthogonal set, and let $v_1^M(m) = v(m:\mathscr{Y}_1)$, $m \in M$, be the function on M defined as in (2.7) with respect to A_1^M and $\mathscr{Y}_1$. Extend v_1^M to a function v_1 on G by setting

$$v_1(mank) = [W(G,A)]^{-1} v_1^M(m), \qquad m \in M, a \in A, n \in N, k \in K. \tag{3.1}$$

This extension is well defined since v_1^M is right K_M-invariant. Since v_1^M if left-invariant under $L_1^M = C_M(A_1^M)$, v_1 is left-invariant under $L_1 = L_1^M A = C_G(A_1)$.

Let H be a θ-stable Cartan subgroup of G with $A_1 \subseteq H_p$. Write $J = H \cap M$. Let $h \in H'$. For $f \in C_c^\infty(G, V)$, define

$$\langle r_1(h), f\rangle = \int_{H_p \backslash G} f(x^{-1}hx) v_1(x)\, d\dot{x}. \tag{3.2}$$

LEMMA 3.3. *For any $h \in H'$ the distribution $r_1(h)$ is tempered. For any $f \in \mathscr{C}(G, V)$, $\int_{H_p\backslash G} f(x^{-1}hx)v_1(x)\, d\dot{x}$ is absolutely convergent and*

$$\begin{aligned}\langle r_1(h), f\rangle &= \int_{H_p\backslash G} f(x^{-1}hx) v_1(x)\, d\dot{x} \\ &= [W(G, A)]^{-1} \Delta_+^G(h)^{-1} \Delta_+^L(h) \int_{J_p\backslash M} \bar{f}^{(P)}(m^{-1}hm) v_1^M(m)\, d\dot{m}.\end{aligned}$$

Proof. Let $f \in \mathscr{C}(G, V)$. Write $h = ja$ where $j \in J'$, $a \in A$. Then using (2.2) and (3.1),

$$\begin{aligned}&\int_{H_p\backslash G} |f(x^{-1}hx) v_1(x)|\, d\dot{x} \\ &\quad = [W(G, A)]^{-1} \int_{J_p\backslash M} |v_1^M(m)| \int_{NK} |f(k^{-1}n^{-1}m^{-1}hmnk)|\, dn\, dk\, d\dot{m} \\ &\quad = [W(G, A)]^{-1} \Delta_+^G(h)^{-1} \Delta_+^L(h) \int_{J_p\backslash M} |v_1^M(m) \bar{f}_a^{(P)}(m^{-1}jm)|\, d\dot{m}\end{aligned}$$

since Δ_+^G and Δ_+^L are invariant under conjugation by M. The lemma now follows since for any $a \in A$, $f \to \bar{f}_a^{(P)}$ is a continuous map from $\mathscr{C}(G, V)$ to $\mathscr{C}(M, V)$ [**2**]. Further, for $g \in \mathscr{C}(M, V)$, $j \in J'$,

$$\int_{J_p\backslash M} g(m^{-1}jm) v_1^M(m)\, d\dot{m}$$

is absolutely convergent and defines a tempered distribution [**1**]. □

COROLLARY 3.4. *Let A' be a special vector subgroup of G with* dim A' $\leq$ dim A. *Let $P' = M'A'N' \in \mathscr{P}(A')$, $\omega' \in \varepsilon_2(M')$. Let f be a wave packet defined as in* (2.6) *with respect to ω' and P'. Then $\langle r_1(h), f\rangle = 0$ unless A' is conjugate to A under K.*

Proof. In this case $\bar{f}^{(P)} = 0$ [**4**]. Thus the result follows from (3.3). □

LEMMA 3.5 *Suppose that* $f = \varphi_\alpha$ *is a wave packet associated to* $\omega \in \varepsilon_2(M)$. *Let* $h \in H'$. *Then*

$$\langle r_1(h), f\rangle = [W(G, A)]^{-1}(-1)^{p_1}\varepsilon(A_1, H)[W(\omega)]^{-1}\Delta_+^G(h)^{-1}\Delta_+^L(h) \cdot \int_{\mathscr{F}}\langle \Theta_{\omega,\nu}, f\rangle \sum_{s\in W(G,A)} (\Theta_{s\omega} \otimes e^{is\nu})(h)\, d\nu$$

where $p_1 = \dim A_1^M$.

Proof. Using (3.3) and (2.4),

$$\langle r_1(h), f\rangle = [W(G,A)]^{-1}\Delta_+^G(h)^{-1}\Delta_+^L(h)F_0 \cdot \int_{J_p\backslash M} v_1^M(m)\int_{\mathscr{F}} e^{i\nu(\log a)}f_\nu^{(P)}(m^{-1}jm)\, d\nu\, d\dot{m}.$$

Since $f_\nu^{(P)} \in \mathscr{C}(M, V)$ and is supported on a compact subset of $\mathscr{F}$, we can interchange the order of integration. Let $W = W(G, A)$, and write $f_\nu^{(P)} = \sum_{s\in W/W(\omega)} g_s$ where $g_s \in L(s\omega)$. Then, using (1.3),

$$\int_{J_p\backslash M} g_s(m^{-1}jm)v_1^M(m)\, d\dot{m} = (-1)^{p_1}\varepsilon(A_1^M, J)\langle \Theta_{s\omega}, g_s\rangle\Theta_{s\omega}(j).$$

But $\varepsilon(A_1^M, J) = \varepsilon(A_1, H)$, and $\langle \Theta_{s\omega}, g_{s'}\rangle = 0$ if $s\omega \neq s'\omega$. Thus using (2.5),

$$\begin{aligned} F_0\int_{J_p\backslash M} & f_\nu^{(P)}(m^{-1}jm)v_1^M(m)\, d\dot{m} \\ &= (-1)^{p_1}\varepsilon(A_1, H) \sum_{s\in W/W(\omega)} F_0\langle \Theta_{s\omega}, f_\nu^{(P)}\rangle\Theta_{s\omega}(j) \\ &= (-1)^{p_1}\varepsilon(A_1, H)[W(\omega)]^{-1} \sum_{s\in W} \langle \Theta_{s\omega,\nu}, f\rangle\Theta_{s\omega}(j). \end{aligned}$$

Now for each $s \in W$,

$$\int_{\mathscr{F}} e^{i\nu(\log a)}\Theta_{s\omega}(j)\langle \Theta_{s\omega,\nu}, f\rangle d\nu = \int_{\mathscr{F}} e^{is\nu(\log a)}\Theta_{s\omega}(j)\langle \Theta_{\omega,\nu}, f\rangle d\nu$$

since $\Theta_{s\omega,s\nu} = \Theta_{\omega,\nu}$. □

Now suppose that H is a Cartan subgroup of G with $H_p = A_1$, and fix $h \in H'$. Let $h_i = x_i h x_i^{-1}$, $1 \le i \le k$, be defined as in (1.6). Then using results from [6, 7], for $\omega \in \varepsilon_2(M)$ and $\nu \in \mathscr{F}$,

$$\Theta_{\omega,\nu}(h) = \sum_{i=1}^{k} \Delta_+^G(h_i)^{-1}\Delta_+^L(h_i)(\Theta_\omega \otimes e^{i\nu})(h_i). \tag{3.6}$$

Fix $1 \le i \le k$. Let $A_i = x_i A_1 x_i^{-1}$, $A_i^M = A_i \cap M$. Let $L_i = C_G(A_i)$. Then A_i^M is a special vector subgroup of M. Let $\mathscr{Y}_i$ be any A_i^M-orthogonal set, and define v_i on G as in (3.1) starting from v_i^M. Then v_i is left L_i-invariant so that $x \to v_i(x_i x)$ is left L_1-invariant. For $x \in G$ define

$$\langle r(h), f\rangle = \int_{H_p\backslash G} f(x^{-1}hx) \sum_{i=1}^{k} v_i(x_i x)\, d\dot{x}. \tag{3.7}$$

THEOREM 3.8. *Let H be a Cartan subgroup of G with $H_p = A_1$, $h \in H'$. Then $r(h)$ is a tempered distribution, and for f a wave packet corresponding to $\omega \in \varepsilon_2(M)$,*

$$\langle r(h), f\rangle = (-1)^{p_1}[W(\omega)]^{-1}\int_{\mathscr{F}} \langle \Theta_{\omega,\nu}, f\rangle \Theta_{\omega,\nu}(h)\, d\nu.$$

Proof. Define $x_1,\dots,x_k$ and $h_1,\dots,h_k$ as in (3.6). Then for $1 \le i \le k$, $H_i = x_i H x_i^{-1}$ is a Cartan subgroup of G with $A_i = (H_i)_p$ so that using (3.3),

$$\begin{aligned}\langle r(h), f\rangle &= \sum_{i=1}^{k} \int_{H_p\backslash G} f(x^{-1}hx) v_i(x_i x)\, d\dot{x}\\ &= \sum_{i=1}^{k} \int_{(H_i)_p\backslash G} f\left(x^{-1}h_i x\right) v_i(x)\, d\dot{x}\\ &= (-1)^{p_1}\varepsilon(A_1, H)[W(\omega)]^{-1}\int_{\mathscr{F}} \langle \Theta_{\omega,\nu}, f\rangle \varphi(\omega, \nu, h)\, d\nu\end{aligned}$$

where

$$\begin{aligned}\varphi(\omega, \nu, h) &= [W]^{-1}\sum_{s\in W}\sum_{i=1}^{k} \Delta_+^G(h_i)^{-1}\Delta_+^L(h_i)\Theta_{s\omega} \otimes e^{is\nu}(h_i)\\ &= [W]^{-1}\sum_{s\in W}\Theta_{s\omega,s\nu}(h) = \Theta_{\omega,\nu}(h). \qquad \square\end{aligned}$$

References

[1] J. Arthur, *The characters of discrete series as orbital integrals*, Inv. Math., **32** (1976), 205–261.

[2] Harish-Chandra, *Harmonic analysis on real reductive groups*, *I*, J. Funct. Anal., **19** (1975), 104–204.

[3] ______, *Harmonic analysis on real reductive groups*, *II*, Inv. Math., **36** (1976), 1–55.

[4] ______, *Harmonic analysis on real reductive groups*, *III*, Ann. of Math., **104** (1976), 117–201.

[5] R. Herb, *Discrete series characters and Fourier inversion on semisimple real Lie groups*, Trans. Amer. Math. Soc., **277** (1983), 241–261.

[6] W. Schmid, *On the characters of the discrete series*, Inv. Math., **30** (1975), 47–144.

[7] J. Wolf, *Unitary representations on partially holomorphic cohomology spaces*, Mem. Amer. Math. Soc., **138** (1974).

Received March 20, 1981. Supported in part by National Science Foundation Grant MCS 77-18723 A04.

UNIVERSITY OF MARYLAND
COLLEGE PARK, MD 20742

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be in typed form or offset-reproduced (not dittoed), double spaced with large margins. Please do not use built up fractions in the text of the manuscript. However, you may use them in the displayed equations. Underline Greek letters in red, German in green, and script in blue. The first paragraph must be capable of being used separately as a synopsis of the entire paper. In particular it should contain no bibliographic references. Please propose a heading for the odd numbered pages of less than 35 characters. Manuscripts, in triplicate, may be sent to any one of the editors. Please classify according to the scheme of Math. Reviews, Index to Vol. 39. Supply name and address of author to whom proofs should be sent. All other communications should be addressed to the managing editor, or Elaine Barth, University of California, Los Angeles, California 90024.

There are page-charges associated with articles appearing in the Pacific Journal of Mathematics. These charges are expected to be paid by the author's University, Government Agency or Company. If the author or authors do not have access to such Institutional support these charges are waived. Single authors will receive 50 free reprints; joint authors will receive a total of 100 free reprints. Additional copies may be obtained at cost in multiples of 50.

The *Pacific Journal of Mathematics* is issued monthly as of January 1966. Regular subscription rate: $190.00 a year (5 Vols., 10 issues). Special rate: $66.00 a year to individual members of supporting institutions.

Subscriptions, orders for numbers issued in the last three calendar years, and changes of address should be sent to Pacific Journal of Mathematics, P.O. Box 969, Carmel Valley, CA 93924, U.S.A. Old back numbers obtainable from Kraus Periodicals Co., Route 100, Millwood, NY 10546.

The Pacific Journal of Mathematics at P.O. Box 969, Carmel Valley, CA 93924 (ISSN 0030-8730) publishes 5 volumes per year. Application to mail at Second-class postage rates is pending at Carmel Valley, California, and additional mailing offices. Postmaster: Send address changes to Pacific Journal of Mathematics, P.O. Box 969, Carmel Valley, CA 93924.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Pacific Journal of Mathematics

Vol. 114, No. 2 June, 1984