

Pacific Journal of Mathematics

A UNIFIED APPROACH TO CARLESON MEASURES AND A_p WEIGHTS. II

FRANCISCO JOSÉ RUIZ AND JOSÉ LUIS TORREA

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In this note we find for each p , $1 < p < \infty$, a necessary and sufficient condition on the pair (μ, v) (where μ is a measure on $\mathbf{R}_+^{n+1} = \mathbf{R}^n \times [0, \infty)$, and v a weight on \mathbf{R}^n) for the Poisson integral to be a bounded operator from $L^p(\mathbf{R}^n, v(x) dx)$ into $L^p(\mathbf{R}_+^{n+1}, \mu)$.

1. Introduction. In this note we find for each p , $1 < p < \infty$, a necessary and sufficient condition on the pair (μ, v) (where μ is a measure on $\mathbf{R}_+^{n+1} = \mathbf{R} \times [0, \infty)$ and v a weight on \mathbf{R}^n) for the Poisson integral to be a bounded operator from $L^p(\mathbf{R}^n, v(x) dx)$ into $L^p(\mathbf{R}_+^{n+1}, \mu)$.

Our proof follows the ideas of Sawyer [7] and the condition we find is

$$(F_p) \quad \int_{\tilde{Q}} \left[\mathcal{M}(v^{1-p'} \chi_Q)(x, t) \right]^p d\mu(x, t) \leq C \int_Q v^{1-p'}(x) dx < +\infty$$

for all cubes in \mathbf{R}^n (cube will always means a compact cube with sides parallel to the coordinate axes).

For \mathcal{M} we denote the maximal operator

$$(*) \quad \mathcal{M}f(x, t) = \sup_Q \frac{1}{|Q|} \int_Q |f(x)| dx, \quad x \in \mathbf{R}^n, t \geq 0,$$

where the supremum is taken over the cubes Q in \mathbf{R}^n , containing x and having side length at least t .

As usual \tilde{Q} denotes the cube in \mathbf{R}_+^{n+1} , with the cube Q as its basis.

Carleson [1] showed that \mathcal{M} is bounded from $L^p(\mathbf{R}^n, dx)$ into $L^p(\mathbf{R}_+^{n+1}, \mu)$ if and only if μ satisfies the so-called "Carleson condition"

$$(1) \quad \mu(\tilde{Q}) \leq C|Q| \quad \text{for each cube in } \mathbf{R}^n.$$

Afterwards, Fefferman and Stein [2] found that

$$(2) \quad \sup_{x \in Q} \frac{\mu(\tilde{Q})}{|Q|} \leq Cv(x) \quad \text{a.e. } x$$

is sufficient for \mathcal{M} to be bounded from $L^p(\mathbf{R}^n, v(x) dx)$ into $L^p(\mathbf{R}_+^{n+1}, \mu)$.

Recently F. Ruiz [6] found the condition

$$(3) \quad \frac{\mu(\tilde{Q})}{|Q|} \left(\frac{1}{|Q|} \int_Q v^{1-p'}(x) dx \right)^{p-1} \leq C$$

to be necessary and sufficient for the boundedness of the operator \mathcal{M} from $L^p(\mathbf{R}^n, v(x) dx)$ into weak- $L^p(\mathbf{R}_+^{n+1}, \mu)$. The condition (3) will be denoted by (C_p) as in [6].

The paper is set out as follows: in §2 we give results and some consequences, whilst §3 contains detailed proofs.

2. Results. Throughout this paper, Q will denote a cube in \mathbf{R}^n with sides parallel to the coordinate planes. For $r > 0$, rQ will denote the cube with the same centre as Q diameter r times that of Q . $|Q|_v$ will denote $\int_Q v(x) dx$.

We shall say that Q is a dyadic cube and we shall write $Q \in \mathcal{D}$, if Q is a subset of \mathbf{R}^n of the form $\prod_{i=1}^n [x_i, x_i + 2^k)$, where $x \in 2^k \mathbf{Z}_+^n$, with k in \mathbf{Z} . We define the dyadic maximal operator \mathcal{N} associated with the Poisson integral by

$$(**) \quad \mathcal{N}f(x, t) = \sup_Q \frac{1}{|Q|} \int_Q |f(x)| dx, \quad x \in \mathbf{R}^n, t \geq 0,$$

where the supremum is taken over the dyadic cubes in \mathbf{R}^n containing x and having side length at least t .

The main results in this paper are the following:

THEOREM A. *Given a weight v in \mathbf{R}^n , a positive measure μ in \mathbf{R}_+^{n+1} , and p , $1 < p < \infty$, the following conditions are equivalent.*

(i) *The operator \mathcal{M} defined in (*) is bounded from $L^p(\mathbf{R}^n, v(x) dx)$ into $L^p(\mathbf{R}_+^{n+1}, \mu)$; i.e.*

$$\int_{\mathbf{R}_+^{n+1}} [\mathcal{M}f(x, t)]^p d\mu(x, t) \leq C \int_{\mathbf{R}^n} |f(x)|^p v(x) dx.$$

(ii) *The pair (μ, r) verifies (F_p) .*

THEOREM B. *Given a weight v in \mathbf{R}^n , a positive measure μ in \mathbf{R}_+^{n+1} , and p , $1 < p < \infty$, the following conditions are equivalent.*

(i) *The operator \mathcal{N} defined in (**) is bounded from $L^p(\mathbf{R}^n, vx dx)$ into $L^p(\mathbf{R}_+^{n+1}, \mu)$.*

(ii) The pair (μ, v) verifies

$$\int_{\tilde{Q}} \left[\mathcal{N}(v^{1-p'} \chi_Q)(x, t) \right]^p d\mu(x, t) \leq C \int_Q v^{1-p'}(x) dx < +\infty$$

for all dyadic cubes Q in \mathbf{R}^n .

The above results have certain consequences. (I) In the particular case in Theorem A where $v(x) \equiv 1$, the condition (F_p) reduces to

$$\int_{\tilde{Q}} \left[\mathcal{M}(\chi_Q)(x, t) \right]^p d\mu(x, t) \leq C \int_Q dx = C|Q|$$

and since $\mathcal{M}(\chi_Q)(x, t) = 1$ for $(x, t) \in \tilde{Q}$, we see that Theorem A gives us Carleson's result mentioned in the introduction.

(II) If the measure μ in $\mathbf{R}_+^{n+1} = \mathbf{R}^n \times [0, \infty)$ is of the form $d(x) = u(x) dx$ concentrated in $\mathbf{R}^n \times \{0\}$, then (F_p) is equivalent to Sawyer's condition

$$(S_p) \quad \int_Q \left[M(v^{1-p'} \chi_Q)(x) \right]^p u(x) dx \leq C \int_Q v^{1-p'}(x) dx < +\infty$$

where Mf denotes the Hardy Littlewood maximal operator.

Since $\mathcal{M}f(x, 0) = Mf(x)$, $x \in \mathbf{R}^n$. Then from Theorem A we obtain

THEOREM (Sawyer [7]). *Let $1 < p < \infty$. Given weights u and v in \mathbf{R}^n the following statements are equivalent:*

- (i) (u, v) satisfies the (S_p) condition
- (ii) $\int_{\mathbf{R}^n} (Mf(x))^p u(x) dx \leq C_p \int_{\mathbf{R}^n} |f(x)|^p v(x) dx$.

(III) Hunt, Kurtz and Neugebauer [3] have shown by a direct proof that if a weight v belongs to the A_p class, $1 < p < \infty$, of Muckenhoupt, i.e.

$$\sup_Q \left(\frac{1}{|Q|} \int_Q v(x) dx \right) \left(\frac{1}{|Q|} \int_Q v^{1-p'}(x) \right)^{p-1} \leq C$$

then v satisfies the (S_p) condition in (II) with $u = v$.

In our case it can be shown, see [6], that if the pair (μ, v) satisfies the (C_p) condition, $1 < p < \infty$, and v belongs to the class A_p of Muckenhoupt, then the operator \mathcal{M} is bounded from $L^p(\mathbf{R}^n, v(x) dx)$ into $L^p(\mathbf{R}_+^{n+1}, \mu)$ and this tells us that in particular (μ, v) will satisfy the (F_p) condition.

In the particular case considered in (II), this suggests that for a pair of weights (u, v) satisfying the A_p condition, $1 < p < \infty$, of Muckenhoupt, i.e.

$$\sup_Q \left(\frac{1}{|Q|} \int_Q u(x) dx \right) \left(\frac{1}{|Q|} \int_Q v^{1-p'}(x) dx \right)^{p+1} \leq C$$

the fact that $v \in A_p$ is sufficient for (u, v) to satisfy the S_p condition.

(IV) If a weight v is given and we call $F_p(v)$ (respectively $C_p(v)$) the set of measures μ on \mathbf{R}_+^{n+1} such that (μ, v) satisfies the F_p condition (respectively the C_p condition) we can state that for $1 < p \leq q$

$$C_1(v) \subset F_p(v) \subset C_p(v) \subset \cdots \subset F_q(v) \subset C_q(v) \subset \cdots.$$

The inclusion $C_p(v) \subset C_q(v)$ is proved in [6]. To see that $F_p(v) \subset C_p(v)$ let us observe that for $(x, t) \in \tilde{Q}$

$$\mathcal{M}(v^{1-p'} \chi_Q)(x, t) \geq \frac{1}{|Q|} \int_Q v^{1-p'}(y) dy$$

and this implies for $\mu \in F_p(v)$

$$\left(\frac{1}{|Q|} \int_Q v^{1-p'}(x) dx \right)^p \mu(\tilde{Q}) \leq C \int_Q v^{1-p'}(x) dx.$$

So $\mu \in C_p(v)$.

Now, given $p < q$, and $\mu \in C_p(v)$, using the Marcinkiewicz interpolation theorem between the boundedness of \mathcal{M} from $L^p(\mathbf{R}^n, v(x) dx)$ into weak- $L^p(\mathbf{R}_+^{n+1}, \mu)$ and the trivial L^∞ boundedness, we obtain $\mu \in F_q(v)$.

REMARK. If, for a given p , v belongs to the A_p class of Muckenhoupt, then it can be shown that $C_p(v) = C_q(v)$, $p \leq q \leq \infty$, see [6]. This fact and the fact that for $v \in A_p$ there exists $\varepsilon > 0$ such that $v \in A_{p-\varepsilon}$ allows us to obtain that

$$F_p(v) = C_p(v) = C_q(v) = F_q(v), \quad p \leq q \leq \infty.$$

3. Detailed proofs. The proof of the implication (i) \Rightarrow (ii) is the same in both Theorems A and B, the only difference being the use of non dyadic or dyadic cubes.

Firstly, let us see that $\int_Q v^{1-p'}(x) dx < +\infty$ for all cubes. If

$$\int_Q v^{1-p'}(x) dx = \int_Q (v^{-1}(x))^{p'} v(x) dx = \infty$$

this would imply the existence of a function $f \in L^p(v)$ such that

$$\int_Q f(x) dx = \int_Q f(x) v^{-1}(x) v(x) dx = +\infty,$$

and in particular $\mathcal{M}f(x, t) = +\infty$ for $(x, t) \in \mathbf{R}_+^{n+1}$ which contradicts the hypothesis:

$$\int_{\mathbf{R}^{n+1}} [\mathcal{M}f(x, t)]^p d\mu(x, t) \leq C \int_{\mathbf{R}^n} f^p(x) v(x) dx < +\infty.$$

To show the inequality in (ii) it is sufficient to choose $f(x) = \chi_Q(x) v^{1-p'}(x)$ in the hypothesis.

Proof of (ii) \Rightarrow (i) in Theorem B. In order to handle a Calderón-Zygmund decomposition we introduce the operators

$$\mathcal{N}^R f(x, t) = \sup_Q \frac{1}{Q} \int_Q |f(x)| dx, \quad x \in \mathbf{R}^n, t > 0,$$

the supremum being taken over all dyadic cubes in \mathbf{R}^n containing x and having side length at least t and at most R .

Observe that $\mathcal{N}^R f(x, t) = 0$ for $t > R$ and that

$$\lim_{R \rightarrow \infty} \mathcal{N}^R f(x, t) = \mathcal{N}f(x, t)$$

with increasing limit.

Let Ω_k be the set

$$\Omega_k = \{(x, t): \mathcal{N}^R f(x, t) > 2^k\}, \quad k \in \mathbf{Z}.$$

LEMMA. For each $k \in \mathbf{Z}$ there exists a family $\{Q_j^k\}$, $j \in J_k$, of dyadic cubes in \mathbf{R}^n such that

- (i) $1/Q_j^k \int_{Q_j^k} |f(x)| dx > 2^k$.
- (ii) The interiors of \tilde{Q}_j^k are disjoint
- (iii) $\Omega_k = \bigcup_{j \in J_k} \tilde{Q}_j^k$.

Proof of the lemma. If $(x, t) \in \Omega_k$ it means that there exists a dyadic cube with $x \in Q$, $l(Q) \geq t$, $l(Q) \leq R$ and $1/|Q| \int_Q |f(x)| dx > 2^k$. This implies the existence of a dyadic maximal Q_j^k such that $Q \subset Q_j^k$, $l(Q_j^k) \leq R$, $l(Q_j^k) \geq t$ and

$$\frac{1}{|Q_j^k|} \int_{Q_j^k} |f(x)| dx > 2^k.$$

In particular, $(x, t) \in \tilde{Q}_j^k$. The fact that the interiors of \tilde{Q}_j^k are disjoint is an obvious consequence of the same property for the Q_j^k 's.

Now let us consider the sets

$$E_j^k = \tilde{Q}_j^k \setminus \{(x, t): \mathcal{N}^R f(x, t) \geq 2^{k+1}\}.$$

Then we have a family of sets $\{E_j^k\}_{j,k}$ with disjoint interiors and

$$\begin{aligned} \int_{\mathbf{R}^{n+1}} [\mathcal{N}^R f(x, t)]^p d\mu(x, t) &\leq \sum_{k,j} \int_{E_j^k} [\mathcal{N}^R f(x, t)]^p d\mu(x, t) \\ &\leq \sum_{j,k} 2^{(k+1)p} \mu(E_j^k) \leq 2^p \sum_{j,k} \mu(E_j^k) \frac{1}{|Q_j^k|} \left(\int_{Q_j^k} |f(x)| dx \right)^p. \end{aligned}$$

Following the ideas of Sawyer [7] and Jawerth [4], we introduce the following notations:

$$\sigma(x) = v^{1-p'}(x), \quad \sigma(Q) = \int_Q \sigma(x) dx$$

$$\gamma_{jk} = \mu(E_j^k) \left(\frac{\sigma(Q_j^k)}{|Q_j^k|} \right)^p,$$

$$g_{jk} = \left(\frac{1}{\sigma(Q_j^k)} \int_{Q_j^k} \frac{|f(x)|}{\sigma(x)} \sigma(x) dx \right)^p$$

$$X = \{(k, j) : k \in \mathbf{Z}, j \in J_k\} \quad \text{with atomic measure } \gamma_{jk}.$$

$$\Gamma(\lambda) = \{(k, j) \in X : g_{jk} > \lambda\}.$$

Then we can write

$$\begin{aligned} \int_{\mathbf{R}^{n+1}} [\mathcal{N}^R f(x, t)]^p d\mu(x, t) &\leq 2^p \sum_{j,k} \gamma_{jk} g_{jk} \\ &= 2^p \int_0^\infty \gamma\{(k, j) : g_{jk} > \lambda\} d\lambda = 2^p \int_0^\infty \left\{ \sum_{(k,j) \in \Gamma(\lambda)} \gamma_{jk} \right\} d\lambda \\ &= 2^p \int_0^\infty \sum_{(k,j) \in \Gamma(\lambda)} \mu(E_j^k) \left(\frac{\sigma(Q_j^k)}{|Q_j^k|} \right)^p d\lambda \\ &= 2^p \int_0^\infty \sum_{(k,j) \in \Gamma(\lambda)} \int_{E_j^k} \left(\frac{\sigma(Q_j^k)}{|Q_j^k|} \right)^p d\mu(x, t) d\lambda, \end{aligned}$$

calling Q_i the maximal cubes of the family $\{Q_j^k : (k, j) \in \Gamma(\lambda)\}$. This is equal to

$$\begin{aligned} &2^p \int_0^\infty \sum_j \sum_{\substack{(k,j) \in \Gamma(\lambda) \\ Q_j^k \subset Q_i}} \int_{E_j^k} \left(\frac{\sigma(Q_j^k)}{|Q_j^k|} \right)^p d\mu(x, t) d\lambda \\ &\leq 2^p \int_0^\infty \sum_i \sum_{\substack{(k,j) \in \Gamma(\lambda) \\ Q_j^k \subset Q_i}} \int_{E_j^k} (\mathcal{N}^R(\sigma \chi_{Q_i})(x, t))^p d\mu(x, t) d\lambda \end{aligned}$$

by the disjointness of the E_j^k 's. This is less than

$$2^p \int_0^\infty \sum_i \int_{\tilde{Q}_i} \left(\mathcal{N}^R(\sigma \chi_{Q_i})(x, t) \right)^p d\mu(x, t) d\lambda.$$

Following hypothesis (i) this is less than

$$\begin{aligned} 2^p \int_0^\infty \sum_i \left(\int_{Q_i} \sigma(x) dx \right) d\lambda &= 2^p \int_0^\infty \sigma\left(\bigcup Q_i\right) d\lambda \\ &= 2^p \int \sigma\left(\bigcup_{(k,j) \in \Gamma(\lambda)} Q_j^k\right) d\lambda. \end{aligned}$$

The definition of $\Gamma(\lambda)$ states that

$$\bigcup_{(k,j) \in \Gamma(\lambda)} Q_j^k \subset \left\{ x: N_\sigma\left(\frac{|f|}{\sigma}\right)(x) > \lambda^{1/p} \right\}$$

where

$$N_\sigma(x) = \sup \frac{1}{\sigma(Q)} \int_Q g(x) \sigma(x) dx,$$

the supremum being taken over all dyadic cubes in \mathbf{R}^n containing x .

Then we have

$$\begin{aligned} \int_{\mathbf{R}^{n+1}} [\mathcal{N}^R f(x, t)]^p d\mu(x, t) &\leq 2^p \int_0^\infty \sigma\left\{ x: \left(N_\sigma\left(\frac{|f|}{\sigma}\right)(x) \right)^p > \lambda \right\} d\lambda \\ &= 2^p \int_{\mathbf{R}^n} \left(N_\sigma\left(\frac{|f|}{\sigma}\right)(x) \right)^p \sigma(x) dx \\ &\leq 2^p \int_{\mathbf{R}^n} \frac{|f(x)|^p}{\sigma(x)^p} \sigma(x) dx \end{aligned}$$

since the dyadic maximal operator with respect to any positive measure ν maps $L^p(d\nu)$, $1 < p < \infty$, into itself.

The proof ends by applying Fatou's lemma and observing that $\sigma^{1-p} = \nu$.

Proof of (ii) \Rightarrow (i) in Theorem A. The proof of this part follows easily from the ensuing lemma due to Sawyer [7].

LEMMA 2. Define for each $y \in \mathbf{R}^n$

$${}_y \mathcal{N} f(x, t) = \sup \frac{1}{|Q|} \int_Q |f(u)| du,$$

the supremum being taken in all cubes Q with $x \in Q$, side length less than t and such that the set $Q - y = \{u - y: u \in Q\}$ is a dyadic cube. Then,

$$\mathcal{M}^{2^k} f(x, t) \leq C \int_{[-2^{k+2}, 2^{k+2}]^n} {}^y \mathcal{N} f(x, t) \frac{dy}{2^{n(k+3)}}$$

where the constant C depends only on the dimension.

By \mathcal{M}^R we mean the maximal operator obtained considering cubes with side length less than R .

Observe that the proof of Theorem B can be repeated for the operator ${}^\lambda \mathcal{N}$ where the dyadic cubes are now of the type $\prod_{i=1}^n [x_i, x_{i+} + 2^k]$ with $x - y \in 2^k \mathbb{Z}^n$.

Then, by Lemma 2 we have

$$\begin{aligned} & \int_{\mathbb{R}_+^{n+1}} [\mathcal{M}^{2^k} f(x, t)]^p d\mu(x, t) \\ & \leq C \int_{\mathbb{R}_+^{n+1}} d\mu(x, t) \left[\int_{[-2^{k+2}, 2^{k+2}]^n} {}^y \mathcal{N} f(x, t) \frac{dy}{2^{n(k+3)}} \right]^p \\ & \leq C \left[\int_{[-2^{k+2}, 2^{k+2}]^n} \frac{dy}{2^{n(k+3)}} \left(\int |f(x)|^p v(x) dx \right)^{1/p} \right]^p \\ & = C \int |f(x)|^p v(x) dx. \end{aligned}$$

By letting $k \rightarrow \infty$ we conclude (i) in Theorem A.

The proof of Lemma 2 follows along the lines of the corresponding result in [7] and is therefore omitted.

Acknowledgments. The authors are grateful to José L. Rubio for his helpful suggestions.

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Received November 21, 1983 and in revised form April 4, 1984.

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