

Pacific Journal of Mathematics

THE SOLUTION TO CRAWLEY'S PROBLEM

ALAN HARVEY MEKLER AND SAHARON SHELAH

THE SOLUTION TO CRAWLEY'S PROBLEM

ALAN H. MEKLER AND SAHARON SHELAH

In this supplement to our paper, ω -elongations and Crawley's problem, we show that if $(V = L)$ every Crawley group is a direct sum of cyclic groups.

A simple argument completes the work begun in [MS] and allows us to show if that if $(V = L)$ then every Crawley group (no matter what its cardinality) is a direct sum of cyclic groups. (See [MS] for definitions and conventions.) We first note that in Theorem 2.2 of [MS] we prove something stronger. Namely

THEOREM 1. *Assume $(V = L)$ and let G be a group of cardinality at most \aleph_1 such that $p^\omega G \simeq Z(p)$. A separable group A is a direct sum of cyclic groups iff for every ω -elongation H of $Z(p)$ by A there is a homomorphism f from H to G such that $f(p^\omega H) \neq 0$.*

LEMMA 2. *Suppose A is a separable group of length ω . There is an ω -elongation H of $Z(p)$ by A and a group G such that $p^\omega G \simeq Z(p)$, $|G| \leq 2^{\aleph_0}$, and there is a homomorphism f from H to G with $f(p^\omega H) \neq 0$.*

Proof. Choose $B \subseteq A$ a basic subgroup and write $B = B_0 \oplus B_1$ where B_0 is countable with elements of arbitrarily large order. Let A^* be the closure of B_1 (i.e. A^* is the maximal subgroup of A so that A^*/B_1 is divisible). The subgroup $A^* + B_0 = A^* \oplus B_0$. Choose H_0 an ω -elongation of $Z(p)$ by B_0 . Let $H_1 = A^* \oplus H_0$. Finally choose H an ω -elongation of $Z(p)$ by A containing H_1 . Since $H \supseteq A^*$, we can let $G = H/A^*$. Let t generate $p^\omega H$. We have the sequence $\langle t + A^* \rangle \twoheadrightarrow G \twoheadrightarrow A/A^*$. Since $t \notin A^*$, to complete the proof we only need to note that A/A^* is separable and $|A/A^*| \leq 2^{\aleph_0}$. Both of these claims are easy to establish by first choosing an independent set of generators of B_0 and then identifying A/A^* with a group of formal sums of multiples of these generators.

THEOREM 3. *Assume $(V = L)$. Every Crawley group is a direct sum of cyclic groups.*

Proof. Suppose A is a separable group which is not the direct sum of cyclic groups. By Lemma 2 we can choose an ω -elongation H of $Z(p)$ by A and a group G such that $p^\omega G \simeq Z(p)$; $|G| \leq 2^{\aleph_0} (= \aleph_1)$; and there is a homomorphism $f: H \rightarrow G$ with $f(p^\omega H) \neq 0$. But by Theorem 2.2 there is an ω -elongation H' of $Z(p)$ by A such that there is no homomorphism $g: H' \rightarrow G$ with $g(p^\omega H') \neq 0$. Hence A is not a Crawley group.

REFERENCES

- [MS] A. Mekler, and S. Shelah, *ω -elongations and Crawley's problem*, Pacific J. Math., **121** (1986), 121–132.

Received June 19, 1985. Research by the first author was supported by NSERC Grant A8948. Research by the second author was supported by NSERC Grant A3040.

SIMON FRASER UNIVERSITY
BURNABY, B.C., CANADA V5A 1S6

AND

THE HEBREW UNIVERSITY
JERUSALEM, ISRAEL

PACIFIC JOURNAL OF MATHEMATICS

EDITORS

V. S. VARADARAJAN (Managing Editor)
University of California
Los Angeles, CA 90024

HEBERT CLEMENS
University of Utah
Salt Lake City, UT 84112

CHARLES R. DEPRIMA
California Institute of Technology
Pasadena, CA 91125

R. FINN
Stanford University
Stanford, CA 94305

HERMANN FLASCHKA
University of Arizona
Tucson, AZ 85721

RAMESH A. GANGOLLI
University of Washington
Seattle, WA 98195

ROBION KIRBY
University of California
Berkeley, CA 94720

C. C. MOORE
University of California
Berkeley, CA 94720

H. SAMELSON
Stanford University
Stanford, CA 94305

HAROLD STARK
University of California, San Diego
La Jolla, CA 92093

ASSOCIATE EDITORS

R. ARENS E. F. BECKENBACH B. H. NEUMANN F. WOLF K. YOSHIDA
(1906–1982)

SUPPORTING INSTITUTIONS

UNIVERSITY OF ARIZONA
UNIVERSITY OF BRITISH COLUMBIA
CALIFORNIA INSTITUTE OF TECHNOLOGY
UNIVERSITY OF CALIFORNIA
MONTANA STATE UNIVERSITY
UNIVERSITY OF NEVADA, RENO
NEW MEXICO STATE UNIVERSITY
OREGON STATE UNIVERSITY

UNIVERSITY OF OREGON
UNIVERSITY OF SOUTHERN CALIFORNIA
STANFORD UNIVERSITY
UNIVERSITY OF HAWAII
UNIVERSITY OF TOKYO
UNIVERSITY OF UTAH
WASHINGTON STATE UNIVERSITY
UNIVERSITY OF WASHINGTON

Om P. Agrawal, Douglas Napier Clark and Ronald George Douglas, Invariant subspaces in the polydisk	1
Christoph Bandt and Gebreselassie Baraki, Metrically invariant measures on locally homogeneous spaces and hyperspaces	13
Marcy Mason Barge, Horseshoe maps and inverse limits	29
Russell Gene Bilyeu, Robert Richard Kallman and Paul Weldon Lewis, Rearrangements and category	41
Jean Bourgain, A problem of Douglas and Rudin on factorization	47
Hernan Cendra, A normal form and integration in finite terms for a class of elementary functions	51
Ky Fan, The angular derivative of an operator-valued analytic function	67
Gerhard Gierz, On the Dunford-Pettis property of function modules of abstract L -spaces	73
Gabriel Katz, On polynomial generators in the algebra of complex functions on a compact space	83
Ridgley Lange, Duality and asymptotic spectral decompositions	93
Anthony To-Ming Lau and Peter F. Mah, Quasinormal structures for certain spaces of operators on a Hilbert space	109
R. Daniel Mauldin, Correction: "The set of continuous nowhere differentiable functions"	119
Alan Harvey Mekler and Saharon Shelah, ω -elongations and Crawley's problem	121
Alan Harvey Mekler and Saharon Shelah, The solution to Crawley's problem	133
Richard Rochberg, Deformation of uniform algebras on Riemann surfaces	135
Joseph Roitberg, On weak epimorphisms in homotopy theory	183
Jesús M. Ruiz, A remark on fields with the dense orbits property	189
Henry Wente, Counterexample to a conjecture of H. Hopf	193
David G. Wright, Rigid sets in E^n	245