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In this paper we generalize a well-known result of Frankel which relates the fundamental group of a complete Riemannian manifold with positive Ricci curvature to the fundamental group of a compact immersed minimal hypersurface. Here we consider the situation in which the Ricci curvature of the ambient manifold is only assumed to be nonnegative, and show that the conclusion of Frankel's theorem can fail only under special circumstances.

1. Introduction. The theory of minimal surfaces has provided a powerful tool for studying the topology of complete Riemannian manifolds of low dimension with nonnegative scalar or Ricci curvature. In general, it is a problem of basic interest to study the topological and geometrical relationships between a minimal submanifold and the manifold in which it is immersed. A well-known result of Frankel [**1**] asserts that if Σ is a compact immersed minimal hypersurface in a Riemannian manifold M with strictly positive Ricci curvature then the homomorphism of fundamental groups: $\Pi(\Sigma) \to \Pi(M)$ induced by inclusion is onto. As the product of spheres $S^1 \times S^2$ shows, the conclusion of Frankel's theorem is false if the Ricci curvature is only assumed to be nonnegative. The purpose of this paper is to study the rigidity of Frankel's theorem, i.e. to study the extent to which Frankel's theorem can fail when the Ricci curvature is only assumed to be nonnegative. Our main theorem, stated below, shows that the theorem can fail only under special circumstances.

THEOREM. *Let M be a complete n-dimensional Riemannian manifold with nonnegative Ricci curvature. Let $\varphi\colon \Lambda \to M$ be a minimal immersion, where Λ is a compact $(n-1)$-dimensional manifold, and let $\Sigma = \varphi(\Lambda)$. Consider the homomorphism of fundamental groups $i_*\colon \Pi(\Sigma) \to \Pi(M)$ induced by the inclusion map $i\colon \Sigma \to M$. Then either* (a) *below holds, or Σ is an imbedded totally geodesic submanifold of M and one of* (b)–(e) *holds.*

(a) *i_* is onto.*

(b) *$\Pi(M)/i_*(\Pi(\Sigma)) = Z_2$. Σ separates M, and the closure of one of the components of $M - \Sigma$ has a double covering which is isometric to $[0, L] \times \Sigma$.*

(c) *M is isometric to the (possibly twisted) product of S^1 and Σ. (More precisely, Σ is two-sided and $M - \Sigma$ is isometric to $(0, L) \times \Sigma$, where the isometry is given by exponentiating normally from one side of Σ to the other).*

(d) *Σ separates M, and a double covering of M is isometric to the (possibly twisted) product of S^1 and Σ.*

(e) *Σ is one-sided, and a double covering of M is isometric to the (possibly twisted) product of S^1 and a double covering of Σ.*

In addition, if i_* is not onto, the Ricci curvature vanishes on all vectors orthogonal to Σ.

We remark that if one assumes M and Σ are orientable then case (e), and only case (e), can be eliminated. There are simple models illustrating each of the possibilities (a)–(e).

Case (a). M = projective 2-space realized as S^2 with antipodal points identified; Σ = equator with antipodal points identified.

Case (b). M = projective 2-space realized as S^2 flattened near the equator, with antipodal points identified; Σ = circle of latitude near the equator.

Case (c) (*a twisted example*). M = flat Klein bottle realized as a square with horizontal sides identified in the opposite direction, and vertical sides identified in the same direction; $\Sigma = S^1$ realized as a horizontal line segment joining the vertical edges.

Case (d). M as in case (c); $\Sigma = S^1$ realized as two vertical line segments symmetrically spaced around the central vertical line.

Case (e). M as in case (c); $\Sigma = S^1$ realized as the central vertical line.

Lawson ([**4**], Theorem 1) has obtained a result closely related to Frankel's. He proves that if M is a compact connected Riemannian manifold with mean convex boundary ∂M and if M has positive Ricci curvature then ∂M is connected and the homomorphism $\Pi(\partial M) \to \Pi(M)$ induced by inclusion is onto. Meeks ([**6**], Proposition 1) has considered the rigidity of Lawson's result in the case M is flat. He shows i_*: $\Pi(\partial M) \to \Pi(M)$ is onto unless ∂M is totally geodesic, and observes that if i_* is not onto then $\Pi(M)/i_*(\Pi(\partial M)) = Z_2$. The proof of our main theorem makes use of an appropriately generalized version of Meek's result. We

would also like to mention here the paper of Meyer [8] which is related to various aspects of our work.

2. The proof. We begin with a number of preliminary results. The first result is the principal geometric tool upon which the proof of the main theorem relies.

LEMMA 1. *Let M be a complete connected Riemannian manifold with mean convex boundary ∂M which has at least two components, Σ_1 and Σ_2. If M has nonnegative Ricci curvature and Σ_1 is compact then $\partial M = \Sigma_1 \cup \Sigma_2$ and M is isometric to $[0, L] \times \Sigma_1$, where the isometry is given by exponentiating normally from Σ_1 to Σ_2. In particular, Σ_1 and Σ_2 are isometric.*

Ichida [2] and Kasue [3] have given proofs of this result by different methods. The proof in a less general setting is implicitly discussed in Meeks ([6], Proposition 1). The following lemma generalizes Proposition 1 in Meeks [6] and Theorem 1 in Lawson [4]. The idea of the proof is similar to that of Proposition 1 in Meeks [6].

LEMMA 2. *Let M be a complete connected Riemannian manifold with compact connected boundary Σ. Assume M has nonnegative Ricci curvature and Σ is mean convex. Then, either*

(a) $i_*\colon \Pi(\Sigma) \to \Pi(M)$ *is onto, or*

(b) $\Pi(M)/i_*(\Pi(\Sigma)) = Z_2$, *and M has a double covering which is isometric to $[0, L] \times \Sigma$. In particular, Σ is totally geodesic and the Ricci curvature vanishes on all vectors orthogonal to Σ.*

Proof of Lemma 2. The proof is an application of Lemma 1. Let $(\tilde{M}, p)$ be the Riemannian covering manifold of M such that $p_*(\Pi(\tilde{M}, \tilde{s})) = i_*(\Pi(\Sigma, s))$. Let $\tilde{\Sigma}_0$ be the component of $p^{-1}(\Sigma)$ passing through $\tilde{s}$. We mention two simple properties of $(\tilde{M}, p)$: (i) Any loop in $\tilde{M}$ based at $\tilde{s}$ is fixed end point homotopic to a loop in $\tilde{\Sigma}_0$ based at $\tilde{s}$, and (ii) $p \mid \tilde{\Sigma}_0\colon \tilde{\Sigma}_0 \to \Sigma$ is injective (and hence an isometry).

Suppose i_* is not onto. Then there exists a component $\tilde{\Sigma}$ of $\partial\tilde{M}$ distinct from $\tilde{\Sigma}_0$. By Lemma 1, $\partial\tilde{M} = \tilde{\Sigma}_0 \cup \tilde{\Sigma}$, and $\tilde{M}$ is isometric to $[0, L] \times \tilde{\Sigma}_0$. It follows that Σ is totally geodesic and that the Ricci curvature vanishes on vectors normal to Σ. Since $p \mid \tilde{\Sigma}\colon \tilde{\Sigma} \to \Sigma$ is a covering map, we have $\mathrm{vol}(\tilde{\Sigma}) = k\,\mathrm{vol}(\Sigma)$, where k is the number of sheets of the covering $(\tilde{\Sigma}, p \mid \tilde{\Sigma})$. Thus, since $\tilde{\Sigma}$ is isometric to $\tilde{\Sigma}_0$, and $\tilde{\Sigma}_0$ is isometric to Σ, $(\tilde{\Sigma}, p \mid \tilde{\Sigma})$ must be a single sheeted covering of Σ, i.e. $p \mid \tilde{\Sigma}\colon \tilde{\Sigma} \to \Sigma$ must be injective.

Since $p^{-1}(\Sigma)$ consists of two copies of Σ we have that $(\tilde{M}, p)$ is a *double* covering of M. Statement (b) now follows from the fact that, for regular coverings, the group of deck transformations (which in the present case is Z_2) is isomorphic to $\Pi(M, s)/p_*\Pi(\tilde{M}, \tilde{s})$.

The following lemma is the basic imbeddedness result needed for the proof of the main theorem.

LEMMA 3. *Let M be an n-dimensional complete connected Riemannian manifold with nonnegative Ricci curvature. Let φ_α: $\Lambda_\alpha \to M$, $\alpha = 1, 2$, be minimal immersions such that $\dim \Lambda_\alpha = n - 1$, and let $\Sigma_\alpha = \varphi_\alpha(\Lambda_\alpha)$. Assume Λ_1 is compact and Σ_2 is closed. If Σ_1 and Σ_2 don't meet then they must be totally geodesic imbedded submanifolds of M, and Σ_2 is compact.*

Proof of Lemma 3. The proof is an application of the maximum principle for the mean curvature equation (cf. Ichida [**2**]). Without loss of generality we can assume that φ_1: $\Lambda_1 \to M$ is a two-sided immersion, by which we mean there exists a smooth nonvanishing normal vector field defined along φ_1, i.e., there exists a smooth map N: $\Lambda_1 \to TM$ such that for each $p \in \Lambda_1$, $N(p)$ is a vector at $T_{\varphi_1(p)}M$ orthogonal to $d\varphi_1(T_p\Lambda_1)$. Indeed, if φ_1 is not two-sided then by standard covering space arguments there exists a two-sheeted covering $(\tilde{\Lambda}_1, p)$ of Λ_1 such that $\varphi_1 \circ p$: $\tilde{\Lambda}_1 \to M$ is two-sided.

Let $L > 0$ be the distance between Σ_1 and Σ_2. Let γ: $[0, L] \to M$ be any unit speed geodesic segment, with $\gamma(0) = q_1 \in \Sigma_1$ and $\gamma(L) = q_2 \in \Sigma_2$, which achieves this minimum. (Since Σ_1 is compact and Σ_2 is closed, at least one such γ exists). γ strikes Σ_1 and Σ_2 orthogonally. In particular, there can be no transverse self-intersections at q_1 and q_2. For each $\alpha = 1, 2$, choose a point $p_\alpha \in \varphi_\alpha^{-1}(q_\alpha)$ and a neighborhood U_α of p_α such that $\varphi_\alpha|_{U_\alpha}$: $U_\alpha \to M$ is an imbedding., Set $V_\alpha = \varphi_\alpha(U_\alpha)$. By the lemma in Section 3 of Ichida [**2**], whose proof is an application of the maximum principle for the mean curvature equation, there is a neighborhood $W_1 \subset U_1$ of p_1 such that $\varphi_1(W_1)$ is totally geodesic and the map E: $W_1 \to M$ defined by $E(p) = \exp_{\varphi_1(p)} LN(p)$, where N is the normal vector field defined along φ_1 which equals $\gamma'(0)$ at q_1, is a smooth isometric imbedding such that $E(W_1) \subset V_2$ and $E(W_1)$ is totally geodesic. (The metric on W_1 is the pullback of the induced metric on V_1).

Now fix a geodesic segment γ: $[0, L] \to M$ which minimizes the distance from Σ_1 to Σ_2, and let N be the normal vector field along φ_1 which agrees with $\gamma'(0)$ at $\gamma(0)$. The set $W = \{ p \in \Sigma$: $\exp_{\varphi_1(p)} LN(p) \in \Sigma_2 \}$ is closed by the continuity of the exponential map, and open by the

discussion in the previous paragraph. Thus, by connectedness, $W = \Lambda_1$ which implies that each geodesic issuing from Σ_1 with initial direction N achieves the minimum distance between Σ_1 and Σ_2. From the discussion in the preceding paragraph Σ_1 is totally geodesic. Furthermore, by another connectedness argument we see that $\Sigma_2 = \{\exp_{\varphi_1(p)} LN(p)\colon p \in \Lambda_1\}$, and hence Σ_2 is compact and totally geodesic, as well.

We have shown that every point in Σ_1 is the minimum distance L from Σ_2 (and vice-versa). Thus, any self-intersections in Σ_1 must be tangential. Let $x_1, x_2 \in \Sigma_1$ be such that $\varphi_1(x_1) = \varphi_1(x_2) = y$. Let S_1, S_2 be neighborhoods of x_1 and x_2, respectively, such that $\varphi_1 : S_\alpha \to M$, $\alpha = 1, 2$, are imbeddings. Since $\varphi_1(S_1)$ and $\varphi_1(S_2)$ meet tangentially at y and are totally geodesic, there exist neighborhoods $Q_\alpha \subset S_\alpha$, of x_α, $\alpha = 1, 2$, such that $\varphi_1(Q_1) = \varphi_1(Q_2)$. Using this observation and the compactness of Λ_1, one can show that for each $p \in \Sigma_1$, there exists an open set o in M containing p and an open set W in Σ such that $\varphi_{1|W}$: $W \to M$ is an imbedding and $\varphi_1(W) = \Sigma_1 \cap o$. It follows that Σ_1 is an imbedded submanifold. A similar argument shows that Σ_2 is imbedded.

COROLLARY. *Let the setting be as in the main theorem. If i_* is not onto then Σ is an imbedded totally geodesic submanifold of M.*

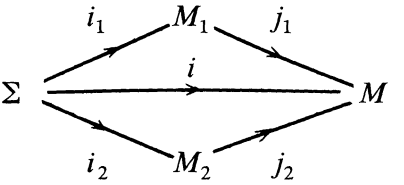
Proof of the Corollary. Let $(\tilde{M}, p)$ be the Riemannian covering manifold of M satisfying $p_*(\Pi(\tilde{M}, \tilde{s})) = i_*(\Pi(\Sigma, s))$. (This covering was introduced in the proof of Lemma 2, but in a slightly different context. The properties mentioned there hold in the present context as well.) Let $\tilde{\Sigma}_0$ be the component of $p^{-1}(\Sigma)$ containing $\tilde{s}$. Since i_* is not onto, there exists at least one other component, $\tilde{\Sigma}$, say, of $p^{-1}(\Sigma)$. Using basic covering space theory (see e.g. Massey [**5**]) there exist immersions φ_1: $\Lambda \to \tilde{M}$ and φ_2: $\tilde{\Lambda} \to \tilde{M}$, where $\tilde{\Lambda}$ is a manifold covering Λ, such that $\varphi_1(\Lambda) = \tilde{\Sigma}_0$, φ_2 is proper, and $\varphi_2(\tilde{\Lambda}) \subset \tilde{\Sigma}$. (Take $(\tilde{\Lambda}, p_1)$ to be the covering manifold of Λ satisfying $(p_1)_*(\Pi(\tilde{\Lambda})) = \varphi^{-1}{}_*(p_*(\Pi(\tilde{M}))$.) Hence, by Lemma 3, $\tilde{\Sigma}_0$ is imbedded and totally geodesic. Since p: $\tilde{M} \to M$ is a local isometry the same conclusions apply to Σ.

Proof of the Theorem. In view of the corollary to Lemma 3 it is sufficient to prove the main theorem under the assumption that Σ is a compact imbedded minimal hypersurface in M.

Assume for the time being that Σ is two-sided. Suppose Σ does not separate M. Since Σ is two-sided we can make a "cut" along Σ to obtain a manifold with boundary M', whose boundary consists of two disjoint copies of Σ, Σ_1 and Σ_2 say, such that any curve in M' from Σ_1 to Σ_2

corresponds to a curve in M from one side of Σ to the other. Lemma 1 implies that the normal exponential map Φ: $[0, L] \times \Sigma_1 \to M'$, where $L = d(\Sigma_1, \Sigma_2)$, is an isometry. Thus, part (c) of the theorem holds.

Suppose now that Σ separates M. Let M_1 and M_2 denote the closures of the two components of $M - \Sigma$. Hence, M_1 and M_2 are manifolds with common boundary Σ. Consider the following diagram of inclusion maps,



Set $G_0 = i_*(\Pi(\Sigma))$, $H_\alpha = (i_\alpha)_*(\Pi(\Sigma))$, $G_\alpha = (j_\alpha)_*(\Pi(M_\alpha))$, $\alpha = 1, 2$. By Lemma 2, either $\Pi(M_\alpha) = H_\alpha$ or $\Pi(M_\alpha)/H_\alpha = Z_2$. It then follows easily that either $G_\alpha = G_0$ or $G_\alpha/G_0 = Z_2$, $\alpha = 1, 2$.

We now consider several cases. Recall by the Seifert-Van Kampen theorem that $\Pi(M)$ is generated by G_1 and G_2.

Case 1. $G_1 = G_0$ and $G_2 = G_0$. In this case, $\Pi(M) = G_0$, and statement (a) of the theorem holds.

Case 2. $G_1 \neq G_0$ and $G_2 = G_0$. In this case $\Pi(M) = G_1$. Statement (b) of the theorem now follows from the equality, $G_1/G_0 = Z_2$, and Lemma 2.

Case 3. $G_1 = G_0$ and $G_2 \neq G_0$. As in Case 2, this leads to statement (b) of the theorem.

Case 4. $G_1 \neq G_0$ and $G_2 \neq G_0$. In this case, $\Pi(M_\alpha)/H_\alpha = Z_2$, $\alpha = 1, 2$. For each $\alpha = 1, 2$, let $(\tilde{M}_\alpha, p_\alpha)$ be the covering manifold of M_α which satisfies, $(p_\alpha)_*(\Pi(\tilde{M}_\alpha)) = H_\alpha$. Then $(\tilde{M}_\alpha, p_\alpha)$ is a double covering of M_α, and $\tilde{M}_\alpha$ is a manifold with boundary $p_\alpha^{-1}(\Sigma)$ which consists of two copies of Σ. By appropriately "gluing" together the boundaries of $\tilde{M}_1$ and $\tilde{M}_2$, we obtain a double covering of M which contains two copies of Σ, neither of which separates. But this situation was considered at the beginning of the proof. From what was done there, one sees that we are now led to part (d) of the theorem.

Thus if Σ is two-sided one of the cases (a)–(d) must hold.

Suppose now that Σ is not two-sided. It is then standard, using Mod 2 intersection theory, to construct a two-sheeted covering $(\hat{M}, p)$ of M such that $(\hat{\Sigma} = p^{-1}(\Sigma), p_{|\hat{\Sigma}})$ is a connected double covering of Σ and $\hat{\Sigma}$ is two-sided. In fact this construction actually implies that $\hat{\Sigma}$ separates $\hat{M}$.

Thus, cases (a)–(d) of the main theorem (and the arguments leading to these cases) apply to $\hat{\Sigma} \subset \hat{M}$. We now observe that if i_*: $\Pi(\hat{\Sigma}, \hat{s}) \to \Pi(\hat{M}, \hat{s})$ is onto, so is i_*: $\Pi(\Sigma, s) \to \Pi(M, s)$ (where $\hat{s} \in p^{-1}(s)$). Let $g = \langle \gamma \rangle$ be any element of $\Pi(M, s)$, where γ is a loop based at s. Let $\hat{\gamma}$ be the lift of γ from $\hat{s} \in \hat{\Sigma}$ to $\hat{x} \in \hat{\Sigma}$. Let $\hat{\sigma}$ be a curve in $\hat{\Sigma}$ from $\hat{x}$ to $\hat{s}$. By assumption, $\langle \hat{\gamma}\hat{\sigma} \rangle \in i_*\Pi(\hat{\Sigma}, \hat{s})$. Projecting via the covering map gives $\langle \gamma \rangle \langle \sigma \rangle = \langle \gamma\sigma \rangle \in i_*\Pi(\Sigma, s)$, where $\sigma = p \circ \hat{\sigma}$. Hence, since $\langle \sigma \rangle \in i_*\Pi(\Sigma, s)$, $g = \langle \gamma \rangle \in i_*\Pi(\Sigma, s)$. Thus, if case (a) holds for $\hat{\Sigma} \subset \hat{M}$ then case (a) holds for $\Sigma \subset M$. Since the two components of $\hat{M} - \hat{\Sigma}$ are isometric (via a deck transformation), the discussion leading to case (b) in the two-sided setting shows that case (b) is not applicable to $\hat{\Sigma} \subset \hat{M}$. Case (c) is also ruled out because $\hat{\Sigma}$ separates. It remains to consider case (d).

By our earlier arguments which led to case (d) we may assume that there exists a double covering $(\tilde{M}, \varphi)$ of $\hat{M}$ such that (1) $\varphi^{-1}(\hat{\Sigma})$ consists of two disjoint copies, $\tilde{\Sigma}_1$ and $\tilde{\Sigma}_2$, say, of $\tilde{\Sigma}$ and (2) each component of $\hat{M} - \hat{\Sigma}$ is double covered by a component of $\tilde{M} - (\tilde{\Sigma}_1 \cup \tilde{\Sigma}_2)$. $(\tilde{M}, \psi)$, where $\psi = p \circ \varphi$, is a four-sheeted covering of M. We use the "cut and paste" method on $\tilde{M}$ to obtain a new double covering of M.

Let $\hat{U}$ be a component of $\hat{M} - \hat{\Sigma}$, and let $\tilde{U}$ be the component of $\tilde{M} - (\tilde{\Sigma}_1 \cup \tilde{\Sigma}_2)$ such that $(\tilde{U}, \varphi \mid_{\tilde{U}})$ is a double covering of $\hat{U}$. Since $\hat{U}$ is one of the two components of $p^{-1}(M - \Sigma)$, $(\hat{U}, p \mid_{\hat{U}})$ is a single covering of $M - \Sigma$. Consequently, $(\tilde{U}, \psi \mid_{\tilde{U}})$ is a double covering of $M - \Sigma$. The closure of $\tilde{U}$, call it D, is a manifold with boundary $\tilde{\Sigma}_1 \cup \tilde{\Sigma}_2 = \psi^{-1}(\Sigma)$. Let V denote the manifold without boundary obtained from D by identifying the points of $\tilde{\Sigma}_1$ and $\tilde{\Sigma}_2$ as follows: For all $x_1 \in \tilde{\Sigma}_1$, $x_2 \in \tilde{\Sigma}_2$, identify x_1 and x_2 if and only if $\psi(x_1) = \psi(x_2)$ and $\varphi(x_1) \neq \varphi(x_2)$. Then V (with covering map essentially given by ψ) is a double covering of M such that the inverse image of Σ with respect to the covering map is a double covering of Σ which is two-sided, but which does not separate V. (To visualize this construction, it is helpful to carry it out explicitly on the example of case (e) given after the statement of the main theorem.) The argument of case (c) applied to the present situation now leads to case (e) of the theorem.

Thus, we have shown that one of the cases (a)–(e) must hold. If i_* is not onto then, by the corollary to Lemma 3, Σ is totally geodesic. The last sentence in the statement of the theorem is a consequence of the product structure in cases (b)–(e). □

The following corollary singles out the situation in which M is noncompact. The imbeddedness result needed for this corollary has already been established in Kasue ([3], Theorem 1).

COROLLARY. *Let M be a complete noncompact n-dimensional Riemannian manifold with nonnegative Ricci curvature. Let $\varphi\colon \Lambda \to M$ be a minimal immersion, where Λ is a compact $(n-1)$-dimensional manifold. Then $\Sigma = \varphi(\Lambda)$ is an imbedded totally geodesic submanifold of M and one of the following must hold.*

(1) *M is isometric to $\mathbf{R} \times \Sigma$.*

(2) *M has one end, and there is a double covering of M which is isometric to the product of $\mathbf{R}$ and a double covering of Σ.*

(3) *M has one end and Σ separates M. The closure of the component of $M - \Sigma$ containing this end is isometric to $[0, \infty) \times \Sigma$ and,*

$$(*) \qquad \Pi(M)/i_*(\Pi(\Sigma)) = 1 \text{ or } Z_2.$$

Proof. We begin by observing that $(*)$ holds in all cases. Indeed, since M is noncompact only cases (a) and (b) of the main theorem are applicable. (However note in cases (1) and (2), i_* is actually onto.) By Theorem 1 in Kasue [3], Σ is imbedded and totally geodesic. Suppose Σ is two-sided. Then Σ must separate M (otherwise M would be compact as in case (c) of the main theorem). M has at most two ends. If M has two ends then (as observed in Meeks, Simon and Yau [7], §9) the splitting theorem of Cheeger and Gromoll implies that (1) holds. If M has only one end then Theorem C in Kasue [3] implies that the closure of the component containing this end is isometric to $[0, \infty) \times \Sigma$.

Suppose now that Σ is not two-sided. As discussed earlier, there is a double covering $(\hat{M}, p)$ of M such that $\hat{\Sigma} = p^{-1}(\Sigma)$ is connected and separates $\hat{M}$. Furthermore each component of $\hat{M} - \hat{\Sigma}$ must be noncompact and hence $\hat{M}$ has two ends. Case (1) then applies to $\hat{\Sigma} \subset \hat{M}$, and thus case (2) holds. □

Again, there are simple models illustrating each of the three cases. For instance, the nontrivial line bundle over S^1 (i.e. the Möbius band) is an example illustrating case (2).

As a final remark note that if M is three dimensional and satisfies the hypotheses of the corollary then, by Gauss-Bonnet, Σ must have genus zero or one. All three cases can occur in dimension three. (In the case M is orientable, Theorem 6 in [7] overlooks the possibility that case (3) can occur.)

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