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# GROUPS OF ISOMETRIES OF A TREE AND THE KUNZE-STEIN PHENOMENON

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In this paper we prove that every group of isometries of a homogeneous or semihomogeneous tree which acts transitively on the boundary of the tree is a Kunze-Stein group. From this, we deduce a weak Kunze-Stein property for groups acting simply transitively on a tree (in particular free groups on finitely many generators).

**1. Introduction.** Let G be a locally compact group, then G is said to satisfy the "Kunze-Stein property" or sometimes G is called a "Kunze-Stein group" if  $L^p(G) * L^2(G) \subset L^2(G)$  for every 1 .

This property was discovered by R. A. Kunze and E. M. Stein for the group  $SL_2(\mathbf{R})$  [15]. Later the same property was proved for every connected semisimple Lie group with finite center by M. Cowling [6]. In this paper we prove that every locally compact group of isometries of a homogeneous or semihomogeneous tree has the Kunze-Stein property provided that G acts transitively on the boundary of the tree. The proof of our Theorem is based on M. Cowling's proof of the Kunze-Stein phenomenon for  $SL_2(\mathbf{R})$  [6]. A weaker property is deduced for discrete groups acting simply transitively on the tree but not on the tree boundary.

It is known that the group  $SL_2(\kappa)$ , where  $\kappa$  is a local field, may be realized as a closed subgroup of the group of all isometries of a homogeneous tree in such a way that  $SL_2(\kappa)$  acts transitively on the boundary [17]. In particular our result implies that  $SL_2(\kappa)$  is a Kunze-Stein group for every local field. This was proved by Gulizia [13] for a local field  $\kappa$  such that the finite residue class field associated with  $\kappa$ is not of characteristic 2.

We follow the terminology and definitions of [6]. In particular A(G) is the Fourier algebra of G as defined in [7];  $C_{00}(G)$  denotes the space of continuous functions with compact support and  $L^{p}(G), 1 \leq p \leq \infty$ , the usual  $L^{p}$ -space with respect to a fixed left Haar measure. As observed in [6], a locally compact group G is a Kunze-Stein group if and only if  $A(G) \subset L^{q}(G)$  for every q > 2. We will also use the theory of representations for groups acting on a tree developed by P. Cartier

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[3], A. Figà-Talamanca and M. A. Picardello [11, 12]. A convenient reference is [12]. In fact the results we quote and use from [12] are all valid with essentially the same proof when a discrete group acting simply transitively on a tree replaces the free group [1].

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2. Notations. We shall give a concise description of the tree and of the group of isometries. We refer the reader to [3, 17, 18] for undefined notions and terminology. Let X be a homogeneous tree of order r; the distance d(x, y) is defined as the length of the unique geodesic [x, y] connecting x to y. Let Aut(X) be the group of all isometries of X. We assume also r > 3 (otherwise, for r = 2, Aut(X) is amenable and noncompact, hence it is not a Kunze-Stein group). Aut(X) is a locally compact separable group and the stability subgroup K of a vertex of X is compact and open in Aut(X). A subgroup  $\Gamma$  of Aut(X) is called simply transitive if it acts transitively on the vertices and  $\Gamma \cap K = \{1\}$ . In other words,  $\Gamma$  acts simply transitively on X iff the map  $\gamma \in \Gamma \to \gamma(x_0) \in X$  is bijective for a fixed vertex  $x_0$  in X. It is known that every such group is isomorphic to the free product of t copies of the integers and s copies of the group of order 2 with 2t + s = r [1, 4]. Since K is open,  $\Gamma$  is discrete in Aut(X). Moreover  $\Gamma \cdot K = \operatorname{Aut}(X)$  and  $\Gamma$  is a lattice. As usual, let  $\langle f, h \rangle = \int f(g)h(g) dg$ .

Let  $\Omega$  be the boundary of the tree, that is the set of equivalence classes of sequences of distinct vertices  $\{s_n: n = 0, 1, 2, ...\}$  such that  $d(s_i, s_{i+1}) = 1$  for every i = 0, 1, 2, ...; two such sequences are said to be equivalent if they have infinitely many common vertices.

 $\Omega$  is a compact metric space; if  $x_0 \in X$  and  $\omega_0 \in \Omega$  there exists a unique sequence of distinct vertices  $\{s_n\}$  in the class  $\omega_0$  such that  $s_0 = x_0$ . In this way,  $\Omega$  can be regarded as the set of infinite sequences starting from a fixed vertex  $x_0$  in X. There exists a unique probability measure  $\nu$  on  $\Omega$ , Aut(X)-quasi invariant and K-invariant. Let  $P(g, \omega)$ be the Poisson kernel, that is, for  $g \in Aut(X)$  and  $\omega \in \Omega$ ,  $P(g, \omega) = d\nu_g/d\nu(\omega)$ , with  $\nu_g(\omega) = \nu(g^{-1}\omega)$ .

For every complex number z, we define the following representation of Aut(X):

$$[\pi_z(g)f](\omega) = P^z(g,\omega)f(g^{-1}\omega).$$

It is known that, for  $t \in \mathbb{R}$ ,  $\pi_{1/2+it}$  are unitary irreducible representations on  $L^2(\Omega)$ ; in fact even the restrictions to  $\Gamma$  are irreducible [12, pg. 76; 1].

For a fixed vertex  $x_0$  in X, let  $X^+ = \{x \in X : d(x, x_0) \text{ is even }\}$  and  $X^- = X \setminus X^+$ . The partition  $X^+$ ,  $X^-$  is independent of the choice of  $x_0$ . If G is a closed unimodular subgroup of Aut(X) acting transitively on  $X^+$  but not on the tree, then the representations  $\pi_{1/2+it}|_G$  are irreducible for  $t \neq (2m+1)\pi/2lg(r-1)$ ,  $m \in Z$  [2, pg. 39, pg. 62]. Let J be the interval  $[0, \pi/lg(r-1)]$  and c(z) the following complex function:

$$c(z) = [(r-1)^{2-2z} - 1]/[(r-1)^{1-2z} - 1].$$

Finally, let *dm* be the following measure:

$$dm(t) = [(r-1)lg(r-1)/4\pi r |c(\frac{1}{2}+it)|^2] dt.$$

3. The results. Let G be a closed noncompact subgroup of Aut(X) acting transitively on  $\Omega$ , and  $K_0 = G \cap K$ . Since  $K_0$  is compact open in G we can assume that its measure is one.

**PROPOSITION 1.**  $K_0$  acts transitively on  $\Omega$ .

*Proof*. Since  $G/K_0$  is countable, Baire's theorem implies that every orbit of  $K_0$  on  $\Omega$  is open. By [17, Prop. 3.4], there exist  $g \in G$ , a sequence  $\{s_n\} \subset X$ ,  $n \in Z$  and  $i_0 \in Z$   $i_0 \neq 0$ , such that  $d(s_n, s_{n+1}) = 1$  and  $g(s_n) = s_{n+i_0}$  for every  $n \in Z$ . In this proof we realize  $\Omega$  as the set of all infinite sequences  $\{t_n\}$  issued from  $t_0 = s_0$ . Therefore the sets:  $E(x) = \{\{t_n\} \in \Omega: t_j = x\}$  with  $x \in X$  and  $d(s_0, x) = j$  form a basis for the topology of  $\Omega$ . Let  $\omega_1 = \{s_0, s_1, \ldots\}$  and  $\omega_2 = \{s_0, s_{-1}, s_{-2}, \ldots\}$ .

Since  $K_0\omega_1$  and  $K_0\omega_2$  are open, it follows that there exists j > 0such that  $E(s_j) \subset K_0\omega_1$  and  $E(s_{-j}) \subset K_0\omega_2$ . Using the automorphism g, it is not hard to show that  $K_0$  acts transitively on  $CE(s_{-1})$  and  $CE(s_1)$ , respectively. Obviously,  $CE(s_{-1}) \cap CE(s_1) \neq \emptyset$  and  $CE(s_{-1}) \cup CE(s_1) = \Omega$ . This means that  $K_0$  acts transitively on  $\Omega$ .

**PROPOSITION 2.** Let G be a closed noncompact subgroup of Aut(X) acting transitively on  $\Omega$ . Then either G acts transitively on the vertices of X, or G has two orbits  $X^+$  and  $X^-$ .

**Proof.** By Proposition 1,  $K_0$  acts transitively on  $\Omega$ , that is,  $K_0$  acts transitively on the set  $S_n^{s_0} = \{y \in X : d(s_0, y) = n\}$  for every  $n \ge 0$ . Moreover for every  $g \in G$ ,  $gK_0g^{-1}$  acts transitively on  $S_n^x$  for every  $n \ge 0$  and  $g(s_0) = x$ . In particular for every  $x \in G(s_0)$ ,  $G(s_0)$  is an

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infinite union of sets  $S_n^x$ . This implies that if  $x, y \in G(s_0)$  d(x, y) = m, then  $S_m^x \cup S_m^y \subset G(s_0)$ . Therefore  $S_m^x \subset G(s_0)$  implies that:

$$\bigcup_{j=0}^{+\infty} S_{jm}^x \subset G(s_0).$$

If  $G(s_0)$  contains vertices x and y with d(x, y) = 1, then  $G(s_0) = X$ and G is transitive on X. Suppose now  $G(s_0) \neq X$ ; thus  $G(s_0) \subset X^+$ . Let  $t = \min\{m > 0 : S_m^{s_0} \subset G(s_0)\}$ . It follows that  $G(s_0) \cap S_m^{s_0} = \emptyset$ for 0 < m < 2t  $m \neq t$  and  $\bigcup_{j=0}^{+\infty} S_{jt}^{s_0} \subset G(s_0)$ . Let  $x \in S_t^{s_0}$  and  $[s_0, x] = \{s_0, x_1, x_2, \dots, x_{t-1}, x\}$  the geodesic connecting  $s_0$  to x; we can choose  $y \in X$  in such a way that d(y, x) = t,  $d(y, s_0) = 2t - 2$  and  $[x, s_0] \cap [x, y] = [x, x_{t-1}] = \{x, x_{t-1}\}$ . Since d(x, y) = t,  $y \in G(s_0)$  but  $y \in S_{2t-2}^{s_0}$  so that  $S_{2t-2}^{s_0} \subset G(s_0)$ . This implies that 2t - 2 = t, that is, t = 2 and  $G(s_0) = X^+$ . Similarly, we can prove that  $G(s_1) = X^-$ , with  $d(s_0, s_1) = 1$ .

The aim of this note is to prove the following Theorem.

**THEOREM** 1. Every closed subgroup G of Aut(X) acting transitively on  $\Omega$  is a Kunze-Stein group.

It is enough to prove the Theorem for noncompact groups. First, we observe that:

$$\int_{J} \|\pi_{1/2+it}\|_{G}(u)\|_{HS}^{2} dm(t) \leq \|u\|_{2}^{2} \quad \text{for every } u \text{ in } C_{00}(G).$$

Indeed  $(G, K_0)$  is a Gelfand pair because  $K_0$  acts transitively on  $\Omega$ and  $g^{-1} \in K_0 g K_0$  for every g in G [9, Prop. 1.2]. The representations  $\pi_{1/2+it}|_G$  are irreducible iff 1 (the function identically one on  $\Omega$ ) is a cyclic vector. By Proposition 2, we have two possibilities: if G is transitive on X, then the representations  $\pi_{1/2+it}|_G$  are irreducible for every  $t \in J$  [12, pg. 76; 1]; otherwise for  $t \in J$ ,  $t \neq \pi/2lg(r-1)$  [2, pg. 39, pg. 62].

Since, for Gelfand pairs, the Plancherel measure on the irreducible unitary representations of G having a  $K_0$ -fixed vector depends only on the right  $K_0$ -invariant functions [9, Th. 4.2; 16, pg. 65], to prove the inequality, it is enough to prove that

$$\int_{J} \|\pi_{1/2+it}\|_{G}(u)\|_{HS}^{2} dm(t) = \|u\|_{2}^{2}$$

for every right  $K_0$ -invariant function u in  $C_{00}(G)$ . To show this, let T be the following projection on  $L^2(\Omega)$ :  $Tf = [\int_{\Omega} f(\omega) d\nu(\omega)]\mathbf{1}$  for

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 $f \in L^2(\Omega)$ . We have  $T = \int_{K_0} \pi_{1/2+it}(k) dk$  (recall that  $K_0$  is transitive on  $\Omega$ ). Let Aut $(X) = \Gamma K$ ; every function u right  $K_0$ -invariant on Gcorresponds to a function  $\tilde{u}$  on  $\Gamma$  in such a way that  $||u||_2 = ||\tilde{u}||_2$  and  $\pi_{1/2+it}|_G(u) = [\pi_{1/2+it}|_{\Gamma}(\tilde{u})]T$ .

Therefore  $\|\pi_{1/2+it}\|_G(u)\|_{HS} = \|\pi_{1/2+it}\|_{\Gamma}(\tilde{u})\mathbf{1}\|_{L^2(\Omega)}$ ; hence the equality follows from [12, pg. 86; 1]. The proof of Theorem 1 is based on the following two Lemmas.

In the next Lemma, we denote by G a locally compact group and by  $L_1^{\infty}(G)$  the space of all functions f in  $L^{\infty}(G)$  such that  $||f||_{\infty} \leq 1$ ; we assume  $\phi$  to be a complex continuous function on the strip  $S = [\alpha, \beta] \times \mathbf{R}$  with  $0 < \alpha < \frac{1}{2} < \beta < 1$ , analytic on  $S^0 = (\alpha, \beta) \times \mathbf{R}$  and such that (1)  $\phi$  is bounded on S; (2)  $|\phi(x + it)| \geq h(x) > 0$  for every  $t \in \mathbf{R}$  and  $\alpha \leq x \leq \beta$ ,  $x \neq \frac{1}{2}$ . With these notations, we have:

**LEMMA 1** (M. Cowling [6]). Let  $F: S \to L_1^{\infty}(G)$  be a continuous map, analytic on  $S^0$  (i.e.  $\langle F_z, u \rangle$  is an analytic function for every u in  $C_{00}(G)$ ). If there exists a positive constant c such that

$$\int_{\mathbf{R}} |\langle F_{1/2+it}, u \rangle|^2 |\phi(\frac{1}{2}+it)| \, dt \le c ||u||_2^2 \quad \text{for every } u \text{ in } C_{00}(G),$$

then the function  $F_{1/2}$  is in  $L^q(G)$  for every q > 2.

*Proof.* This Lemma is obtained from Lemma 2.1 of [6, pg. 215] where  $S = [\alpha, \beta] \times \mathbf{R}$ , q = q' = 2, X = G and  $X_0$  is a singleton, observing that the function  $(z/z-2)^n$  could be replaced with a general analytic function  $\phi$  with the properties (1) and (2).

**LEMMA** 2. The coefficients of the quasi-regular representation on  $\Omega$ , that is the functions:

$$\langle \pi_{1/2}(g)\xi,\eta\rangle = \int_{\Omega} P^{1/2}(g,\omega)\xi(g^{-1}\omega)\overline{\eta(\omega)} \,d\nu(\omega)$$
 for  $\xi,\eta$  in  $L^{2}(\Omega)$  and g in G

are in  $L^q(G)$  for every q > 2.

*Proof*. Since  $|\langle \pi_{1/2}(g)\xi,\eta\rangle| \leq \langle \pi_{1/2}(g)|\xi|,|\eta|\rangle$  it is enough to prove the Lemma for  $\xi \geq 0$ ,  $\eta \geq 0$  and  $\|\xi\|_2 = \|\eta\|_2 = 1$ . Define  $\xi_z = \xi^{2z}$  and  $\eta_z = \eta^{2-2z}$  for  $\xi(\omega) \neq 0 \neq \eta(\omega)$ ,  $\xi_z(\omega) = 0$  for  $\xi(\omega) = 0$ ; similarly  $\eta_z(\omega) = 0$  for  $\eta(\omega) = 0$ . In particular  $\xi_{1/2} = \xi$  and  $\eta_{1/2} = \eta$ . Let  $z = \delta + it \in S$  and  $p = 1/\delta > 1$ ,  $q = p/(p-1) = 1/(1-\delta)$  the conjugate index of p; it is easy to see that:

(1)  $\xi_z \in L^p(\Omega), \|\xi_z\|_p = 1.$ 

- (2)  $\eta_z \in L^q(\Omega), \|\eta_z\|_q = 1.$
- (3)  $\|\pi_z(g)u\|_p = \|u\|_p$  for every u in  $L^p(\Omega)$  and g in G.

Let  $\psi(z) = \exp(z^2 - 1)$ ;  $|\psi(z)| \leq 1$  on S and the map  $F_z = \psi(z)\langle \pi_z(\cdot)\xi_z, \eta_z \rangle$  is a continuous map on S into  $L_1^{\infty}(G)$ , analytic on  $S^0$ . Since  $F_{1/2} = \exp(-\frac{3}{4})\langle \pi_{1/2}(\cdot)\xi, \eta \rangle$ , to prove the Lemma, it suffices to show that:

$$\int_{\mathbf{R}} |\langle F_{1/2+it}, u \rangle|^2 |\phi(\frac{1}{2}+it)| \, dt \le c ||u||_2^2 \quad \text{for every } u \text{ in } C_{00}(G)$$

and some analytic function  $\phi$ .

Let  $\phi(z) = (r-1)lg(r-1)/[4\pi rc(z)c(1-z)]$  where c(z) is the function defined in the preliminaries.  $\phi(z) = \phi(z + \pi i/lg(r-1))$  and so  $\phi$  is bounded. Since  $\phi(z) \neq 0$  for Re  $z \neq \frac{1}{2}$ , it follows that:  $|\phi(x+it)| \ge \min\{|\phi(x+it)|: t \in \mathbf{R}\} > 0$ , for every  $x \neq \frac{1}{2}$ ,  $\alpha \le x \le \beta$ . We have  $|\phi(\frac{1}{2}+it)| dt = dm(t)$ . Let  $J_k$  be the interval

$$J_k = [k\pi/lg(r-1), (k+1)\pi/lg(r-1)]$$
 for  $k \in \mathbb{Z}$ ;

therefore  $J_0 = J$ . The functions  $\|\pi_{1/2+it}(u)\|_{HS}$  and dm(t) are periodic; hence, for every  $k \in \mathbb{Z}$ :

$$\int_{J_k} \|\pi_{1/2+it}(u)\|_{HS}^2 \, dm(t) = \int_J \|\pi_{1/2+it}(u)\|_{HS}^2 \, dm(t).$$

Let  $h_k$  be the maximum of the function

$$|\psi(\frac{1}{2}+it)|^2 = \exp(-3/2-2t^2)$$
 on  $J_k$  and  $\sum_{-\infty}^{+\infty} h_k = c < +\infty.$ 

Finally, we have:

$$\begin{split} \int_{\mathbf{R}} |\langle F_{1/2+it}, u \rangle|^2 |\phi(\frac{1}{2}+it)| \, dt \\ &= \sum_{-\infty}^{+\infty} \int_{J_k} |\psi(\frac{1}{2}+it)|^2 |\langle \pi_{1/2+it}(u) \xi_{1/2+it}, \eta_{1/2+it} \rangle|^2 \, dm(t) \\ &\leq \sum_{-\infty}^{+\infty} h_k \int_{J_k} \|\pi_{1/2+it}(u)\|_{HS}^2 \, dm(t) = c \int_J \|\pi_{1/2+it}(u)\|_{HS}^2 \, dm(t) \\ &\leq c \|u\|_{2}^2, \end{split}$$

(recall that  $\|\xi_{1/2+it}\|_2 = \|\eta_{1/2+it}\|_2 = 1$ ).

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**Proof of Theorem 1.** If G acts transitively on  $\Omega$ , then  $\Omega \simeq G/G_0$  where  $G_0$  is the stability subgroup of a fixed point  $\omega_0$  in  $\Omega$ . By the "principe de majoration" of C. Herz [14], for every f in A(G) there exists a coefficient of  $\pi_{1/2}$  such that:  $|f(g)| \leq \langle \pi_{1/2}(g)\xi, \eta \rangle$  for every g in G. Hence, from Lemma 2,  $A(G) \subset L^q(G)$  for every q > 2 and G is a Kunze-Stein group.

**REMARK.** We shall say that a vertex v of a tree is of homogeneity l if v belongs to exactly l edges. Let  $X_{l,q}$  be a semihomogeneous tree, that is, a tree such that every vertex is of homogeneity l or q and two adjacent vertices are of homogeneity l and q, respectively. We suppose  $l \neq q$ , otherwise X is a homogeneous tree. Let  $S_l$  and  $S_q$  be the subsets of vertices of homogeneity l and q, respectively. Theorem 1 is true for semihomogeneous trees, with the same proof.

Indeed, if G is a closed noncompact subgroup of  $\operatorname{Aut}(X_{l,q})$  acting transitively on the boundary of  $X_{l,q}$ , then  $G \cap K_{v_0}$  acts transitively on the boundary for every vertex  $v_0$ . Moreover  $G(v_0) = S_l$  and  $G(w_0) =$  $S_q$  for every  $v_0 \in S_l$  and  $w_0 \in S_q$ . Hence, without loss of generality, we can suppose that l < q. The representations  $\pi_{1/2+it}|_G$  are irreducible [2, pg. 62] and the Plancherel measure of the Gelfand pair  $(G, G \cap K_{v_0})$ is a multiple of  $|c(\frac{1}{2}+it)|^{-2}$  for an analytic function c(z) [10, pg. 153]. The proof proceeds in the same fashion as for homogeneous trees.

4. Simply transitive subgroups. Let  $\Gamma$  be a simply transitive subgroup of Aut(X); for  $\eta \in L^2(\Omega)$  we define, as in [11; 1], the Poisson transform of  $\Gamma: \wp(\eta)(x) = \langle \pi_{1/2}(x) \mathbf{1}, \eta \rangle$ .

COROLLARY.  $\wp(L^2(\Omega)) \subset l^q(\Gamma)$  for every q > 2.

*Proof.* By Theorem 1,  $f(g) = \langle \pi_{1/2}(g) \mathbf{1}, \eta \rangle \in L^q(\operatorname{Aut}(X))$  for every q > 2. Let  $\operatorname{Aut}(X) = \Gamma K$  and g = xk with  $x \in \Gamma$  and  $k \in K$ ; therefore  $\pi_{1/2}(g)\mathbf{1} = \pi_{1/2}(x)\mathbf{1}$  because  $\nu$  is K-invariant and so

$$\|\varphi(\eta)\|_{l^q(\Gamma)} = \|f\|_{L^q(\operatorname{Aut}(X))}.$$

The Corollary follows.

 $\Gamma$  is not a Kunze-Stein group (in a discrete Kunze-Stein group every amenable subgroup is finite); nevertheless, we can prove a "weak Kunze-Stein property":

 $l_r^p(\Gamma) *_{\Gamma} l^2(\Gamma) \subset l^2(\Gamma)$  for every  $1 , where <math>l_r^p$  is the space of radial functions in  $l^p$ , that is, the functions which depend only on the length of the words of  $\Gamma$  and  $*_{\Gamma}$  means the convolution product of

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 $\Gamma$ . It is easy to see that the "weak Kunze-Stein property" is equivalent to the following:  $A_r(\Gamma) \subset l^q(\Gamma)$  for every q > 2. This was proved in [5] for free groups on finitely many generators. Notice that  $A_r(\Gamma) = l_r^2(\Gamma) *_{\Gamma} l_r^2(\Gamma)$ .

**THEOREM 2.** The following hold: (1)  $l_r^2(\Gamma) *_{\Gamma} l^2(\Gamma) \subset l^q(\Gamma)$  for every q > 2. (2)  $l^p(\Gamma) *_{\Gamma} l_r^2(\Gamma) \subset l^2(\Gamma)$  for every 1 .

**Proof.** It is enough to prove (2); (1) follows by duality argument. Putting  $\dot{f}(xk) = f(x)$  with  $x \in \Gamma$  and  $k \in K$ , it is possible to identify the functions f on  $\Gamma$  with the right K-invariant functions  $\dot{f}$  on Aut $(X) = \Gamma K$ . The radial functions on  $\Gamma$  correspond to the bi K-invariant functions on Aut(X). Let  $f \in l^p(\Gamma)$  for  $1 and <math>\phi \in l_r^2(\Gamma)$ , then the function  $\dot{f} * \dot{\phi}$  is right K-invariant; hence, by Theorem 1, the restriction to  $\Gamma$  is in  $l^2(\Gamma)$ . Moreover:  $(\dot{f} * \dot{\phi})|_{\Gamma} = f *_{\Gamma} \phi$  and, from this, Theorem 2 follows.

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