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CORRECTION TO: "SUMS OF PRODUCTS OF POWERS OF GIVEN PRIME NUMBERS"

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shown to be rigid in 5.2, this is correct. Thus the applications in the remainder of the proof of Proposition 5.3 are valid.

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CORRECTION TO SUMS OF PRODUCTS OF POWERS OF GIVEN PRIME NUMBERS

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Lemma 3(b) is false and hence the proof of Theorem 3 needs revision. We present a corrected version of Lemma 3(b) and a proof of Theorem 3 based on it.

Lemma 3(b). *If* $3^b \mid 2^a + 1$, *then* $a \geq 3^{b-1}$.

Proof. If $3^b \mid 2^a + 1$, then $2^{2a} - 1 = (2^a + 1)\,(2^a - 1) \equiv 0 \pmod{3^b}$. Since 2 is a primitive root of 3^b for any $b \in \mathbf{N}$, $\varphi(3^b) \mid 2a$ where $\varphi(x)$ is the Euler's function. Hence $3^{b-1} \mid a$. □

Proof of Theorem 3. Without loss of generality we may assume that $x \geq 1$, $y \geq 0$, $z \geq 2$, $w \geq 1$. By (1.3) and Lemma 3(b), we have $x \leq z$ and $z \geq 3^{\min(y,w)-1}$. We derive from (1.3) that $2^x \mid 3^w - 1$ and therefore $2^{x-2} \leq w$. Hence

$$x < (\log 2)^{-1} \log w + 2.$$

We distinguish between two cases.

Case 1. $y \le w$. Since (1.3) implies $3^w < 2^z$, we have $w < 0.631z$ and

$$(1.11) \qquad |2^z - 3^w| < 2^x 3^y < 4w3^y < 2.524z3^y.$$

If $z > 11$, then, from (1.11) and Lemma 1, we obtain for nonexceptional pairs (z, w),

$$\frac{\exp(3^{y-1}(\log 2 - 0.1))}{3^{y-1}} \le \frac{\exp(z(\log 2 - 0.1))}{z} < 2.524 \times 3^y.$$

Thus we have $3^y < 11.2y$ and hence $y \le 3$. From (1.11) and Lemma 1 we see that

$$z(\log 2 - 0.1) < \log z + 4.3$$

and so $z \le 11$, which yields a contradiction. For each exceptional pair (z, w), the number $2^z - 3^w + 1$ has some prime factor greater than 3. Thus there are no solutions in this case with $z > 11$.

If $2 \le z \le 11$, then $0 \le w < 0.631z < 6.95$, hence $1 \le x \le 4$. By checking these ranges for x, y, z, w we find the solutions: $(1,0,2,1)$, $(1,1,3,1)$, $(1,1,5,3)$, $(3,0,4,2)$, $(3,1,5,2)$, $(4,1,7,4)$, $(4,3,9,4)$.

Case 2. $w < y$. It follows from (1.3) that

$$|2^{z-x} - 3^y| \le |3^w - 1|/2.$$

If $z-x > 11$, then we obtain from Lemma 1 for non-exceptional pairs $(z-x, y)$ that

$$(z-x)(\log 2 - 0.1) \le w \log 3,$$

and so

$$3^{w-1} \le 2(w + \log w + 1).$$

Thus $w \le 3$, and $x \le 3$, $|2^{z-x} - 3^y| \le 13$. Therefore

$$z \le \frac{3 \log 3}{\log 2 - 0.1} + x < 9,$$

which yields a contradiction. It is easy to check that $|2^{z-x} - 3^y| > 13$ for each exceptional pair $(z-x, y)$. Thus each solution of (1.3) in this case satisfies $z - x \le 11$, hence $z \le 14$. If $2 \le z \le 14$, then by (1.3), $0 \le y \le 9$. We find only one solution with $y > w$, namely $(1,5,9,3)$.

We conclude that (1.3) has exactly eight non-trivial solutions $(x, y, z, w) \in \mathbf{N}_0^4$, namely

$$(1.12) \qquad \begin{array}{llll} (1,0,2,1), & (1,1,3,1), & (1,1,5,3), & (1,5,9,3), \\ (3,0,4,2), & (3,1,5,2), & (4,1,7,4), & (4,3,9,4). \end{array}$$

The argument for solutions with some negative values is similar to that in the proof of Theorem 1. Using (1.12) we obtain only one additional non-trivial solution in $\mathbf{Z}^4$, namely $(3, -1, 1, -1)$. □

Finally we note that the following should be added to reference [3] (W. J. Ellison): *On a theorem of S. Sivasankaranarayana Pillai*, Same Séminaire, Exp. 12, 10 pp.

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