Pacific Journal of Mathematics

REDUCTION OF TOPOLOGICAL STABLE RANK IN INDUCTIVE LIMITS OF C^*-ALGEBRAS

Marius Dadarlat, Gabriel Nagy, András Némethi and Cornel Pasnicu

Vol. 153, No. 2 April 1992

PACIFIC JOURNAL OF MATHEMATICS
Vol. 153, No. 2, 1992

REDUCTION OF TOPOLOGICAL STABLE RANK IN INDUCTIVE LIMITS OF C^*-ALGEBRAS

MARIUS DĂDĂRLAT, GABRIEL NAGY, ANDRÁS NÉMETHI AND CORNEL PASNICU

We consider inductive limits A of sequences $A_1 \to A_2 \to \cdots$ of finite direct sums of C^*-algebras of continuous functions from compact Hausdorff spaces into full matrix algebras. We prove that A has topological stable rank (tsr) one provided that A is simple and the sequence of the dimensions of the spectra of A_i is bounded. For unital A, $\operatorname{tsr}(A) = 1$ means that the set of invertible elements is dense in A. If A is infinite dimensional, then the simplicity of A implies that the sizes of the involved matrices tend to infinity, so by general arguments one gets $\operatorname{tsr}(A_i) \leq 2$ for large enough i whence $\operatorname{tsr}(A) \leq 2$. The reduction of tsr from two to one requires arguments which are strongly related to this special class of C^*-algebras.

The problem of reduction of real rank (see **[6]**) for these algebras was recently studied in **[2]** in connection with some interesting features revealed in several papers (**[3]**, **[1]**, **[15]**, **[5]**, **[12]**, **[11]**). The reduction of tsr and real rank for other classes of C^*-algebras was studied in **[22]**, **[21]**, **[8]**, **[24]**, **[17]**, **[25]**.

The paper consists of three sections:

1. Preliminaries and Notation
2. Local aspects of the connecting homomorphisms
3. The Main Result.

1.

1.1. For a unital C^*-algebra A and a finitely generated projective A-module E, we denote by $\operatorname{End}_A(E)$ the algebra of A-linear endomorphisms of E and by $\operatorname{GL}_A(E)$ the group of units of $\operatorname{End}_A(E)$. For $E = A^n$ we shall write $\operatorname{GL}(n, A)$ for $\operatorname{GL}_A(A^n)$ and $\operatorname{GL}^0(n, A)$ for the connected component of 1. Let $U(A)$ denote the unitary group of A and $U(n) := U(\mathbf{C}^n)$. A selfadjoint idempotent element of a C^*-algebra will be simply called projection.

Recall some definitions from **[23]**. For a unital C^*-algebra A and a natural number n let $Lg_n(A)$ denote the set of n-tuples of elements of A which generate A as a left ideal. The topological stable rank of A is the least n (if it does not exist it will be taken by definition to be ∞) such that $Lg_n(A)$ is dense in A^n. One denotes by $\operatorname{csr}(A)$ the least integer n such that $\mathrm{GL}^0(m, A)$ acts transitively by right multiplication on $Lg_m(A)$ for any $m \geq n$. (If no such integer exists one takes $\operatorname{csr}(A) = \infty$.) For nonunital A one takes $\operatorname{tsr}(A) := \operatorname{tsr}(\widetilde{A})$ and $\operatorname{csr}(A) := \operatorname{csr}(\widetilde{A})$ where $\widetilde{A}$ is the algebra obtained from A by adjoining a unit.

For a compact Hausdorff space X of finite covering dimension one has:

$$\operatorname{tsr}(C(X)) = \left[\frac{\dim X}{2}\right] + 1,$$
$$\operatorname{csr}(C(X)) \leq \left[\frac{\dim X + 1}{2}\right] + 1$$

(see **[23]** and **[18]**).

1.2. We consider C^*-inductive limits

$$A = \varinjlim (A_i, \Phi_{ij}).$$

The A_i's are C^*-algebras of the form

$$A_i = \bigoplus_{t=1}^{s(i)} C(X_{it}) \otimes M_{n(i,t)}$$

where X_{it} is a Hausdorff compact space, $s(i)$, $n(i, t)$ are positive integers and $M_{n(i,t)}$ is the C^*-algebra of complex $n(i, t) \times n(i, t)$ matrices. The $*$-homomorphisms $\Phi_{ij}\colon A_i \longrightarrow A_j$ are not assumed to be unital or injective. We denote by Φ_i the natural map $A_i \longrightarrow A$ and by $X_i = \bigsqcup_{t=1}^{s(i)} X_{it}$ the spectrum of A_i.

We begin with a brief discussion on the $*$-homomorphisms between certain homogeneous C^*-algebras.

1.3. For given C^*-algebras C, D we denote by $\operatorname{Hom}(C, D)$ the space of all $*$-homomorphisms from C to D with the point-norm topology. $\operatorname{Hom}^1(C, D)$ stands for the subspace of unital $*$-homomorphisms. We shall identify

$$\operatorname{Hom}(C(X), C(Y) \otimes M_n) \quad \text{with } \operatorname{Map}(Y, \operatorname{Hom}(C(X), M_n))$$

where for topological spaces Y, Z, $\operatorname{Map}(Y, Z)$ denotes the space of continuous functions from Y to Z endowed with the compact-open topology.

Each $\psi \in \operatorname{Hom}(C(X), M_n)$ has the form

$$\psi(f) = \sum f(x_r) p_r, \qquad f \in C(X),$$

for suitable points $x_r \in X$ and mutually orthogonal projections p_r in M_n. Let L_ψ be the set of all x_r's that appear in the above formula. More generally, each $\Phi \in \mathrm{Hom}(C(X), C(Y) \otimes M_n)$ is identified with a map $\Phi\colon Y \to \mathrm{Hom}(C(X), M_m)$ and we define for each $y \in Y$, $L_\Phi(y) := L_{\Phi(y)}$. In the same way for given

$$\Phi \in \mathrm{Hom}\left(\bigoplus C(X_\alpha) \otimes M_{n(\alpha)}, \bigoplus C(Y_\beta) \otimes M_{m(\beta)}\right)$$

and $y \in Y$ we define

$$L_\Phi(y) = \bigsqcup_\alpha L_{\Phi_{\alpha,\beta}}(y)$$

where $\Phi_{\alpha,\beta}$ denotes the component of Φ acting from $C(X_\alpha) \subset C(X_\alpha) \otimes M_{n(\alpha)}$ to $C(Y_\beta) \otimes M_{m(\beta)}$.

Note that $\Phi(f)(y) = \Phi(g)(y)$ whenever $f = g$ on $L_\Phi(y)$.

The map $y \mapsto L_\Phi(y)$ has useful semicontinuity properties:

(a) if $L_\Phi(y)$ is contained in some open set U then $L_\Phi(z) \subset U$ for any z in some neighborhood of y,

(b) the set $\{y\colon L_\Phi(y) \cap U \neq \varnothing\}$ is open for each open set U (see **[9]** and **[19]**).

2. We begin by giving two criteria of simplicity for C^*-algebras A as above, which extend the corresponding results for AF-algebras **[4]** and Bunce-Deddens algebras [7].

2.1. PROPOSITION. *Let $A = \varinjlim (A_i, \Phi_{ij})$ be as in* 1.1 *and assume that the connecting homomorphisms Φ_{ij} are injective. Then the following conditions are equivalent*:

(i) *A is simple.*

(ii) *For any positive integer i and any open nonempty subset U of X_i there is a j_0 such that $L_{\Phi_{ij}}(x) \cap U \neq \varnothing$ for any $j \geq j_0$ and $x \in X_j$.*

(iii) *For any nonzero $a \in A_i$ there is a j_0 such that*

$$\Phi_{ij}(a)(x) \neq 0 \quad \textit{for each } j \geq j_0 \textit{ and } x \in X_j.$$

Proof. (ii) $\Leftrightarrow$ (iii). This is clear since for given $a \in A_i$ one has

$$\Phi_{ij}(a)(x) = 0 \quad \text{if and only if} \quad a = 0 \text{ on } L_{\Phi_{ij}}(x).$$

(i) $\Rightarrow$ (ii). Assume that (ii) does not hold for some i and some open nonempty $U \subsetneqq X_i$. Passing to a subsequence, if necessary, we may assume that for any $j \geq i$ the set $F_j = \{x \in X_j ; L_{\Phi_{ij}}(x) \cap U = \varnothing\}$

is nonempty and $F_j \neq X_j$. By the last part of 1.3 F_j is closed. Therefore the family $(J_j)_{j\geq i}$ where

$$J_j = \{a \in A_j : a = 0 \text{ on } F_j\}$$

defines a closed two sided ideal J in A. (Note that $\Phi_{jk}(J_j) \subset J_k$ since $L_{\Phi_{ij}}(y) \subset L_{\Phi_{ik}}(x)$ for any $y \in L_{\Phi_{jk}}(x)$.) Also $J \neq A$ since if e_i is the unit of A_i then $\operatorname{dist}(\Phi_{ij}(e_i), J_j) = 1$ for any $j \geq i$ and so $e_i \notin J$. The existence of J contradicts (i).

(iii) $\Rightarrow$ (i). Let J be a two-sided closed nonzero ideal of A. One has $J = \overline{\bigcup(J \cap A_i)}$ (see [4]). We shall prove that $J \cap A_j = A_j$ for large enough j. Take $a \in J \cap A_i$, $a \neq 0$. By (iii) there is a j_0 such that $\Phi_{ij}(a)(x) \neq 0$ for all $j \geq j_0$ and $x \in X_j$. Since $\Phi_{ij}(J \cap A_i) \subset J \cap A_j$ we find that $\Phi_{ij}(a) \in J \cap A_j$ for $j \geq j_0$. Since $\Phi_{ij}(a)$ does not vanish at any point of X_j this forces $J \cap A_j = A_j$. □

Let $A = \varinjlim (A_i, \Phi_{ij})$ be as above. For a noninvertible element $a \in A_i$ there are $x_0 \in X_i$, $u \in U(A_i)$ and a projection $p \in A_i$ (both u and p "scalars") such that $ua(x_0)p = pua(x_0) = 0$.

For simple A the following two lemmas enable us to obtain something similar for $\Phi_{ij}(a)$ (for some $j \geq i$) locally around any point of X_j, after a small perturbation of a.

2.2. LEMMA. *Let $\Phi \in \operatorname{Hom}\left(\bigoplus_{i=1}^{s} C(X_i) \otimes M_{n(i)}, C(Y) \otimes M_m\right)$, let $k \geq 1$, let U be an open subset of X_1 and let $y \in Y$ such that $L_\Phi(y) \cap U$ has at least k points. Then there is $p_W \in C(Y) \otimes M_m$ such that $p_W(z)$ is a projection of rank greater than or equal to k for all z in some neighborhood W of y and*

$$\Phi(a)p_W = p_W\Phi(a)$$

for any $a \in \bigoplus_{i=1}^{s} C(X_i) \oplus M_{n(i)}$ satisfying

$$a(x)e_{11} = e_{11}a(x) = 0$$

for all $x \in U$. (Here (e_{ij}) stands for a system of matrix units of $M_{n(1)}$.)

Proof. Take U_1, U_2 open subsets of $X = \bigcup_{i=1}^{s} X_i$ having disjoint closures such that

$$L_\Phi(y) \cap U \subset U_1 \subset U, \qquad L_\Phi(y) \cap (X_1 - U) \subset U_2.$$

Using the continuity of L_Φ (see 1.3) we find a neighborhood W of y such that $L_\Phi(z) \subset U_1 \cup U_2$ for all $z \in W$. Take a continuous map $g: X_1 \to [0, 1]$ such that $g = 1$ on U_1 and $g = 0$ on U_2 and define $p_W = \Phi(g \otimes E_{11})$. If $z \in W$ then $p_W(z) = p_W^2(z) = p_W^*(z)$ since $g = g^2 = g^*$ on $L_\Phi(W)$. One has $\operatorname{rank} p_W(z) \geq k$ since $L_\Phi(y) \cap U_1$ has at least k elements and $g = 1$ on U_1. Finally if $a(x)e_{11} = e_{11}a(x) = 0$ for all $x \in U$ then $(g \otimes e_{11})a = a(g \otimes e_{11}) = 0$. This implies $p_W \Phi(a) = \Phi(a) p_W = 0$. □

2.3. LEMMA. *Let $C = C(X) \otimes M_n$ and let $a \in C$ such that $\det a(x) = 0$ for some $x \in X$. Then for any $\varepsilon > 0$ there exist u, $v \in \mathrm{GL}(C)$ and $b \in C$ such that*

$$\|uav - b\| < \varepsilon \quad \text{and} \quad be_{11} = e_{11}b = 0 \text{ on a neighbourhood of } x.$$

Proof. Take u, $v \in \mathrm{Gl}(n, \mathbf{C})$ such that the matrix $ua(x)v$ has only zero entries on the first row and on the first column. Now b is easily found since continuous functions vanishing at x can be uniformly approximated by continuous functions vanishing on a neighbourhood of x. □

3. The next step toward the main result is based on the following theorem which follows from Michael's paper **[16]**.

3.1. THEOREM. *Let X be a Hausdorff compact space of dimension d, let T be a complete metric space and let Y be a map from X to the family of the nonempty closed subsets of T.*

Suppose that

(a) *Y is lower semicontinuous, i.e. for each open subset U of T the set $\{x \in X : Y(x) \cap U \neq \varnothing\}$ is open;*

(b) *Each $Y(x)$ is $(d+1)$-connected;*

(c) *There is an $\varepsilon > 0$ such that for any $0 < r < \varepsilon$ and $x \in X$ the intersection of $Y(x)$ with any closed ball of radius r in T is a contractible space.*

Then there is a continuous map $\sigma: X \to T$ such that $\sigma(x) \in Y(x)$ for all $x \in X$.

Proof. The theorem follows from Theorem 1.2 in **[16]** using the comments from the second part of the same paper.

3.2. PROPOSITION. *Let X be a Hausdorff compact space, let $k' \geq k \geq 1$ integers, let $\mathscr{W}$ be an open cover of X and assume that for each $W \in \mathscr{W}$ there is given a continuous projection valued map $p_W: W \to M_n$ such that* $\operatorname{rank} p_W(x) \geq k'$ *for $x \in W$. If* $\dim(X) \leq 2(k' - k) - 1$

then there is a continuous projection valued map $p\colon X \longrightarrow M_n$ *such that for* $x \in X$:

$$\operatorname{rank} p(x) \geq k\,,$$
$$p(x) \leq \bigvee\{p_W(x)\colon W \in \mathscr{W}\,,\ x \in W\}\,.$$

Proof. For $x \in X$ define $\mathscr{W}(x) = \{W \in \mathscr{W} : x \in W\}$ and $H(x) = \operatorname{span}\{p_W(x)\mathbf{C}^n\colon W \in \mathscr{W}(x)\}$.

For any linear subspace H of $\mathbf{C}^n$ let $V(H\,,k)$, $k \leq \dim(H)$, denote the Stiefel manifold of k-orthogonal frames in H (see **[14]**). For any $x \in X$ define $Y(x) = V(H(x)\,,k) \subset V(\mathbf{C}^n\,;k)$. We check that Y satisfies the conditions of Theorem 3.1.

(a) The lower semicontinuity of Y follows from the lower semicontinuity of the map $x \mapsto H(x) \subset \mathbf{C}^n$ which is almost obvious having in mind the definition of $H(x)$.

(b) $V(H\,,k)$ is $2(\dim(H) - k)$-connected (see **[14]**). Therefore $V(H(x)\,,k)$ is $2(k' - k)$-connected since $\dim H(x) \geq k'$.

(c) For any m, $n \geq m \geq k$, there is $\varepsilon_m > 0$ such that any closed ball of radius at most ε_m in $V(\mathbf{C}^m\,,k)$ is contractible. (We consider $V(\mathbf{C}^m\,,k)$ with the metric induced by the restriction of a $\mathrm{U}(n)$-invariant Riemann structure on $V(\mathbf{C}^n\,,k)$.) In this situation $V(\mathbf{C}^m\,,k)$ is a totally geodesic submanifold of $V(\mathbf{C}^n\,,k)$ and the same is true for any $V(H\,,k)$ with $H \subset \mathbf{C}^n$. Therefore the induced metric form from $V(\mathbf{C}^n\,,k)$ coincides with the metric given by the induced Riemann structure of $V(H\,,k)$ (see **[13]**). Having also the $\mathrm{U}(n)$-invariance of this metric one can take

$$\varepsilon = \min\{\varepsilon_m\colon k \leq m \leq n\}\,. \qquad \square$$

We also need the following approximation results:

3.3. Lemma. *Let* B *be a unital* C^**-algebra and let*

$$k \geq \max(\operatorname{tsr}(B)\,,\operatorname{csr}(B))\,.$$

Then for any positive integer m *and any* $a \in M_m(B)$, *the matrix* $\left(\begin{smallmatrix} a & 0 \\ 0 & 0_k \end{smallmatrix}\right)$ *belongs to the closure of* $\mathrm{GL}(m+k\,,B)$.

Proof. If $m \leq k$ one can take

$$b_\varepsilon = \begin{pmatrix} a & \varepsilon 1_m & 0 \\ \varepsilon 1_m & 0_m & 0 \\ 0 & 0 & \varepsilon 1_{k-m} \end{pmatrix} \in \mathrm{GL}(m+k\,,B)$$

and $b_\varepsilon \to a$ as $\varepsilon \to 0$.

For $m \geq k$ we proceed by induction. Assume the statement holds for a fixed $m \geq k$ and let a $a \in M_{m+1}(B)$. Since

$$m \geq \max(\operatorname{tsr}(B), \operatorname{csr}(B))$$

it follows from [23] that for each $\varepsilon > 0$ there are $t \in \mathrm{GL}(m+1, B)$, $a_1 \in M_m(B)$ and $b \in B^m$ such that

$$\left\| a - \begin{pmatrix} 1 & 0 \\ b & a_1 \end{pmatrix} \cdot t \right\| < \varepsilon .$$

By the induction hypothesis one can approximate

$$\left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline b & a_1 & 0 \\ 0 & 0 & 0_k \end{array}\right)$$

with an invertible matrix of the form

$$\left(\begin{array}{c|c} 1 & 0 \quad 0 \\ \hline b & \\ 0 & c \end{array}\right)$$

Hence $\begin{pmatrix} a & 0 \\ 0 & 0_k \end{pmatrix}$ will be approximated by

$$\left(\begin{array}{c|c} 1 & 0 \quad 0 \\ \hline b & \\ 0 & c \end{array}\right) \cdot \begin{pmatrix} t & 0 \\ 0 & 1_k \end{pmatrix} . \qquad \square$$

3.4. Remark. Suppose B, k are as above. Let F, G, H be finitely generated projective B-modules and put $E = F \oplus G \oplus H$. If F, G are free and $G \simeq B^k$, then a slight modification of the above arguments shows that $\mathrm{End}_B(F) \subset \overline{\mathrm{GL}_B(E)}$.

In the proof of the main result we shall invoke the following straightforward approximation device:

3.5. Lemma. *Let $B = \overline{\bigcup B_i}$ where the B_i's form an increasing sequence of unital C^*-algebras. Let e_i be the unit of B_i. If for any $a \in B_i$ and $\varepsilon > 0$ there is $j \geq i$ and $b \in \mathrm{GL}(e_i B_j e_i)$ such that $\|a - b\| < \varepsilon$ then $\operatorname{tsr}(B) = 1$.*

Proof. Let $\widetilde{B} = B + \mathbf{C} \cdot 1$ be the algebra obtained by adjoining a unit to B. Let $x + \lambda 1 \in \widetilde{B}$ with $x \in B_i$. By hypothesis there is $j \geq i$ and $y \in \mathrm{GL}(e_i B_j e_i) \subset \mathrm{GL}(e_i B e_i)$ such that $\|x + \lambda e_i - y\|$ is small. Choosing a non zero scalar λ' close to λ, the element $y + \lambda'(1 - e_i)$ is invertible and approximates $x + \lambda \cdot 1$. Therefore $\mathrm{GL}(\widetilde{B})$ is dense in $\widetilde{B}$ which means $\operatorname{tsr}(B) = 1$. $\square$

3.6. THEOREM. *Let* $A = \varinjlim (A_i, \Phi_{ij})$ *where* $A_i = \bigoplus_{t=1}^{s(i)} C(X_{it}) \otimes M_{n(i,t)}$, *each* X_{it} *being a Hausdorff compact space such that* $d = \sup \dim(X_{it}) < \infty$.

If A *is simple then* $\operatorname{tsr}(A) = 1$.

Proof. Replacing each A_i by its image in A one may suppose that all the Φ_{ij}'s are injective. We shall verify the conditions from Lemma 3.5. Let $a \in A_i$ be a noninvertible element and put $Z = \{x \in X_i : \det a(x) = 0\}$. If Z consists only of isolated points of X_i then it is obvious that $a \in \overline{\mathrm{GL}(A_i)}$. Thus we may assume that there is $x \in Z$ such that each neighbourhood of x is an infinite set.

Moreover by Lemma 2.3 we may suppose that $ae_{11}^t = e_{11}^t a = 0$ on some neighbourhood U of x for some t. Fix integers k', k such that

$$k \geq 2d + 4, \qquad 2(k' - k) + 1 \geq d.$$

Since U is an infinite open set and the C^*-algebra A is simple it follows by Proposition 2.1 that there is $j \geq i$ such that $L_{\Phi_{ij}}(y) \cap U$ has at least k' elements for any $y \in X_j$. This enables us by using Lemma 2.2 to find an open covering $\mathscr{W}$ of X_j such that for each $W \in \mathscr{W}$ there is $p_W \in A_j$ satisfying

(1) p_W is projection valued on W,
(2) $\operatorname{rank} p_W(y) \geq k'$ for any $y \in W$,
(3) $p_W \Phi_{ij}(a) = \Phi_{ij}(a) p_W = 0$ on W,
(4) $p_W \leq \Phi_{ij}(e_i)$ where e_i is the unit of A_i.

Proposition 3.2 provides us a projection $p \in A_j$ such that
(a) $p(x) \leq \bigvee\{p_W(x) : W \in \mathscr{W},\ x \in W\}$ for all $x \in X_j$.
(b) $\operatorname{rank} p(x) \geq k$ for all $x \in X_j$.

Of course (4) and (a) imply that $p \leq \Phi_{ij}(e_i)$.

We have also
(c) $\Phi_{ij}(a)p = p\Phi_{ij}(a) = 0$
as a consequence of (3) and (a).

Let $b := \Phi_{ij}(a)$ have the components (b_t) with $b_t \in C(X_{jt}) \otimes M_{n(j,t)}$. We shall use Remark 3.4 in order to approximate each b_t by invertible elements in $\operatorname{End}_{C(X_{jt})}(E_t)$ where $E_t := \Phi_{ij}(e_i) C(X_{jt})^{n(j,t)}$. Consider also the finitely generated projective $C(X_{jt})$-modules

$$P_t = pC(X_{jt})^{n(j,t)}, \qquad Q_t = (\Phi_{ij}(e_i) - p) C(X_{jt})^{n(j,t)}.$$

It is clear that $E_t \simeq P_t \oplus Q_t$.

Since $\operatorname{rank} P_t \geq k \geq 2d + 4$, by using the stability properties of vector bundles (see [14]), one can split P_t as a direct sum of finitely generated projective $C(X_{jt})$-modules $P_t = R_t \oplus G_t \oplus H_t$ such that $Q_t \oplus R_t$ and G_t are free and

$$\operatorname{rank} G_t \geq [(d+1)/2] + 1 \geq \max\{\operatorname{tsr} C(X_{jt}), \operatorname{csr} C(X_{jt})\}.$$

Let $F_t = Q_t \oplus R_t \oplus G_t$. By equation (c) above one can regard b_t as an element of $\mathrm{End}_{C(X_{jt})}(F_t)$ that vanishes on G_t. Since both F_t and G_t are free it follows from Lemma 3.3 that b_t belongs to the closure of $\mathrm{GL}(F_t)$. As F_t is a direct summand in E_t, this implies that b_t belongs to the closure $\mathrm{GL}(E_t)$. It follows that $\Phi_{ij}(a)$ belongs to the closure of $\mathrm{GL}(\bigoplus_t E_t) = \mathrm{GL}(\Phi_{ij}(e_i)A_j\Phi_{ij}(e_i))$. The proof is complete by virtue of Lemma 3.5. □

REFERENCES

[1] B. Blackadar, *Symmetries of the CAR algebra*, preprint, 1988.
[2] B. Blackadar, O. Brattelli, G. A. Elliott and A. Kumjian, *Reduction of real rank and inductive limits of C^*-algebras*, preprint.
[3] B. Blackadar and A. Kumjian, *Skew products of relations and the structure of simple C^*-algebras*, Mat. Z., **189** (1985), 55–63.
[4] O. Brattelli, *Inductive limits of finite-dimensional C^*-algebras*, Trans. Amer. Math. Soc., **171** (1972), 195–234.
[5] O. Brattelli, G. A. Elliott, D. E. Evans and A. Kishimoto, *Finite group actions on* AF *algebras obtained by folding the interval*, preprint, 1989.
[6] L. B. Brown and G. K. Pedersen, *C^*-algebras of real rank zero*, preprint, 1989.
[7] J. Bunce and J. Deddens, *A family of simple C^*-algebras related to weighted shift operators*, J. Funct. Anal., **19** (1975), 12–34.
[8] M.-D. Choi and G. A. Elliott, *Density of the self-adjoint elements with finite spectrum in an irrational rotation C^*-algebra*, preprint, 1988.
[9] M. Dădărlat, *On homomorphisms of certain C^*-algebras*, preprint, 1986.
[10] M. Dădărlat and A. Némethi, *Shape theory and connective K-theory*, to appear in J. Operator Theory.
[11] G. A. Elliott, *On the classification of C^*-algebras of real rank zero*, preprint.
[12] D. E. Evans and A. Kishimoto, *Compact group actions on* UHF *algebras obtained by folding the interval*, J. Funct. Anal., (to appear).
[13] S. Helgason, *Differential Geometry, Lie Groups and Symmetric Spaces*, Academic Press, 1978.
[14] D. Husemoller, *Fibre Bundles*, 2nd ed., Springer Verlag, 1966.
[15] A. Kumjian, *An involutive automorphism of the Bunce-Deddens algebra*, C.R. Math. Rep. Acad. Sci. Canada, **10** (1988), 217–218.
[16] E. Michael, *Continuous selections* II, Ann of Math., **64**, no. 3, (1956), 562–580.
[17] G. Nagy, *Some remarks on lifting invertible elements from quotient C^*-algebras*, J. Operator Theory, **21** (1989), 379–386.
[18] V. Nistor, *Stable range for tensor products of extensions of $\mathcal{K}$ by $C(X)$*, J. Operator Theory, **16** (1986), 387–396.
[19] C. Pasnicu, *On inductive limits of certain C^*-algebras of the form $C(X) \otimes F$*, Trans. Amer. Math. Soc., **310** (1988), 703–714.
[20] G. K. Pedersen, *C^*-algebras and their Automorphism Groups*, Academic Press, London/New York, 1979.

[21] I. F. Putnam, *The invertible elements are dense in the irrational rotation C^*-algebras*, preprint, 1989.
[22] N. Riedel, *On the topological stable rank of irrational rotation algebras*, J. Operator Theory, **13** (1985), 143–150.
[23] M. A. Rieffel, *Dimension and stable rank in the K-theory of C^*-algebras*, Proc. London Math. Soc., **46** (1983), 301–333.
[24] M. Rordam, *On the structure of simple C^*-algebras tensored with a* UHF-*algebra* I, II, preprints.
[25] S. Zhang, *C^*-algebras with real rank zero and the internal structure of their corona and multiplier algebras* I, II, III, IV, preprints.

Received October 5, 1990 and in revised form June 7, 1991.

UNIVERSITY OF CALIFORNIA
LOS ANGELES, CA 90024-1555

UNIVERSITY OF CALIFORNIA
BERKELEY, CA 94720

OHIO STATE UNIVERSITY
COLUMBUS, OH 43210

AND

INSTITUTE OF MATHEMATICS
BD. PACII 220
79622 BUCHAREST, ROMANIA

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be in typed form or offset-reproduced (not dittoed), double spaced with large margins. Please do not use built up fractions in the text of the manuscript. However, you may use them in the displayed equations. Underline Greek letters in red, German in green, and script in blue. The first paragraph must be capable of being used separately as a synopsis of the entire paper. In particular it should contain no bibliographic references. Please propose a heading for the odd numbered pages of less than 35 characters. Manuscripts, in triplicate, may be sent to any one of the editors. Please classify according to the 1991 *Mathematics Subject Classification* scheme which can be found in the December index volumes of *Mathematical Reviews*. Supply name and address of author to whom proofs should be sent. All other communications should be addressed to the managing editor, or Elaine Barth, University of California, Los Angeles, California 90024-1555-05.

There are page-charges associated with articles appearing in the Pacific Journal of Mathematics. These charges are expected to be paid by the author's University, Government Agency or Company. If the author or authors do not have access to such Institutional support these charges are waived. Single authors will receive 50 free reprints; joint authors will receive a total of 100 free reprints. Additional copies may be obtained at cost in multiples of 50.

The *Pacific Journal of Mathematics* (ISSN 0030-8730) is published monthly except for July and August. Regular subscription rate: $190.00 a year (10 issues). Special rate: $95.00 a year to individual members of supporting institutions.

Subscriptions, orders for numbers issued in the last three calendar years, and changes of address should be sent to Pacific Journal of Mathematics, P.O. Box 969, Carmel Valley, CA 93924, U.S.A. Old back numbers obtainable from Kraus Periodicals Co., Route 100, Millwood, NY 10546.

The Pacific Journal of Mathematics at P.O. Box 969, Carmel Valley, CA 93924 (ISSN 0030-8730) is published monthly except for July and August. Second-class postage paid at Carmel Valley, California 93924, and additional mailing offices. Postmaster: send address changes to Pacific Journal of Mathematics, P.O. Box 969, Carmel Valley, CA 93924.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

Pacific Journal of Mathematics

Vol. 153, No. 2 April, 1992